An Application of the Harmonic Oscillator Model to Verify Dunning's Theory of the Economic Growth

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Abstract

Analogies with mechanisms ruling the natural world have often been sought in the course of economic phenomena. This paper is also an attempt to combine the physical phenomenon of a harmonious oscillator with the theory of economic growth by J. H. Dunning (1981). In his theory, Dunning distinguished stages of economic growth of countries that imply the dependency between the investment position of countries and their GDP per capita, while the graph presenting this dependency reminds a trajectory of oscillating motion of a damped harmonic oscillator. This analogy has given inspiration to reinterpret the theory of economy on the grounds of the mechanism of a physical model. In this paper, the harmonious oscillator motion equation was adapted to the description of dependencies shown in the theory of economic growth by J. H. Dunning. The mathematical solution of this equation is properly parameterised and parameters are estimated with the use of the Gauss-Newton algorithm. The main objective of this paper is to allocate a specific stage in the economic growth to each country on the basis of the values of parameter estimations of the proposed cyclical models of changes in the net investment indicator.

Keywords	***JEL code***
Dunning's theory, convergence, Gauss-Newton algorithm	*C51*

INTRODUCTION

Dependencies between the country's economic growth and the level of inward and outward foreign direct investments are the subject of numerous research works. There is a range of economic theories attempting to describe such dependencies. One of these theories which is really worth attention is the theory of economic growth formulated by J. H. Dunning (1981). In his theory, Dunning distinguished stages of economic growth that imply the dependency between the investment position of countries (NOI *per capita*[2]) and their GDP *per capita*, while the graph showing this dependency reminds a tra-

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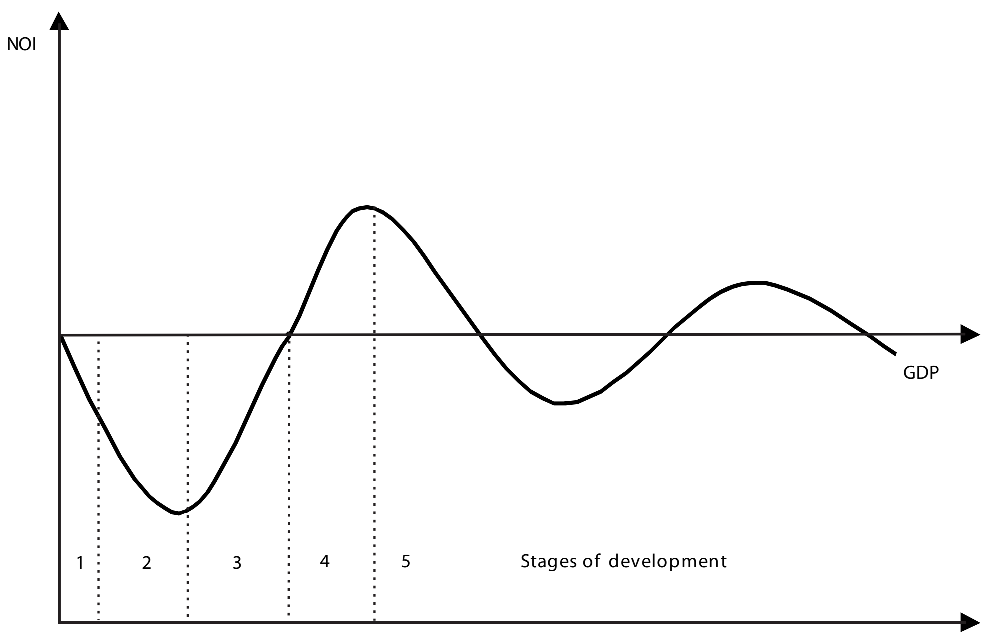
[2] Net outward investment (NOI) is the difference between foreign investments located outside the borders of a given country by companies based in this country and foreign investments executed by foreign companies within the territory of this country (Dunning, 1981).

jectory of oscillating motion of a damped harmonic oscillator. This analogy has inspired the author of this article to reinterpret the theory of economy on the grounds of the mechanism of the operation of a physical model. In this article, the harmonic oscillator motion equation was adapted to describe the dependencies presented in J. H. Dunning's economic growth theory. Through the proper parameterisation, the proposed econometric model describes the path of the country's investment development in the dynamic perspective, which allows for allocating the country to the relevant stage of economic growth distinguished in J. H. Dunning's theory. An advantage of this model lies also in its capacity to detect convergence or divergence phenomena in the country's investment development. The elementary objective of this article is to allocate the relevant stages in the economic growth to the examined countries on the basis of the estimated parameters of the proposed model of cyclical changes in the net investment index. The model was also estimated separately for each economy sector, thanks to which it is possible to compare the dynamics of economic growth from the sector perspective. The results of the ordering of economy sectors to individual stages of development was then subjected to the cluster analysis, which allowed for indicating a group of countries most similar in terms of the level of economic growth in the sector perspective. The research covered 10 selected countries from Central and Eastern Europe being EU members. Eurostat data for the period 1995–2012 was used in calculations.

1 REVIEW OF LITERATURE

In his theory, J. H. Dunning (1981) distinguished five stages of the country's economic growth, which are strictly connected with the level of GDP *per capita* as well as values and directions of changes in NOI *per capita*. These stages occur consecutively, creating a sort of an investment development path. The course of such a pattern path is illustrated in Figure 1.

Figure 1 Pattern of the path of the country's economic growth according to J. H. Dunning's theory



Source: Dunning, Narula (2002).

The stages of economic growth distinguished by J. H. Dunning may be characterised as follows (Stawicka, 2008):

– Countries with the weakest economy with GDP *per capita* below USD 400[3] in which the value of NOI *per capita* is close to zero and negative undergo the first stage of development. These countries are characterised by the deficit of capital, so they do not export foreign direct investments themselves but at the same time do not attract foreign capital, either;
– The second stage of growth involves countries with GDP *per capita* from the range of USD 400–1 500 and is characterised by the negative value of NOI *per capita*. Countries being at this stage of growth are to a certain extent more attractive to foreign investors but still have too low supply of capital to export foreign direct investments;
– The third stage of growth involves countries with GDP *per capita* from the range of USD 2 000–4 750 having a negative but increasing value of NOI *per capita*. These countries conduct their own foreign direct investments more and more often but the scale of these investments is lower than inward foreign direct investments in these countries;
– The fourth and fifth stage involves highly developed countries with GDP *per capita* above USD 2 600 in which NOI *per capita* is usually positive, which shows that these countries are active exporters of foreign direct investments.

The course of the curve illustrating the dependence of NOI *per capita* on the value of GDP *per capita* presented in Figure 1 reminds the oscillating motion of a damped harmonic oscillator. The application of the harmonic oscillator model in describing economic phenomena (especially the modelling of economic development) is known in the theory of economics (Chiang, 1994) but few researchers were able to use it in the empirical verification of J. H. Dunning's theory of economic growth (Kayam, Hisarciklilar, 2009). In the research conducted so far that verifies this theory there is predominance of polynomial regression models (Barry, Görg, McDowell, 2002, Buckley, Castro, 1998). However, solutions of this kind are not free of defects, including limitations in the possibility of interpreting the parameters of polynomial models and the resulting difficulties with ordering countries to a specific stage of economic growth. The proposed model is based on the mechanism of a harmonious oscillator, so it seems to be a solution competitive to traditional approaches to the verification of J. H. Dunning's theory of economic growth.

2 RESEARCH METHODS

In general, the oscillator motion equation takes the following form (Zill, 1986):

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \varpi_0^2 x = 0\,, \tag{1}$$

where:

x – displacement of the oscillator from the equilibrium position,
t – time,
β – elasticity index,
ω_0 – frequency of free vibrations.

Equation (1) is commonly used to describe the motion of a mechanical oscillator or an electromagnetic oscillator dampened by external factors (forces of resistance, friction, etc.).

[3] Boundary values of GDP per capita specified by J. H. Dunning are no longer valid (they were determined on the basis of data for the period 1967–1978). However, the very mechanism of identifying the stages of economic growth is constantly used in research works devoted to economic growth and in reports of institutions monitoring economic growth (e.g. UNCTAD).

Equation (1) is a homogeneous second-order differential equation the solution of which in case of weak dampening (i.e. when β< ω0) takes the form:

$$x = x_0 e^{-\beta t} \cos(\omega t + \varphi), \tag{2}$$

where:

$\omega = \sqrt{\omega_0^2 - \beta^2}$,

φ – phase shift,

x_0 – initial displacement of the oscillator.

If one assumes that values of NOI *per capita* change approximately according to equation (1), then with the use of function (2) one can formulate the following model describing cyclical changes of this index:

$$NOI_t = \alpha e^{-\beta PKB_t} \cos(\gamma PKB_t) + \varepsilon_t . \tag{3}$$

In model (3) there are three parameters requiring estimation: α, β, γ, that characterise the course of dependence between NOI and GDP in the following manner:

1. Parameter α determines the initial value of amplitude (with $\beta = 0$) and the starting direction of changes in NOI (increase towards the positive orientation of the vertical axis of the Cartesian coordinate system, when α>0 or decrease towards the negative orientation of this axis when α<0). The value of parameter α estimator may also suggest the capacity of the economy to absorb foreign direct investments or export own foreign direct investments. High and negative estimation value of parameter α estimator may suggest strong investment competition of the economy of a given country which is effective in attracting foreign capital. High and positive estimation value of this parameter, in turn, may indicate high economic potential (with considerable supply of capital) of the country actively conducting foreign direct investments.

2. Parameter determines the intensity of vibration dampening. Depending on the sign of parameter β, displacements from the equilibrium position can have threefold character:

– β >0 indicates decreasing displacements,
– β < 0 indicates increasing displacements,
– β = 0 indicates the constant amplitude of displacements.

The positive value of parameter β can indicate symptoms of convergence[4] which results in the economy of a given country becoming similar to the economies of highly developed countries in the range of the pattern of the investment development path. The negative value of parameter β may be interpreted as the occurrence of a phenomenon opposite to convergence, i.e. divergence, which results in greater distance in the investment development in relation to the economies of developed countries.

3. Parameter γ indicates the length of one cycle in oscillatory motion (distance between two successive upper or lower turning points), whereas the full cycle period is $2\pi\gamma^{-1}$. A longer cycle period is characteristic of better developed economies while the shorter one occurs in general in economies at a weaker level of development.

The analysis of a specific configuration of the values of estimations of the discussed parameters makes it possible to locate a considered country in one of the stages of economic growth defined by J. H. Dunning. The expected stages in the development of countries are presented in Figure 1, depending on the estimators of parameters α and β.

[4] Convergence involves the penetration of economy patterns of countries with a similar level of wealth and differentiation of patterns in countries with different wealth levels (Woźniak, 2005).

When analysing the content of table 1, it should be stressed that if parameter α does not differ considerably from zero, then the country may be in the first stage of development or at the beginning of the fourth stage (the amplitude of oscillation is inconsiderable then). In this situation, the familiarity with the sign of parameter α estimator can be helpful in classifying a country into one of the stages: a positive sign indicates the fourth stage while the negative one suggests the first stage.

It must be also stated that the classification of countries into the second or third stage may not be strict (in case $\alpha < 0$ and $\beta < 0$). In that case, when allocating to one of the stages, one can also use the estimator of parameter γ which – if known – enables the calculation of the length of the cycle ($2\pi\gamma^{-1}$). The results of empirical research confirm that a higher level of GDP is associated with a longer stage of the cycle (so a lower value of the estimator of parameter γ) (Kayam, Hisarciklilar, 2009). Low and statistically significant values of the estimators of parameter γ may suggest a higher level of the country's economic growth.

Table 1 Stages of economic growth depending on the signs of parameters α and β in model (3)

Estimator signs		Stage of economic growth
α	β	
0*)	any	1 or 4
–	–	2 or 3
–	+	3
+	–	4
+	+	5

*) Zero value is to be understood as a statistically insignificant result.
Source: Own elaboration

3 RESULTS AND DISCUSSION

3.1 Results of the estimation of the harmonic oscillator model for the countries of Central and Eastern Europe

The parameters of model (3) were estimated for ten countries of Central and Eastern Europe being member states of the European Union on the basis of the data for the period 1995–2012, with the use of the non-linear method of least squares – the Gauss–Newton algorithm. This method is an iterative procedure of conducting a range of successive implementations of the least squares method whereas each model iteration is substituted with a linear approximation obtained in the development of the Taylor's theorem with the accuracy to the first derivatives around the approximation of parameters (Goryl, 2004). The application of the Gauss–Newton algorithm requires the specification of starting values of parameters (in the literature on the subject, the application of the method of m points or m sums is proposed), which are then "corrected" in the course of successive iterations until the convergence of the algorithm is obtained (the degree of convergence may be controlled with the use of stopping criteria). In this research, the non-linear estimation of model (3) was conducted with the use of the Gauss–Newton algorithm programmed in the STATISTICA computer package (version 9.1).

The results of the estimation of parameters of model (3) are presented in Table 2 (in brackets under estimations of parameters test probabilities p are to be found). The chart also contains values of the model's coefficients of determination and numbers of stages in the economic growth ordered for individual countries on the basis of the values of parameter estimations.

When analysing the results from Table 2, it should be stated that some results are not statistically significant, therefore despite the generally good match of the estimated model (3) to empirical data, conclusions about the unambiguous allocation of individual countries to specific stages in the development must be formulated carefully in some cases.

Table 2 Results of estimation of the parameters of model (3) for the countries of Eastern and Central Europe

Country	Parameter			Coefficient of determination R^2 (%)	Stage of development
	α	β	γ		
Bulgaria	–10.898	–0.002	0.000	55.47	2–3
	(0.030)	(0.982)	(0.995)		
Czech Republic	–3.372	0.002	0.154	89.21	3
	(0.035)	(0.021)	(0.310)		
Estonia	–8.191	–0.004	0.000	60.47	2–3
	(0.005)	(0.083)	(0.954)		
Hungary	–2.067	–0.005	0.000	80.11	2–3
	(0.014)	(0.001)	(0.894)		
Latvia	–3.946	–0.000	0.240	48.72	1
	(0.977)	(0.998)	(0.000)		
Lithuania	–3.487	–0.004	0.000	83.47	2–3
	(0.008)	(0.025)	(0.947)		
Poland	–15.201	0.011	0.110	75.19	3
	(0.043)	(0.186)	(0.000)		
Romania*)	5.621	0.001	0.153	---	---
	(0.803)	(0.957)	(0.109)		
Slovakia	1 958.736	–0.005	0.000	49.55	4
	(0.667)	(0.000)	(0.934)		
Slovenia	–156.906	0.000	0.000	90.97	3
	(0.028)	(0.012)	(0.048)		

*) Stable estimations of parameters with the use of the Gauss-Newton algorithm were not obtained for Romania.
Source: Own elaboration

On the basis of the results showed in Table 2, it seems that the highest fourth stage of the economic growth was achieved by Slovakia. But this classification must be treated with caution as parameters α and γ were not statistically significant in model (3) estimated for Slovakia and the very model is adjusted rather moderately well to statistical data.

Czech Republic, Poland and Slovenia may be classified in the third stage of economic growth. But while in case of Czech Republic and Slovenia, parameter β turned out to be statistically significant (at the significance level of 0.05), then in case of Poland this parameter is not significant, so the classification of Poland to the third stage of growth must be done with certain caution.

The results of the estimation of parameters in model (3) most frequently indicated the second or third stage of growth in the examined countries. This situation occurred in case of Bulgaria, Estonia, Lithuania and Hungary. It should also be pointed out that model (3) estimated only for Lithuania and Hungary had both parameters α and β statistically significant and in relation to these countries the conclusion concerning the classification to the second or third stage seems particularly "strong". However, parameter γ estimated for these countries was not statistically significant, which makes it impossible to determine the length of the cycle stage in a reliable way and to allocate each of these countries to one of two stages (second or third) in an unambiguous way.

The lowest level of growth was allocated to Latvia (first stage of growth) but here it should be emphasised that key parameters decisive for categorising the country to a specific stage of growth are not statistically significant. Stable estimations of the parameters of model (3) were not obtained only in case of Romania, so it was not possible to allocate this country to the relevant stage of economic growth.

Parameter β achieved a positive and statistically significant value in model (3) estimated for the Czech Republic and Slovenia. This suggests that clear symptoms of decreasing proportions in the investment development may occur in these countries in relation to the majority of "old" EU countries. In the model estimated for Poland, parameter β is positive (and may suggest convergence) but statistically insignificant. In other countries, the negative value of the parameter β estimators suggests symptoms of divergence, i.e. the possibility of increasing distance in the investment development compared to economically developed countries. However, only in model (3) built for Lithuania, Slovakia and Hungary, parameter β turned out to be negative and statistically significant, which suggests that divergence may be distinct there.

3.2 Results of the estimation of the harmonious oscillator model for economy sectors in the countries of Central and Eastern Europe

J. H. Dunning's theory is usually verified with reference to the whole economy of the country. As economy is a complex structure in which different sections or sectors can develop at a different pace and usually bring different contributions to the growth of the country, it seems that the verification of J. H. Dunning's theory is justified, also in the sector perspective of economy. To this end, model (3) was estimated for Central and Eastern Europe countries separately for agriculture, processing industry, construction and services.[5] This allowed for tracking and comparing investment development paths of the examined countries in the elementary sectors of economy. In the calculations for individual sectors, the values of GDP *per capita* were substituted with the gross value added (GVA) generated in individual sectors of economy in the period 1995–2012. Table 3 presents the results of estimation for model (3) describing the development of agriculture in the countries of Central and Eastern Europe.

Table 3 Results of the estimation of the parameters of the investment development path in agriculture in the countries of Central and Eastern Europe

Country	Parameter			Coefficient of determination R^2 (%)	Stage of development
	α	β	γ		
Bulgaria	−15.201 (0.003)	−0.011 (0.686)	0.110 (0.000)	28.02	2–3
Czech Republic	−10.898 (0.040)	0.000 (0.982)	0.000 (0.995)	27.31	3
Hungary	−1 958.736 (0.037)	0.005 (0.000)	0.000 (0.934)	73.10	3
Estonia	−3.372 (0.365)	−0.002 (0.821)	0.154 (0.000)	30.16	1
Latvia	−3.946 (0.977)	0.000 (0.998)	0.240 (0.000)	42.63	1
Lithuania	−8.191 (0.005)	−0.004 (0.083)	0.000 (0.954)	20.54	2–3
Poland	3.487 (0.008)	−0.004 (0.025)	0.000 (0.947)	65.79	2–3
Romania	−2.067 (0.014)	−0.005 (0.001)	0.000 (0.894)	43.51	2–3
Slovakia	5.621 (0.803)	−0.001 (0.957)	0.153 (0.000)	43.36	4
Slovenia	−156.906 (0.028)	0.000 (0.012)	0.000 (0.048)	78.60	3

Source: Own elaboration

5 Usually 3 economy sectors are distinguished: agriculture (with forestry and fishery), industry (with construction industry) and services. For the purpose of this research, this division was particularised by separating processing industry from construction.

On the basis of the results contained in Table 3, it can be deduced that Slovakia achieved the highest fourth stage in the development of agriculture. This conclusion is weakened by the lack of significance of key parameters of model (3) built for the Slovak agriculture. The third stage of the investment development of agriculture is observed in the Czech Republic, in Slovenia and in Hungary.

Parameters of model (3) estimated for Bulgaria, Lithuania, Poland and Romania suggest that the agriculture of these countries may be classified in the second or third stage of economic development. However, it is difficult to specify clearly a concrete stage in the development of these countries as parameter γ determining the length of the cycle stage in case of Lithuania, Poland and Romania is not statistically significant.

On the basis of the estimation results of model (3), it can be stated that the youngest, first stage of economic growth is represented by agriculture in Latvia and Estonia. However, the last result of classification should be treated as "approximate" as parameter α is not statistically significant both in the model estimated for Latvia and Estonia. Positive and statistically significant estimators of parameter β were obtained only for Slovenia and Hungary, which means that the agriculture of these countries may undergo convergence.

Table 4 presents the results of the estimation of model (3) describing the development of processing industry in the countries of Central and Eastern Europe.

Table 4 Results of the estimation of the parameters of the model of the investment development path in processing industry in the countries of Central and Eastern Europe

Country	Parameter			Coefficient of determination R^2 (%)	Stage of development
	α	β	γ		
Bulgaria	−50.065 (0.029)	−0.001 (0.642)	0.510 (0.000)	58.11	2–3
Czech Republic	2.579 (0.038)	−0.005 (0.413)	0.039 (0.000)	47.14	4
Hungary	−2 411.874 (0.032)	0.000 (0.000)	0.000 (0.952)	89.75	3
Estonia	−67.547 (0.016)	−0.001 (0.000)	0.000 (0.406)	76.69	2–3
Latvia	−1.294 (0.044)	−0.005 (0.061)	0.043 (0.000)	37.98	2–3
Lithuania	−20.051 (0.524)	−0.002 (0.020)	0.011 (0.000)	28.47	1
Poland	0.491 (0.079)	−0.003 (0.503)	0.018 (0.000)	52.21	4
Romania	−880.422 (0.435)	0.000 (0.907)	0.005 (0.000)	45.52	1
Slovakia	−221.924 (0.000)	0.000 (0.045)	0.000 (0.952)	88.97	3
Slovenia	−187.044 (0.028)	0.000 (0.012)	0.000 (0.048)	75.14	3

Source: Own elaboration

Table 4 shows that the highest level of investment development (fourth stage) in processing industry was achieved by the Czech Republic and Poland, whereas only in case of the Czech Republic this conclusion is strong since the index α calculated in model (3) for this country was statistically significant. Slovakia, Slovenia and Hungary may be classified in the third stage of investment development in processing industry. Processing industry of Bulgaria, Estonia and Latvia is in the second or third stage of development but Latvia seems to be the country most advanced towards the third stage (with the duration of the cycle longer than in case of Bulgaria; for Estonia, it is impossible to calculate a reliable length of the development cycle stage as parameter γ is not statistically significant). The weakest level of processing industry development (stage one) is characteristic of Lithuania and Romania while this conclusion should be treated with caution as parameter α is not statistically significant both in the model estimated for Lithuania and Romania. In the light of the obtained results, the phenomenon of convergence in processing industry is distinct in Slovakia, Slovenia and Hungary (for these countries only, parameter β was positive and statistically significant).

Table 5 Results of the estimation of the parameters of the investment development path in construction in the countries of Central and Eastern Europe

Country	Parameter			Coefficient of determination R^2 (%)	Stage of development
	α	β	γ		
Bulgaria	−23.420 (0.005)	−0.001 (0.526)	0.000 (0.935)	66.21	2–3
Czech Republic	−19.027 (0.019)	−0.007 (0.000)	0.000 (0.931)	89.09	2–3
Hungary	304.985 (0.058)	0.146 (0.257)	−2.608 (0.000)	62.17	4
Estonia	−1.814 (0.696)	−0.003 (0.309)	0.032 (0.000)	40.74	1
Latvia	−8.907 (0.000)	−0.002 (0.002)	0.000 (0.843)	61.97	2–3
Lithuania	−7.726 (0.008)	−0.002 (0.001)	0.000 (0.746)	71.46	2–3
Poland	−13.149 (0.001)	−0.002 (0.201)	0.000 (0.900)	87.74	2–3
Romania*)	−7.624 (0.860)	−0.003 (0.752)	0.000 (0.934)	---	---
Slovakia	−19.167 (0.045)	−0.001 (0.402)	0.000 (0.970)	35.31	2–3
Slovenia	14.998 (0.810)	0.047 (0.815)	−0.420 (0.949)	33.04	4

*) Stable estimations of parameters with the use of the Gauss-Newton algorithm were not obtained for Romania.
Source: Own elaboration

Table 5 suggests that in the majority of the compared countries, i.e. Bulgaria, Czech Republic, Lithuania, Latvia, Poland and Slovakia, the construction industry is in the second or third stage of development. On the basis of the evaluation of parameter γ in these countries it was also hard to state towards which stage of development they are shifted as in none of the countries categorised in the second or third stage of development parameter γ was statistically insignificant.

Slovenia and Hungary exhibit the highest, fourth stage of development in construction but parameter α was not statistically significant for any of the countries, so the conclusion formulated here should be treated carefully. The first and least mature stage of investment development is observed in Estonia's construction industry but this is a weak conclusion as it is based on statistically insignificant parameter α. Only Romania was not classified to any stage of development as the Gauss-Newton algorithm in model (3) estimated for this country did not achieve convergence. None of the countries did observe significant convergence in construction while the phenomenon of distinct divergence took place in the Czech Republic, Lithuania and Latvia.

Table 6 presents the results of the estimation of model (3) describing the development of services in the countries of Central and Eastern Europe.

Table 6 Results of the estimation of the parameters of the investment development path model in services in the countries of Central and Eastern Europe

Country	Parameter			Coefficient of determination R^2 (%)	Stage of development
	α	β	γ		
Bulgaria	–55.325 (0.008)	–0.001 (0.000)	0.002 (0.000)	71.28	2–3
Czech Republic	1.802 (0.710)	–0.003 (0.027)	0.006 (0.000)	79.73	4
Hungary	–1 817.228 (0.031)	0.000 (0.703)	0.000 (0.866)	72.84	3
Estonia	–40.680 (0.645)	0.000 (0.829)	0.000 (0.154)	31.63	1
Latvia	–71.453 (0.481)	0.000 (0.114)	0.001 (0.000)	39.67	1
Lithuania	–37.516 (0.007)	–0.001 (0.000)	0.001 (0.000)	42.74	2–3
Poland	–263.832 (0.000)	0.000 (0.000)	0.000 (0.000)	72.58	3
Romania	–9 470.170 (0.111)	0.000 (0.275)	0.000 (0.208)	21.99	1
Slovakia	–757.479 (0.046)	0.000 (0.698)	0.000 (0.907)	57.01	3
Slovenia	–917.951 (0.043)	0.000 (0.871)	0.000 (0.024)	68.08	3

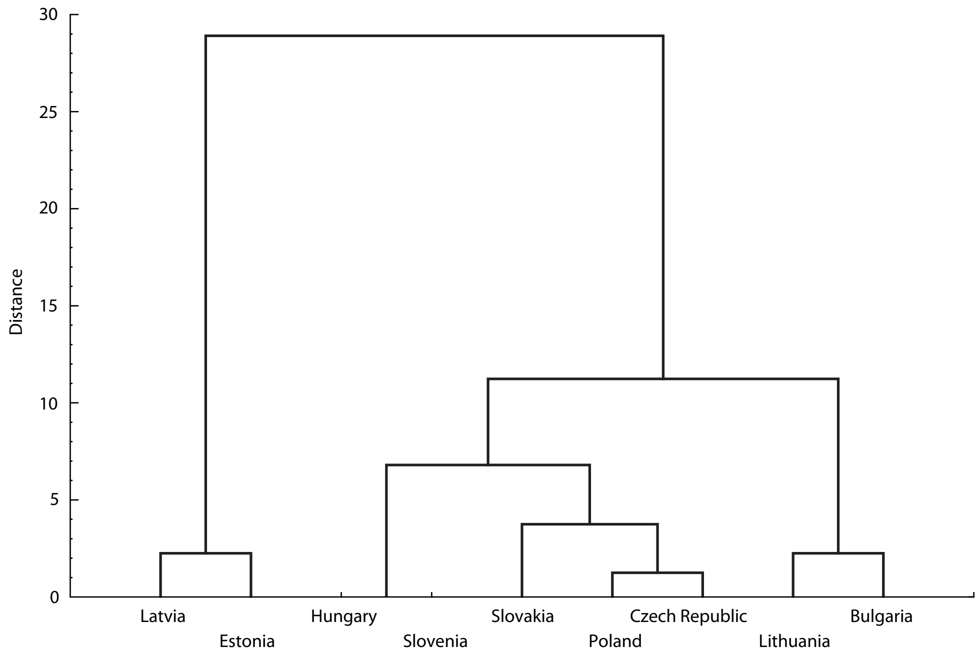
Source: Own elaboration

Table 6 suggests that among the compared countries the predominant ones are in the third stage of investment development of the service sector. The highest, fourth stage of the development of services was achieved in the Czech Republic but this classification should be treated with caution as parameter α estimated for this country was not statistically significant. In the second or third stage of the development of the service sector, there is Bulgaria and Lithuania, while the lowest value of parameter γ for Lithuania may suggest that it is shifted towards the third stage of development than in case of Bulgaria. Negative and statistically insignificant evaluations of parameter α for Estonia, Latvia and Romania indicate that these countries are in the first stage of investment development of the service sector. Among the compared countries, only Poland observed significant convergence in the service sector and the distinct divergence occurred in Bulgaria, the Czech Republic and Lithuania.

3.3 Comparison of the level of economic growth of the countries of Central and Eastern Europe

In order to determine which countries are most similar in terms of the level of economic growth, they were grouped with the use of the Ward's method with squared Euclidean distance. When choosing this method of grouping objects, their popularity and effectiveness confirmed with numerous empirical research works were taken as a criterion (Sokołowski, 1992). The results of the classification of countries in particular stages of economic growth were used while grouping (Tables 3–6). Characteristics used in the cluster analysis included levels of economic growth (1–4) attributed to individual countries in each of the following economy sectors: agriculture, processing industry, construction and services. In case of non-acute classification to a specific stage in the economic growth, an arithmetic mean of the neighbouring values representing numbers of development stages (e.g. in case of classification to stages 2–3, the mean equals 2.5) was applied as the value of the grouping characteristic. The analysis did not take Romania into account as the Gauss-Newton algorithm will not achieve convergence for the parameters of model (3) estimated for this country. The results of groupings are presented in Figure 2.

Figure 2 Dendrogram of the grouping of the countries of Central and Eastern Europe with the use of Ward's method due to the level of economic growth by economy sectors



Source: Own elaboration

The optimal number of clusters was determined with the use of the criterion of the first distinct increase in agglomerative distance (Sokołowski, 1992) by obtaining the following groups of countries similar in terms of the course of the investment development path in various economy sectors:

- group 1: Latvia, Estonia,
- group 2: Hungary, Slovenia, Slovakia, Poland, Czech Republic,
- group 3: Lithuania, Bulgaria.

The degree of diversification of the created groups and the "ability" of the grouping variable to diversify the created groups, i.e. the level of economic growth in individual sectors, were evaluated with the use of a single-factor analysis of variance and the results of this analysis are presented in Table 7. It contains aggregated levels of economic growth in the featured groups of countries and p-values of a single-factor analysis of variance ANOVA.

Table 7 Results of a single-factor analysis of variance ANOVA by economy sectors

Economy sector	Average values in groups			*p*-value
	1	2	3	
Agriculture	1.00	3.10	2.50	0.0041
Processing industry	2.50	3.40	1.75	0.0458
Construction	1.75	3.10	2.50	0.2038
Services	1.00	3.20	2.50	0.0011

Source: Own elaboration

Table 7 suggests that countries categorised in the second cluster achieved the highest average level of development in each of the four economy sectors. Countries from the first group have the weakest result in the range of development in agriculture, construction and services. Countries from the third group, in turn, are characterised by the lowest level of development of processing industry. The analysis of Table 7 also suggests that economic growth of agriculture, processing industry and services contributed to the significant diversification of the created clusters while the development of construction did not considerably diversify the created clusters.

To present the current situation in the area of the economic development in compared countries the average GDP *per capita* (or GVA *per capita* by the sectors of economy), NOI *per capita* were calculated for the data from the period 2008–2012. Table 8 shows the values both for the whole economy and by sectors: agriculture, processing industry, construction and services. Taking into account the value of GDP per capita, it is clear that the strongest economies in recent years were the Czech Republic and Slovenia. However, the Slovenia seems to have a higher investment position, because it has a more advantageous ratio of NOI to GDP. In the light of the presented results in Table 8 Slovakia and Bulgaria have the weakest economies. They are characterized by relatively low values of GDP and relatively low values of NOI. In the agriculture sector, the highest economic position have Hungary and Romania. The manufacturing industry has the highest development in the Czech Republic, and in Slovenia. The highest level of construction development was recorded in Slovenia and the Czech Republic.

Table 8 The average GDP, GVA and NOI per capita in years 2008–2012 (in euros) in the countries of Central and Eastern Europe

Country	Whole economy		Agriculture		Processing industry		Construction		Services	
	GDP	NOI	GVA	NOI	GVA	NOI	GVA	NOI	GVA	NOI
Bulgaria	3 931.1	−3 872.8	220.6	−18.5	923.6	−621.0	320.6	−285.6	2 466.2	−2 815.7
Czech Republic	13 871.7	−6 791.6	339.6	−12.7	4 374.5	−2 625.3	944.7	−44.9	8 187.2	−3 470.2
Hungary	9 474.5	−4 723.9	399.4	−33.5	2 444.7	−1 333.7	466.8	−55.0	6 163.5	−3 017.0
Estonia	9 745.6	−5 217.5	300.9	−61.2	2 027.5	−1 038.1	910.3	−70.9	6 099.9	−3 240.5
Latvia	7 763.2	−3 216.3	238.9	−67.8	1 057.6	−358.4	640.1	−59.7	5 826.5	−2 317.9
Lithuania	7 680.3	−2 288.8	308.3	−21.1	1 771.5	−467.4	746.6	−33.3	4 853.9	−1 476.3
Poland	9 141.0	−2 528.1	320.5	−14.2	2 177.7	−869.0	607.2	−51.7	6 035.6	−1 545.0
Romania	5 647.6	−2 167.1	347.1	−20.2	1 545.0	−692.5	604.2	−29.4	3 151.3	−1 189.5
Slovakia	3 018.0	−6 302.7	36.0	−11.6	1 060.8	−2 176.0	656.9	−168.3	1 247.5	−2 870.9
Slovenia	13 692.2	−2 499.4	126.9	10.3	3 763.6	−817.3	1 012.6	14.8	8 589.0	−1 689.0

Source: Own study based on Eurostat data

We note, Slovenia shows the highest investment competitiveness in terms of NOI position. Also, the Czech Republic and Slovenia in the services sector have received the highest GVA, which may indicate a relatively high level of development of this sector in both countries. Of course, the complexity of business processes is so high that the same measure of GDP (or GVA) *per capita*, NOI *per capita* are certainly not sufficient to fully assess the economic and investment position of individual countries. In the discussion on the economic position of countries it should be taken into account the sources of comparative advantage of economies, the level of technological advancement, the quality of human capital and other factors (Grossman, Elhanan, 1993, Stokey, 1991, Woźniak, 2008). However, J. H. Dunning (1981) built his theory of economic development on based two variables only: GDP *per capita* and NOI *per capita* so these variables were the basis for the verification of this theory of economic development in this article.

This description of the economic situation with GDP (or GVA), NOI *per capita* is obviously static, which significantly hinders the identification of the economic development stage. The proposed approach in this paper is free from these limitations.

CONCLUSION

The approach to the examination of relations between the level of NOI *per capita* and the level of GDP *per capita* proposed in this article in the context of the verification of J. H. Dunning's theory of economic growth seems to be an interesting alternative for the most frequently applied solutions in this field. So far, the most frequent method of examining the investment development path involves the polynomial models of regression describing the dependence between NOI and GDP. Even though they enable the approximation of the examined dependence, they also have several important limitations. First of all, it is difficult to give substantive interpretation to parameters in such models, which does not allow for specifying unambiguously the stages of the country's economic development and for stating whether an investment development path expires or develops.

The proposed model using the operating mechanism of a harmonious oscillator seems to cope well with these difficulties. An elementary advantage of this model is an easy interpretation of its parameters, which enables the creation of simple rules of allocating a country to a specific stage of economic growth. The proposed approach makes it also possible to model the path of investment development in the dynamic perspective and the analysis of its fluctuations allows for evaluating the degree of convergence or divergence as well as the length of the very investment cycle. Classical polynomial regression models did not give such possibilities in the range of the analysis of the investment development path. It should also be added that potential difficulties of numerical character connected with the estimation of the proposed harmonious oscillator model are relatively easy to overcome with the use of suitable computer packages with the programmed Gauss-Newton algorithm.

The sector perspective of the problem of economic growth proposed in this paper allowed to go beyond classical frameworks of J. H. Dunning's model of economic growth. The decomposition of economy into basic sectors allows for determining their contribution into economic growth (by measuring the added value generated by them). For this reason, sector perspective of the issue of economic growth seems to offer wider outlook on the economic development than it is possible from the perspective of the economy as a whole.

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