Measuring the Efficiency of Public Road Transport Companies in the Slovak Republic Using DEA and SFA

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Abstract

The paper measures and compares the efficiency of companies providing public road transport in the Slovak Republic. For this purpose two rather complementary methods, namely data envelopment analysis (DEA) and stochastic frontier analysis (SFA), are used. An input-oriented slack based model under variable returns to scale is applied as a DEA efficiency measure. The validity of DEA results is confirmed by the stability analysis consisting of re-calculation of DEA under different combinations of inputs and outputs. Identified efficient decision making unites are ranked using super-efficiency. A SFA model is based on the well-known Cobb-Douglas function type, assuming normally distributed errors and half-normally distributed inefficiencies. In order to overcome the multicolinearity problem principal component analysis is applied. Finally, we identify transport companies efficient with respect to both methods.

Keywords	***JEL code***
Data envelopment analysis, stochastic frontier analysis, efficiency measurement, public road transport	*L91, C67, C44*

INTRODUCTION

Transport is one of the key factors in the development of any modern society and in itself it is not a goal but a means of economic development and a prerequisite for achieving social and regional cohesion (Kitnerová, 2008, p. 18). The transport sector (H branch of the Statistical classification of economic activities SK NACE Rev. 2) is one of the largest spheres of economy and because of its importance and role in the national economy it is an equal partner of agriculture or information and communication sector (see Figure 1). In the Slovak Republic, for example, transport (land, air, water and pipeline transport) contributed 1.591 billion € to Slovak's Gross Domestic Product (GDP) in 2010, representing around 2.57 per

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cent of the Slovak economy. According to SLOVSTAT database (Statistical Office of the SR, 2012), the average number of employed persons was 140 420 in the third quarter of 2012, representing around 6.42 per cent of the total average number of employed persons.

The functioning of the transport market is influenced by national economic and social policy. In this sense transport companies are understood not only as part of the economy, but also as part of the infrastructure. Proportions of market principles and government interventions are one of the traits that characterize the transport market. These macroeconomic and microeconomic aspects are the main driver of many discussions aimed at achieving efficiency of the transport sector.

The scientific community works with a number of quantitative approaches to measure the efficiency of the transformation process. These can be classified into two groups. Group of parametric methods is characterized by the stochastic nature represented by including at least one random component, i.e. stochastic frontier analysis (SFA), thick frontier analysis (TFA), distribution free approach (DFA), etc. The second group of non-parametric methods is characterized by the deterministic nature and thus they do not effectively eliminate the negative influence of random errors, errors in measurement or imperfect data to measure efficiency, i.e. data envelopment analysis (DEA) and free disposal hull (FDH). Two methods DEA and SFA are linked with both strengths and weaknesses. The solution of DEA model does not generate any error estimation and creates no space for classical hypothesis testing of the statistical significance of the results. In addition, any deviation from the production possibility frontier (PPF) is considered as inefficiency, i.e. there is no possibility of random shocks as well as measurement errors. SFA, on the other hand, takes into account that the deviation from the PPF is not necessarily a manifestation of production unit inefficiency. It can be caused by some noise in the data, or unspecified error (Kočišová, 2008, p. 379) or as a result of accident (luck) or measurement errors (Vincová, 2005, p. 24). Unfortunately, it requires strict parametric functional form and distributional assumptions. At the present state of the art of the two approaches should primarily be viewed as complements rather than substitutes (Kooreman, 1994, p. 345).

1 LITERATURE REVIEW

The literature related to the efficiency measurement of transport sector has developed rapidly over the last few years. The main impetus was the need to eliminate of inefficiency in transport performance in terms of society-wide interest, as well as growing competition between transport companies. Given the relatively specific group of inputs used and outputs achieved in the transport sector, the need arose to apply the approaches allowing the inclusion of variables expressed not always in a financial nature.

In earlier research the non-parametric method DEA and parametric method SFA were used separately. In recent years, the issue of measuring transport efficiency by DEA is elaborated, i.e. in Barnum et al. (2007), Sampaio et al. (2008), Agarwal (2009), Klieštik (2009) and Ozbek et al. (2009). Barnum et al. (2007) applied DEA in measuring the efficiency of public transport in Chicago. The authors simultaneously examined the effects of external environmental factors on the efficiency of decision making units (DMUs). Sampaio et al. (2008) analyzed technical efficiency of 19 transport systems of Europe and Brazil by means of the radial output-oriented BCC model of Banker, Charnes and Cooper (1984) (hence the acronym BCC) and Agarwal (2009) examined the differences in technical efficiency and scale efficiency of 29 state transport undertakings in India. Klieštik (2009) applied input and output-oriented CCR model of Charnes, Cooper and Rhodes (1978) (hence the acronym CCR) to evaluate the efficiency of 15 transport companies in the Slovak Republic. Using Malmquist index Klieštik evaluated the efficiency change in two successive periods. Ozbek et al. (2009) primarily focused on the DEA methodology and utilized CCR model to compare the efficiency of state transportation departments in the maintenance of highways.

Farsi et al. (2006) and Holmgren (2012) are dealing with the quantification of the efficiency of transport using SFA. Farsi et al. (2006) quantified cost and scale efficiencies of Switzerland's regulated rural bus companies operating in regional networks using 4 alternative SFA models. The final dataset involved 985 observations including 94 operators over a 12-year period from 1986 to 1997. Holmgren (2012) evaluated the efficiency of public transport operations undertaken in 26 Swedish counties by the public transport authorities, in the period 1986–2009, taking into account substantial differences in operating conditions between countries.

Additionally, both DEA and SFA methods have been applied simultaneously in transport sector, e.g. Lan and Lin (2003), Michaelides et al. (2009), Margari et al. (2007). Lan and Lin (2003) adopted these methods to estimate productive efficiency of 74 railway systems in 1999. Lan and Lin (2003) used CCR and BCC DEA models and the SFA with translog production function for the half-normal and truncated-normal distributions. Michaelides et al. (2009) performed an independent comparison of DEA and SFA results in measuring technical efficiency in International Air Transport. Using a panel set of the world's 24 largest network airlines, for the period 1991–2000, Michaelides et al. (2009) concluded that SFA results are comparable to those from DEA. Margari et al. (2007) used a special three-stage DEA-SFA approach using panel of 42 Italian public transit companies for the period 1993–1999. Authors decomposed DEA inefficiency measures into three components: exogenous effects, pure managerial inefficiency and statistical noise.

2 METHODS

In measuring the efficiency of the 20 transportation companies[3] of the Slovak Republic via DEA in 2010 it is necessary to select appropriate set of inputs and outputs. One of the established aspects of selection variables is to fulfil an initial condition regarding the number of inputs and outputs in relation to the number of DMUs. In this context, Ozbek et al. (2009) postulate the following rule for the minimal number of DMUs (n):

$$n \geq 2ms, \tag{1}$$

where m is the number of inputs and s is the number of outputs.

Table 1 presents 8 possible combinations of 5 inputs and 2 outputs, which relatively precise characterize operations of the transport companies. It is clear that the total number of inputs and outputs fulfils the condition (1).

In order to choose an appropriate DEA model one has to specify the orientation of the model, form of identified technical efficiency and the assumption of returns to scale. Concerning the purpose of the analysis it is appropriate to consider the quantification of input-oriented Pareto-Koopmans technical efficiency under assumption of the variable returns to scale. Input orientation is due to the nature of variables considered, i.e. within the frame of increasing efficiency a potential reduction in the level of inputs relative to a given level of outputs is considered. The score of Pareto-Koopmans technical efficiency can be quantified by non-radial DEA models and assumption of the variable returns to scale takes into account the different scale of transport operations. All these arguments are satisfied by using input-oriented Slack Based Model under variable returns to scale assumption – hereafter SBM-I-V model (Tone, 2001).

[3] Totally 14 companies of the Slovak Bus Transport (SBT), i. e. SBT Banská Bystrica Inc., SBT Dunajská Streda Inc., SBT Humenné Inc., SBT Lučenec Inc., SBT Michalovce Inc., SBT Nové Zámky Inc., SBT Poprad Inc., SBT Prešov Inc., SBT Trenčín Inc., SBT Trnava Inc., SBT Žilina Inc., SBT Liorbus Inc., Veolia Transport Nitra Inc. and Slovak Lines Inc. and 6 City Transport Companies (CTC), i. e. CTC Bratislava Inc., CTC Banská Bystrica Inc., CTC Košice Inc., CTC Prešov Inc., CTC Žilina s.r.o. and CTC Považská Bystrica Inc.

Let $\mathbf{X}\in R_{+}^{m\times n}$ represents a matrix of m inputs of n DMUs and $\mathbf{Y}\in R_{+}^{s\times n}$ represents a matrix of s outputs of n DMUs. Any DMU o, $o\in\{1, ..., n\}$ transforms m inputs $\mathbf{x}_o\in R_{+}^{m}$ into s outputs $\mathbf{y}_o\in R_{+}^{s}$. Consider a vector of potential disproportional slacks of inputs – *excesses* $\mathbf{s}_o\in R_{+}^{m}$ that shift up DMU o to the production possibility frontier. Then a potential input inefficiency of DMU o can be expressed as average percentage slacks of inputs $\frac{1}{m}\sum_{i=1}^{i=m}\frac{s_{io}}{x_{io}}$. The optimization task of SBM-I-V model is then formulated as:

$$\min_{\mathbf{s}_o,\boldsymbol{\lambda}} \ \rho = 1-\frac{1}{m}\sum_{i=1}^{i=m}\frac{s_{io}}{x_{io}} \qquad \textit{subjekt to} \qquad \begin{aligned} &\mathbf{s}_o = \mathbf{x}_o - \mathbf{X}\boldsymbol{\lambda} \geq \mathbf{0},\\ &\mathbf{Y}\boldsymbol{\lambda} - \mathbf{y}_o \geq \mathbf{0},\\ &\boldsymbol{\lambda} \geq \mathbf{0},\\ &\mathbf{e}'\boldsymbol{\lambda} = 1, \end{aligned} \tag{2}$$

where $\boldsymbol{\lambda}\in R^{n}$ is a vector of weights, $\mathbf{e}\in R^{n}$ is a corresponding unit vector, $\mathbf{s}_o\in R_{+}^{m}$ is a vector of potential disproportional input slacks and ρ is a score of efficiency taking values from the interval (0, 1). As an optimal solution of the model (2) is ($\mathbf{s}_o^{*}$, $\boldsymbol{\lambda}^{*}$), the DMU o ($\mathbf{x}_o$, $\mathbf{y}_o$) is considered efficient if $\rho = 1$, i.e., $\mathbf{s}_o^{*} = \mathbf{0}$.

In order to solve the problem of many DMUs efficiency being 1, we can use a slack-based measure of super efficiency ρ^{*} to estimate DMUs efficiency (Tone, 2002). Super-efficiency model discriminates between these efficient DMUs. The corresponding super SBM-I-V model is the following:

$$\min_{\bar{x},\bar{y},\lambda} \ \rho^{*} = \frac{1}{m}\sum_{i=1}^{i=m}\frac{\bar{x}_i}{x_{io}} \qquad \textit{subjekt to} \qquad \begin{aligned} &\bar{\mathbf{x}} \geq \sum_{j=1,\neq o}^{n}\lambda_j\mathbf{x}_j,\\ &\bar{\mathbf{y}} \leq \sum_{j=1,\neq o}^{n}\lambda_j\mathbf{y}_j,\\ &\sum_{j=1,\neq o}^{n}\lambda_j = 1,\\ &\boldsymbol{\lambda} \geq \mathbf{0}, \bar{\mathbf{x}} \geq \mathbf{x}_o, \bar{\mathbf{y}} = \mathbf{y}_o. \end{aligned} \tag{3}$$

SFA is a parametric method of measuring the relative efficiency of production units based on the cost and production functions. We assume that these functions have a specific functional form with unknown parameters. In the presented study we restrict ourselves to the well-known Cobb-Douglas function type:

$$y = \beta_0 x_1^{\beta_1} x_2^{\beta_2} \cdots x_m^{\beta_m}, \tag{4}$$

where y is an output, $x_1, x_2,...,x_m$ are inputs and $\beta_o\in R_{+}$, $\beta_1, \beta_2,..., \beta_m\in R$ are unknown production technology parameters.

The underlying assumption of SFA is that the deviations from the production frontier are results of both inefficiency and noise. Assuming an additive specification we use the following base model (Bogetoft, 2011):

$$y^k = f\left(x^k;\beta\right) + v^k - u^k,\ k = 1,\ldots,n \tag{5}$$

where $f(x^k; \beta)$ is the logarithm of Cobb-Douglas function type (4), $v^k \sim N(0, \sigma^2_v)$ is the random error, $u^k \sim N_{+}(0, \sigma^2_u)$ is the possible inefficiency (N_{+} denotes a half-normal probability distribution) and v^k, u^k are independent. The model can be reparametrized using $\sigma^2 = \sigma_u^2 + \sigma_v^2$, $\lambda = \sqrt{\frac{\sigma_u^2}{\sigma_v^2}}$ (Kumbhakar, 2003). As $\lambda \to 0$ either $\sigma^2_v \to \infty$ or $\sigma^2_u \to \infty$, i.e. the random error dominates the inefficiency and we have the ordinary regression. As $\lambda \to \infty$ either $\sigma^2_v \to 0$ or $\sigma^2_u \to \infty$, i.e. the inefficiency dominates the random error. The parameters λ and σ are estimated along with the parameters β using the maximum likelihood method (Kumbhakar, 2003).

The DMU-specific technical efficiency TE is then given by (Bogetoft, 2011):

$$TE^k = 1 - \frac{\hat{u}}{f\left(x^k, \hat{\beta}\right)}, k = 1, \ldots, n. \tag{6}$$

SFA provides us with significance test for parameters of stochastic frontier function coefficients. They are analogous to the tests used in multiple linear regression. More details can be found in (Bogetoft, 2011).

3 RESULTS

The data set consists of a cross-sectional data extracted from Annual Reports 2010 provided by fourteen companies of the Slovak Bus Transport (SBT) and six city transport companies (CTC). Three SBT companies were omitted because they did not provide data. The available variables were split into inputs (the average number of employees (IN1), total kilometres driven (IN2), total number of vehicles (IN3), tangible fixed assets (IN4), operation costs (IN5)) and outputs (total number of passengers (OUT1) and total sales (OUT2)).

One of the main claims for the selection of variables is a relatively high between-group correlation, i.e. all outputs should be directly generated by inputs. For the purpose to quantify the intensity of dependence between the set of inputs and the set of outputs the canonical correlation analysis was applied. The stated criterion speaks in favor of the first combination of inputs and outputs, i.e. 1st possible combination in Table 1. The correlation matrix of all considered variables is displayed in Table 2. Listed correlation coefficients indicate the problem with multicollinearity in the case of inputs. It can negatively influence the results of SFA. Also the assumed outputs are highly correlated. Therefore we applied principal component analysis (PCA) and replaced the original inputs in SFA by the first two principal components representing 96.8% of variance and the original outputs by the first principal component representing 96.5% of variance.

Applying model (2) we obtained results listed in Table 3. It is easy to identify the high number of technically efficient companies (twelve). Moreover, three companies are relatively close to the production frontier and there is small difference in technical efficiency among inefficient companies with low technical efficiency. DEA results are generally quite sensitive to the selection of the DMUs and the selection of inputs and outputs. Ozbek et al. (2009) emphasize the need of a sensitivity analysis in the form of re-calculating the DEA model with omitted variables or some DMUs. In this case, the DEA sensitivity analysis was performed as re-calculating of the SBM-I-V model (2) for different combinations of inputs and outputs according to Table 1. To compare difference between the efficiency scores, Pearson's and Spearman's rank correlation coefficients were used. The relatively high values of correlation coefficients can be a sign that two approaches generate very similar values of efficiency scores. As Table 4 shows, it can be concluded that the DEA results were not highly influenced by the selection of inputs and outputs.[4] Then applying model (3) we ranked efficient DMUs (column ρ^* in Table 3).

Applying (5) and (6) we obtained results listed in Table 3 (column TE) and Table 6 (part a). According to them approximately 98% ($100 \times \frac{\lambda^2}{1+\lambda^2}$) of the total error variance is due to inefficiency. However, λ and the parameter corresponding to the second component of inputs are not statistically significant (p-value = 0.268, p-value = 0.1, respectively). It was probably caused by the small number of DMUs. If we omit the second principal component of inputs (Table 6, part b), λ remains insignificant (p-value = 0.209). Moreover, the resulting efficiencies are very similar (column TE* in Table 3). There is only one interesting difference, namely CTC Bratislava and CTC Košice exchanged their positions

[4] Only the 2nd combination of inputs and outputs provided the relatively different results. In almost all cases, relatively-low values of correlation coefficients were reported.

(TE = 0.915, TE* = 0.723 for CTC Bratislava, TE = 0.707, TE* = 0.966 for CTC Košice). Because DEA methods and SFA were applied to different data sets (principal components versus original variables), results are not directly comparable. Applying SBM-I-V model to principal components we got results listed in Table 3, column ρ' (for two inputs and one output) and column ρ" (for one input and one output). For two inputs and one output two DMUs are efficient (CTC Kosice and CTC Bratislava). Moreover, almost all other DMUs have efficiencies between 0.5 and 0.7. For one input and one output, one DMU is efficient (CTC Bratislava) and all other DMUs have efficiencies between 0.16 and 0.244. If we omit CTC Bratislava, we can get results similar to SFA model with one input and one output.

Using boxplots and multidimensional scaling we can identify CTC Bratislava and CTC Košice as outliers. Omitting these companies and applying model (2) we can get very similar set of efficient DMUs. The previously efficient DMUs remain efficient and SBT Trenčín transforms to an efficient DMU.

CONCLUSION

The presented paper is the initial stage of our research devoted to efficiency of public transport companies in Slovakia. The set of twelve efficient companies resulting from the slack based input-oriented DEA model with variable returns to scale was further ranked using super slack based input-oriented DEA model. Moreover, obtained results were compared to SFA model based on the well-known Cobb-Douglas type of production function. Due to highly correlated inputs and outputs we used simple SFA models with one output and two inputs based on principal components or one output and two inputs, respectively. The presented SFA models are not statistically significant but are in general coherent with results obtained applying DEA methods to the same data set. Models can be negatively affected by the insufficient number of decision making units and by the presence of outliers in our data set. According to our analysis the set of efficient DMUs includes all city transport companies. A thorough analysis of identified efficient companies (separately for CTCs), as well as a comparative application of alternative DEA and SFA models will be the object of our future research.

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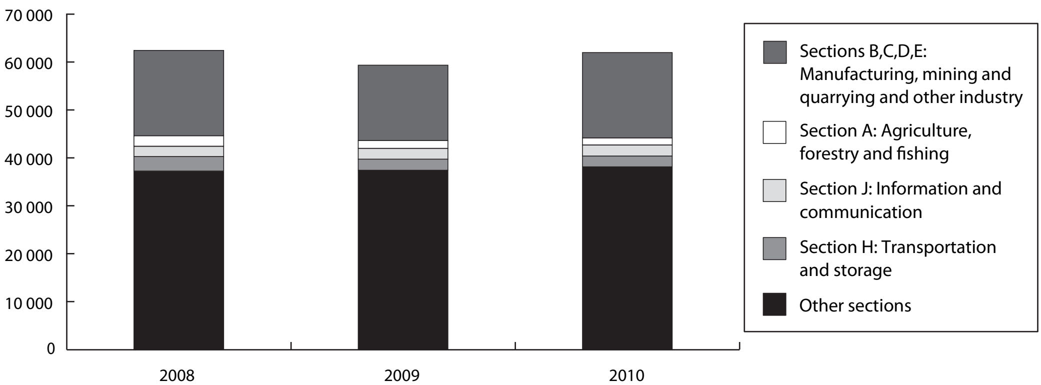
ANNEX – TABLES AND FIGURES

Figure 1 GDP (in mil. EUR at constant prices: chain-linked volumes with reference year 2005) by branches of statistical classification of economic activities SK NACE Rev.2



Source: Statistical Office of the Slovak Republic

Table 1 The possible combinations of inputs and outputs

Variables	Notation	Unit of measure	Possible combinations							
			1.	2.	3.	4.	5.	6.	7.	8.
Inputs considered										
Average number of employees	IN_1	Number	•	•	•	•	•	•	•	
Total kilometres driven	IN_2	Km	•	•	•	•	•	•		•
Total number of vehicles	IN_3	Number	•	•	•	•	•		•	•
Tangible fixed assets	IN_4	€	•	•	•	•		•	•	•
Operation costs	IN_5	€	•	•	•		•	•	•	•
Outputs considered										
Total number of passengers	OUT_1	Thousand	•	•		•	•	•	•	•
Total sales	OUT_2	€	•		•	•	•	•	•	•
First canonical correlation			**0.995**	0.987	0.992	0.990	0.994	0.997	0.992	0.995

Source: Own construction, annual reports 2010 of the transport companies

Table 2 The correlation matrix of the variables considered

	IN_1	IN_2	IN_3	IN_4	IN_5	OUT_1	OUT_2
IN_1	1	0,8984	0,7704	0,9434	0,9673	0,9691	0.9297
IN_2	0.8984	1	0.8268	0.9148	0.9537	0.8871	0.9815
IN_3	0.7704	0.8268	1	0.7162	0.7934	0.7976	0.8425
IN_4	0.9434	0.9148	0.7162	1	0.9521	0.9444	0.9295
IN_5	0.9673	0.9537	0.7934	0.9521	1	0.9670	0.9770
OUT_1	0.9691	0.8871	0.7976	0.9444	0.9670	1	0.9305
OUT_2	0.9297	0.9815	0.8425	0.9295	0.9770	0.9305	1

Source: Own construction, R (R Core Team, 2012)

Table 3 SFA efficiency scores and DEA efficiency scores with perceptual excesses

DMU	TE	TE*	ρ	ρ*	ρ′	ρ″	Excesses (in %)				
							IN_1	IN_2	IN_3	IN_4	IN_5
CTC Ban. Bystrica	0.769	0.718	1	1.064	0.566	0.166	0	0	0	0	0
CTC Bratislava	0.915	0.723	1	1	1	1	0	0	0	0	0
CTC Košice	0.707	0.966	1	1.176	1	0.244	0	0	0	0	0
CTC Pov. Bystrica	0.768	0.695	1	1.893	0.554	0.163	0	0	0	0	0
CTC Prešov	0.974	0.894	1	1.627	0.600	0.175	0	0	0	0	0
CTC Žilina	0.869	0.794	1	1.080	0.579	0.172	0	0	0	0	0
SBT Ban. Bystrica	0.736	0.673	1	1.153	0.561	0.165	0	0	0	0	0
SBT Dun. Streda	0.964	0.933	1	1.278	0.638	0.183	0	0	0	0	0
SBT Prešov	0.907	0.882	1	1.002	0.644	0.189	0	0	0	0	0
SBT Žilina	0.964	0.935	1	1.063	0.694	0.208	0	0	0	0	0
Slovak Lines	0.949	0.886	1	1.019	0.636	0.193	0	0	0	0	0
Veolia Tran. Nitra	0.986	0.962	1	1.303	0.701	0.205	0	0	0	0	0
SBT Humenné	0.811	0.814	0.999		0.645	0.186	0.01	0.02	0.01	0	0.01
SBT Trenčín	0.958	0.933	0.803		0.7188	0.219	15.34	21.04	19.23	42.81	0
SBT Nové Zámky	0.848	0.881	0.772		0.689	0.198	28.81	13.45	20.56	38.56	12.60
SBT Trnava	0.925	0.921	0.722		0.678	0.198	37.38	8.49	25.69	61.85	5.38
SBT Liorbus	0.934	0.899	0.710		0.642	0.189	54.51	10.05	16.40	55.10	9.19
SBT Michalovce	0.921	0.903	0.694		0.654	0.191	41.32	24.33	20.47	44.20	22.64
SBT Poprad	0.835	0.815	0.642		0.621	0.180	34.04	37.65	35.78	54.33	17.36
SBT Lučenec	0.883	0.863	0.618		0.647	0.191	44.18	43.08	26.00	62.21	15.49

Source: Own construction, DEA Solver Pro, R (package Benchmarking (Bogetoft, 2013))

Table 4 The stability results of DEA

Possible combinations		1.	2.	3.	4.	5.	6.	7.	8.
1.	Pearson correlation coefficient	x	0.552	0.811	0.953	0.683	0.937	0.901	0.995
	Spearman rank correlation coefficient		0.553	0.824	0.934	0.779	0.928	0.883	0.960
2.	Pearson correlation coefficient	0.552	x	0.242	0.630	0.475	0.640	0.550	0.550
	Spearman rank correlation coefficient	0.553		0.251	0.650	0.440	0.6581	0.428	0.463
3.	Pearson correlation coefficient	0.811	0.242	x	0.744	0.672	0.730	0.690	0.808
	Spearman rank correlation coefficient	0.824	0.251		0.728	0.761	0.744	0.736	0.823
4.	Pearson correlation coefficient	0.953	0.630	0.744	x	0.616	0.882	0.846	0.954
	Spearman rank correlation coefficient	0.934	0.650	0.728		0.694	0.852	0.830	0.906
5.	Pearson correlation coefficient	0.683	0.475	0.672	0.616	x	0.612	0.573	0.682
	Spearman rank correlation coefficient	0.779	0.440	0.761	0.694		0.694	0.577	0.657
6.	Pearson correlation coefficient	0.937	0.640	0.730	0.882	0.612	x	0.833	0.929
	Spearman rank correlation coefficient	0.928	0.660	0.744	0.852	0.694		0.822	0.902
7.	Pearson correlation coefficient	0.901	0.550	0.690	0.846	0.573	0.833	x	0.890
	Spearman rank correlation coefficient	0.883	0.428	0.736	0.830	0.577	0.822		0.924
8.	Pearson correlation coefficient	0.995	0.550	0.808	0.954	0.682	0.929	0.890	x
	Spearman rank correlation coefficient	0.960	0.463	0.823	0.906	0.657	0.902	0.924	

Source: Own construction, R (R Core Team, 2012)

Table 5 Loadings of principal components for inputs and outputs

	Comp.1	Comp.2	Comp.3	Comp.4	Comp.5
IN_1	–0.456	0.237	–0.612	0.344	0.493
IN_2	–0.457		0.767	0.223	0.39
IN_3	–0.405	–0.870	–0.191	–0.197	
IN_4	–0.451	0.394		–0.800	
IN_5	–0.465	0.176		0.391	–0.775
OUT_1	0.707	–0.707			
OUT_2	0.707	0.707			

Source: Own construction, R (R Core Team, 2012)

Table 6 SFA model based on principal components – a) 2 inputs, b) 1 input

2 inputs	Parameters	Std.err	t-value	Pr(>|t|)
(Intercept)	3.904	0.324	12.036	0.0
xComp.1	–0.739	0.081	–9.098	0.0
xComp.2	–0.701	0.182	–3.864	0.1
lambda	6.396	5.571	1.148	0.268
sigma2	0.028			
sigma2v	0.001			
sigma2u	0.027			
log likelihood	19.003			
1 input	**Parameters**	**Std.err**	**t-value**	**Pr(>|t|)**
(Intercept)	3.072	0.345	8.907	0.0
xComp.1	–0.776	0.141	–5.482	0.0
lambda	2.996	2.293	1.306	0.209
sigma2	0.048			
sigma2v	0.005			
sigma2u	0.043			
log likelihood	11.365			

Source: Own construction, R (package Benchmarking (Bogetoft, 2013))