Spatial Variations in the Educational Performance in Slovak Districts

Michaela Chocholatá [1] | *University of Economics in Bratislava, Bratislava, Slovakia*

Abstract

This paper deals with the spatial analysis of the educational performance measured by the average percentage Maths scores achieved in individual districts of Slovakia in Testing 9 during the school year 2018/2019. Besides identification of the spatial patterns in test scores achievements, the paper tries to investigate the impact of selected socio-economic variables (average nominal monthly wage and unemployment rate) onto the test scores achievements. Since we suppose the significant impact of the socially disadvantaged background onto the test results, corresponding dummy variable was taken into consideration as well. The ordinary least squares (OLS) estimation of the global linear regression model was followed by the local spatial approach using the geographically weighted regression (GWR) to capture the geographical variability of estimated parameters. Spatial variations in the relationship among the educational performance and the selected socio-economic variables were confirmed.

Keywords

Educational performance, geographically weighted regression, spatial variations, Slovak districts

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INTRODUCTION

The issue of educational performance is very attractive from the political, economic as well as from the social point of view. Improvement of the educational quality, identification of the strengths and weaknesses of the educational process as well as revealing of the disparities in educational performance are crucial factors incorporated in majority of the national development strategies. Disparities in educational performance can be connected with many factors including the pupil's home and family background, various teacher characteristics and school characteristics (see e.g. Qiu and Wu, 2011). As pointed out e.g., by Fotheringham, Charlton and Brunsdon (2001), Naidoo, van Eeden and Munch (2014), and Vidyattama, Li and Miranti (2019), by analysing the educational performance inequalities it is important to investigate the socio-economic characteristics of the analysed region.

There have been published various studies dealing with the educational performance testing the impact of different socio-economic variables (family income, unemployment rate, families with a single parent,

[1] Department of Operations Research and Econometrics, Faculty of Economic Informatics, University of Economics in Bratislava, Dolnozemská cesta 1/b, 852 35 Bratislava, Slovakia. E-mail: michaela.chocholata@euba.sk, phone: (+421)267295832.

parents' education etc.).[2] However, the degree to which socio-economic factors matter differs across individual countries (Haahr et al., 2005). Some researchers (Fotheringham, Charlton and Brunsdon, 2001; Naidoo, van Eeden and Munch, 2014; Gutiérrez, Sánchez, and Giorguli, 2011; Qiu and Wu, 2011; Chocholatá, 2019) accent that the spatial variations in the educational performance can be masked by estimating only a classic global model, i.e. that some socio-economic variables can have a significant effect in some regions while in other regions their effect may be insignificant. The geographically weighted regression (GWR) approach developed by Brunsdon, Charlton and Fotheringham (see e.g., Fotheringham, Brunsdon and Charlton, 2002) enables to reveal the spatial variations in modelled relationships.

Educational performance both at the international and national level is usually measured by the test scores achieved by pupils or students. Slovakia is involved in various international test measurements, e.g. PISA (OECD's Programme for International Student Assessment), TIMSS (Trends in International Mathematics and Science Study), PIRLS (Progress in International Reading Literacy Study), TALIS (Teaching and Learning International Survey), ICILS (The International Computer and Information Literacy Study).[3] Monitoring and assessment of the quality of educational process at the national level, is measured by the external testing of the primary school pupils (Testing 5, Testing 9) and the secondary school students (external part of secondary school graduation exam). The test achievement scores also enable to have the detailed view on the pupils' and students' knowledge as well as to identify the strengths and weaknesses of the educational process.

This paper examines the average percentage Maths scores achieved in individual districts of Slovakia in Testing 9 (T9)[4] during the school year 2018/2019 in order to assess some spatial patterns in test scores achievements as well as to identify the impact of selected socio-economic variables (average nominal monthly wage and unemployment rate) and that of a socially disadvantaged background onto the test scores achievements. Besides the classic global linear regression approach, the instruments of spatial analysis and GWR are used as well.

The paper is organized as follows: the introduction is followed by the section devoted to GWR methodology issues, the second section comprises the data, the third section deals with the empirical results of analysis and the last section concludes the paper.

1 GEOGRAPHICALLY WEIGHTED REGRESSION – METHODOLOGICAL ISSUES

To assess the impact of selected socio-economic variables onto the educational performance, the first step is usually to estimate the parameters of the global linear regression model using the ordinary least squares (OLS) approach. Taking into account the spatial character of modelled data, the application of the OLS on such data is usually connected with the violation of statistical assumption of independent residuals and quite often also with the violation of the assumption of residual constant variance (Qiu and Wu, 2011). Concerning the spatial data analysis, two types of the spatial effects can be distinguished, the spatial autocorrelation and the spatial heterogeneity. While the significant tendency towards clustering of similar (dissimilar) values in space is known as positive (negative) spatial autocorrelation, the presence of spatial heterogeneity indicates that parameters can vary across regions depending on their location (Chocholatá, 2018a, 2018b; Furková, 2018; Qiu and Wu, 2011). As pointed out e.g., by Abreu, De Groot and Florax

[2] For a survey, see e.g. Chocholatá and Furková (2017).
[3] For more information see: <*https://www.nucem.sk/sk/merania*>.
[4] Testing 9, i.e. external testing (in national language and Mathematics) of pupils of the 9th year of primary schools as well as those of the 4th year of grammar schools/sport schools with an eight-year educational program, is performed by the National Institute for Certified Educational Measurements (Národný ústav certifikovaných meraní vzdelávania „NÚCEM") in order to monitor pupils' level of knowledge and skills and to obtain relevant information about their performance at the end of lower secondary education.

(2005), and Anselin (2001), using the cross-sectional models, it is quite problematic to distinguish between these two spatial effects since these often come together. Fotheringham (2009) asserts that in some cases the spatial autocorrelation among residuals can be caused by the spatial heterogeneity and in such cases it is a good solution to use the local approach.[5] The local approach, i.e., the GWR approach, can be used to alleviate problems from both spatial effects in a global linear regression model (Qui and Wu, 2011).

The GWR approach enables to estimate local parameter values for each region in the data set and thus let us see the spatial heterogeneity of the analysed relationships. The corresponding model can be written as follows (Wheeler and Páez, 2010):

$$y_i = \beta_{i0} + \sum_{k=1}^{p-1} \beta_{ik} x_{ik} + \varepsilon_i \,, \tag{1}$$

where index $i = 1, \ldots, n,$ denotes the i-th region, y_{i} is the value of dependent variable at region i, x_{ik} denotes the values of the k-th independent variable at region i, β_{i0} is the intercept, β_{ik} is the regression parameter for the k-th independent variable, p is the number of regression terms, and ε_i denotes the error term at region i.

The GWR model expressed in matrix notation is as follows (Wheeler and Páez, 2010; Furková, 2018):

$$y_i = \boldsymbol{x}_i \boldsymbol{\beta}_i + \boldsymbol{\varepsilon}_i \,, \tag{2}$$

where $\mathbf{x_i}$ is a row vector of independent variables and $\boldsymbol{\beta_i}$ is a column vector of regression parameters at region i. The local regression parameters are functions of region i and can be estimated by the weighted least squares:

$$\widehat{\boldsymbol{\beta}}_i = \left(\boldsymbol{X}^T \boldsymbol{W}_i \boldsymbol{X} \right)^{-1} \boldsymbol{X}^T \boldsymbol{W}_i \boldsymbol{y} \,, \tag{3}$$

where $\boldsymbol{\beta_i}$ is the vector of p local regression parameters at region i, $\boldsymbol{y}$ denotes the $n \times 1$ vector of dependent variables, $\boldsymbol{X}$ is the $n \times k$ matrix of independent variables (including a column of ones for the intercept) and $\boldsymbol{W_i} = diag\,(W_{i1}, W_{i2}, \ldots, W_{in})$ is the $n \times n$ diagonal weight matrix at region i (Wheeler and Páez, 2010).

The weights are linked to the proximity of the region i to all the other regions. Regions closer to the region i have a higher weight in local regression in comparison to regions more distant in space (Fotheringham, Brunsdon and Charlton, 2002). The calculation of weights is based on the spatial kernel function (in general we can distinguish adaptive and fixed spatial kernel functions) and specification of its bandwidth. The optimal value of bandwidth can be calculated e.g., by minimising a cross validation score – CV or by the corrected Akaike Information Criterion – AICc (see e.g., Fotheringham, Brunsdon and Charlton, 2002; Nakaya, 2016). The difference between the adaptive and fixed spatial kernel function is as follows: since the adaptive spatial kernel function is based on the use of the same number of regions in each local kernel, the fixed spatial kernel function uses the same spatial range in each local kernel (for more information see e.g., Wheeler and Páez, 2010). GWR results in estimation of n vectors of local parameters, i.e. one for each region. Analysts often map the estimated local parameters to uncover something which is hidden in the global linear regression model and try to assess the spatial pattern of the estimated parameters (Fotheringham, Charlton and Brunsdon, 2001; Wheeler and Páez, 2010). To test whether the local GWR model describes the data significantly better than a global linear regression model, the GWR ANOVA test can be used. For the further testing procedures dealing e.g. with the spatial variation of the estimated local regression parameters see Leung, Mei and Zhang (2000), and Nakaya (2016).

[5] See e.g., Mur, López and Angulo (2008), Qiu and Wu (2011) for issues regarding the links between the spatial dependence and spatial heterogeneity.

2 DATA

The empirical part of the paper is based on the data set which comprises the regional data for the 79 districts of Slovakia. The dependent variable is the T9 average percentage Maths scores for the 79 Slovak districts retrieved from the website of NÚCEM[6] for the school year 2018/2019. The shape file of the Slovak districts was downloaded from the website Freemap Slovakia.[7] To assess the impact of socio-economic variables on the districts' school performance (measured by T9 average percentage Maths scores), the independent variables – average nominal monthly wage (in Euro) and unemployment rate (in %) in a district (for the year 2018) were downloaded from the DATAcube database of the Statistical Office of the Slovak Republic.[8] One more independent variable, a dummy (0/1) variable, indicating districts with more than 5% pupils with the socially disadvantaged background was retrieved from the above mentioned NÚCEM website. The whole analysis was performed in the free downloadable softwares GeoDa and GWR4.

Testing 9 from Mathematics in the school year 2018/2019 was performed at April 3, 2019 by 40 452 pupils with the Slovak average percentage Maths scores' achievement of 63.1%. Box plots and descriptive statistics for the T9 average percentage Maths scores (denoted as *mat*),[9] average nominal monthly wage (*w18*) and unemployment rate (*un18*) are depicted in Figure 1.

Figure 1 Box plots of the average percentage T9 test scores in Maths (*MAT*), average nominal monthly wage in Euro (*w18*) and unemployment rate in % (*un18*)

Note: Figure available in the online version of Statistika: Statistics and Economy Journal No. 2/2020.
Source: Author's calculations in GeoDa

Besides the mean values of analysed indicators (calculated as the average of the district values) there is possible to identify various upper outliers and one lower outlier. As for the dependent variable, the average percentage Maths scores, the upper outlier was the district of Bratislava I and lower outliers were the districts of Revúca and Gelnica. Extremely high average nominal monthly wages – upper outliers – were detected in the districts of Bratislava I, Bratislava II, Bratislava III and Bratislava IV. Upper outliers with regard to the high unemployment rates were identified for the districts of Rimavská Sobota, Kežmarok, Rožňava and Revúca. Minimum and maximum values further confirm the that there are huge differences across analysed districts concerning the average percentage Maths scores with minimum of 44.7% (Gelnica) and maximum of 78.8% (Bratislava I). The average nominal monthly wages of 726 Euro in Bardejov district and of 1 696 Euro in Bratislava II district, illustrate the enormous difference between the minimum and the maximum values. Regarding the unemployment rates, the difference of almost 14.5 percentage points between the lowest 1.68% unemployment rate (Hlohovec) and 16.15% (Rimavská Sobota) clearly indicates the existence of the substantial regional differences, as well.

Figure 2 illustrates the box maps[10] for the analysed variables (T9 average percentage Maths scores – *mat*, average nominal monthly wage – *w18* and unemployment rate – *un18*) in order to visualise the unequally distribution of analysed variables over space and to detect possible clusters of similar or dissimilar values. Figure 2 incorporates the unique values map indicating localization of the 11 districts with more than 5% pupils with the socially disadvantaged background (*szp*) as well.

[6] *<https://www.nucem.sk/dl/4422/S_T9_2019_Priloha_4.1.pdf>.*
[7] *<http://wiki.freemap.sk/HraniceAdministrativnychUzemi>.*
[8] *<http://datacube.statistics.sk/>.*
[9] Descriptive statistics are calculated based on district values of the Maths test scores.
[10] Box map consists of six categories and it is a special form of a quartile map. However it is worth mentioning, that the first and the last quartile no longer correspond to exactly one fourth of the observations, since the lower and upper outliers, respectively, are depicted as extra categories (Anselin, Kim and Syabri, 2010).

Figure 2 Box maps of the average percentage T9 test scores in Maths (*MAT*), average nominal monthly wage in Euro (*w18*), unemployment rate in % (*un18*) and unique values map indicating 11 districts with more than 5% pupils with the socially disadvantaged background (*szp*)

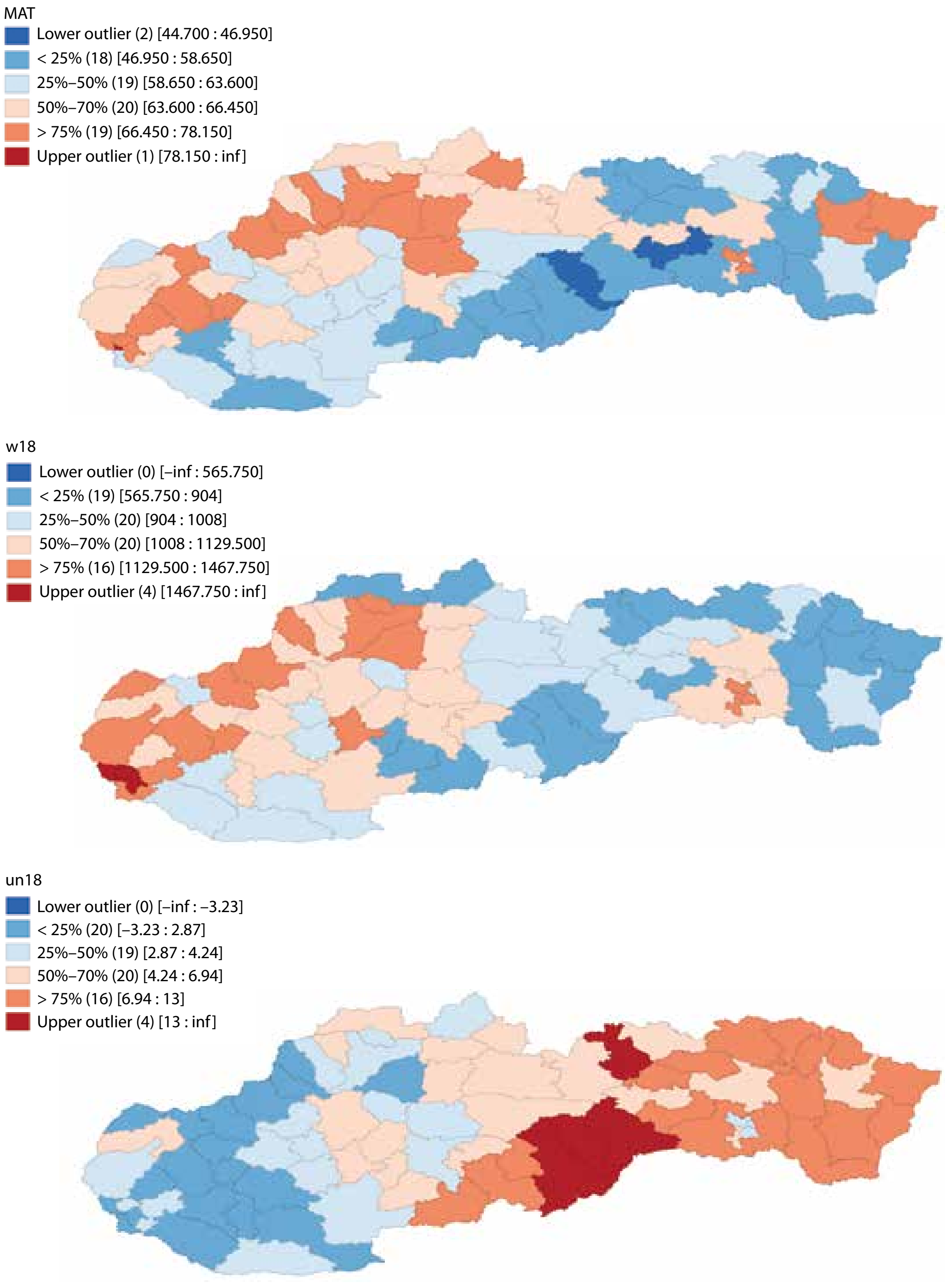


Figure 2 (continuation)

Source: Author's calculations in GeoDa

Regarding the dependent variable, average percentage Maths scores, the best results were detected in region Bratislava I followed by other 19 districts located in western, middle and eastern part of Slovakia. Districts with the worst results were Gelnica and Revúca. Low test scores achievements were detected also in districts located mostly in southern and eastern part of Slovakia.[11] Significant polarisation between western and eastern districts in also visible in case of both independent variables – average nominal monthly wage and unemployment rate. The unique values map indicates 11 districts with more than 5% pupils with the socially disadvantaged background located in the southern part of middle Slovakia and eastern part of Slovakia. Based on the above-mentioned results of the spatial analysis indicating the presence of the huge spatial heterogeneity, it could be hardly supposed that the same relationship can hold across all the regions in the data set under investigation (Chocholatá, 2018b).

3 EMPIRICAL RESULTS

As the first step, the global linear regression model was estimated using the classic OLS technique:

$$mat_i = \beta_0 + \beta_1 w18_i + \beta_2 un18_i + \beta_3 szp_i + \varepsilon_i \,, \tag{4}$$

where the dependent variable (*mat* – average percentage T9 test scores in Maths) is a function of independent variables (*w18* – average nominal monthly wage in 2018, *un18* – unemployment rate in 2018 and *szp* – dummy variable indicating districts with more than 5% pupils with the socially disadvantaged background), β_0, β_1, β_2 and β_3 are unknown parameters and ε_i represents an error term. Estimates of individual global parameters (i.e. without regional differentiation) are in Table 1 (column: Linear model). The estimated parameters corresponding to the average nominal monthly wage and unemployment rate were statistically significant at the 1 percent level of significance indicating the positive impact of average nominal monthly wages and negative impact of the unemployment rate on the analysed average percentage Maths scores. The negative impact of the socially disadvantaged background was confirmed at the 10 percent level of significance.

[11] With regard to the test results of previous five years, i.e. 2014–2018, districts of Gelnica, Rožňava and Rimavská Sobota belong to the five districts with the worst Maths scores, on the other hand, only the district of Bratislava I belongs to the top five districts with the best Math scores achievements during this five years' period.

Table 1 Estimation results of OLS regression and of GWR

Model	Linear model OLS	GWR				
		Minimum	Lower Quartile	Median	Upper Quartile	Maximum
β_0	54.6945***	45.8446	50.5667	52.5663	63.3869	68.9600
β_1 (*w18*)	0.0117***	0.0018	0.0061	0.0142	0.0150	0.0201
β_2 (*un18*)	−0.8056***	−1.2958	−1.1706	−1.0132	−0.9153	−0.8537
β_3 (*szp*)	−3.1972*	−2.5232				
AICc	461.313	453.578				
Adjusted R^2	0.5794	0.6332				

Notes: Symbols ***, * indicate the rejection of H_0 hypotheses at 1% and 10% level of significance, respectively.
Source: Author's calculations in GeoDa and GWR4

Since we deal with the spatial data, the regression residuals were further tested for the presence of the spatial autocorrelation by calculation of the spatial diagnostic test statistics – the Moran's *I* (the formula for calculation see e.g., Getis, 2010). Corresponding Moran's scatterplot with the estimated Moran's *I*[12] of 0.1298 indicating the presence of the statistically significant positive spatial autocorrelation is in Figure 3.[13]

Figure 3 Moran's scatterplot of the OLS residuals

Note: Figure available in the online version of Statistika: Statistics and Economy Journal No. 2/2020.
Source: Author's calculations in GeoDa

To capture the spatial heterogeneity across analysed regions, the global analysis based on model (4) was followed by the local spatial analysis based on model (5). Model (5) is an extension of the GWR model (1) with the mixture of globally fixed and locally varying parameters. It was supposed that while the variables of average nominal monthly wage and unemployment rate have the locally varying impact (with the corresponding geographically varying, i.e. local parameters β_{i1} *and* β_{i2}), the variable of *szp* is expected to be the global variable[14] (with the fixed, i.e. global parameter β_3):

$$mat_i = \beta_{i0} + \beta_{i1} w18_i + \beta_{i2} un18_i + \beta_3 szp_i + \varepsilon_i \,. \tag{5}$$

Parameters of model (5) were estimated based on GWR technique using the adaptive bi-square kernel with 61 nearest neighbours (Nakaya, 2016). The GWR estimation results (minimum, lower quartile, median, upper quartile, maximum) are gathered in Table 1 (columns: GWR). The estimated parameters confirmed the positive impact of average nominal monthly wages and negative impact of both the unemployment rate and socially disadvantaged background on the analysed average percentage Maths scores. Figure 4

[12] The Moran's *I* values were calculated based on the queen contiguity matrix of the first order.
[13] Randomization with 999 permutations was used to prove the statistical significance of results.
[14] Testing geographical variability of local parameters proved the global character of the szp variable. Results are available from the author upon request.

illustrates the spatial variation of local coefficients of determination R_i^2 and those of estimated local parameters from the GWR fit, respectively. The values of local R_i^2 spanning from moderate 0.549 (Bytča) to good 0.735 (Lučenec) indicate an acceptable goodness-of-fit and clearly confirm the different model performance across individual regions. The impact of the average nominal monthly wage on the Maths test scores was positive across all the Slovak districts however having the different intensity in eastern regions and regions located in the south of western and middle part of Slovakia. Considerable spatial variation (especially between districts located in western and eastern part of Slovakia) is visible for the parameter values of the second local variable, the unemployment rate, confirming its overall negative effect.

Figure 4 Local coefficients of determination R_i^2 (*localR2*) and local parameters of average nominal monthly wage (est_*w18*) and unemployment rate (est_*un18*) from the GWR fit

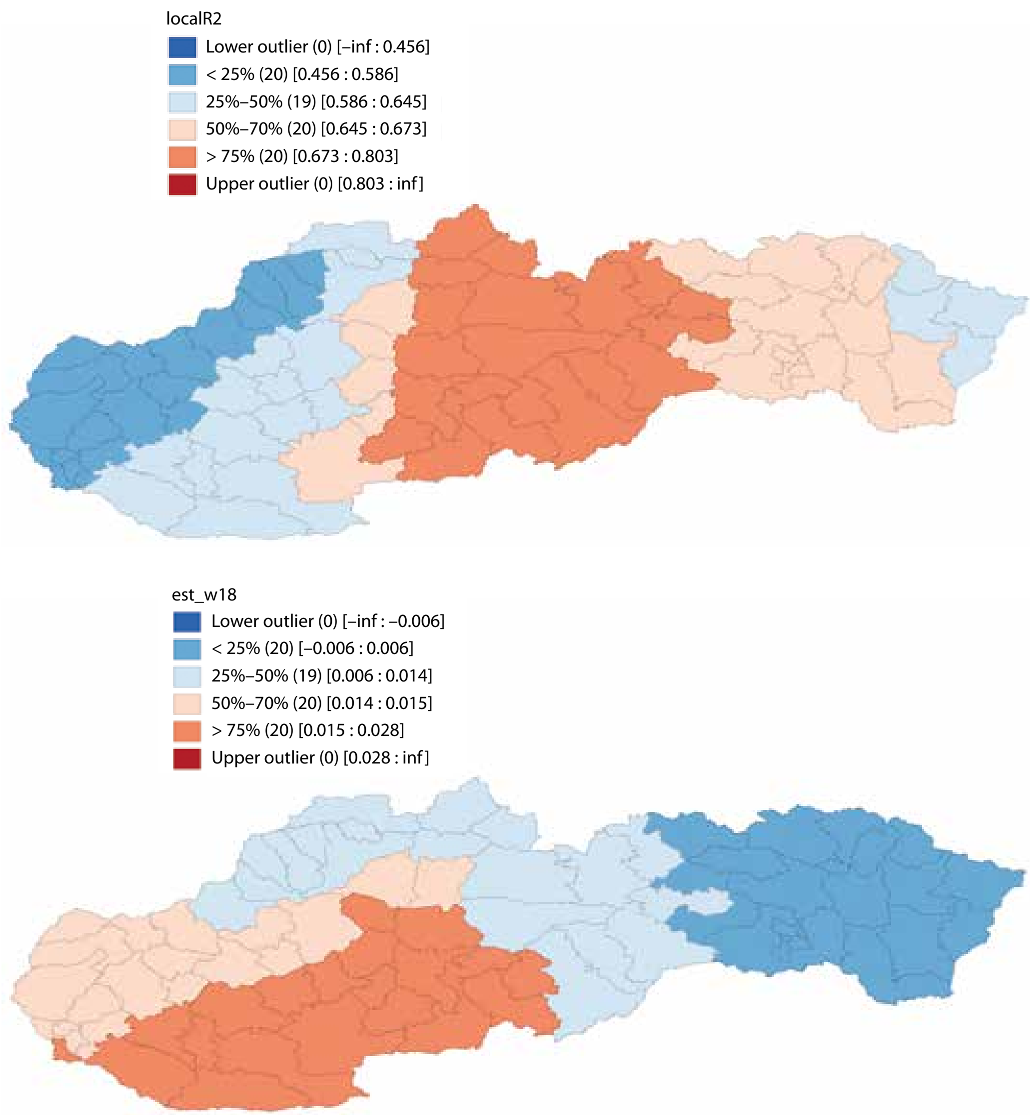
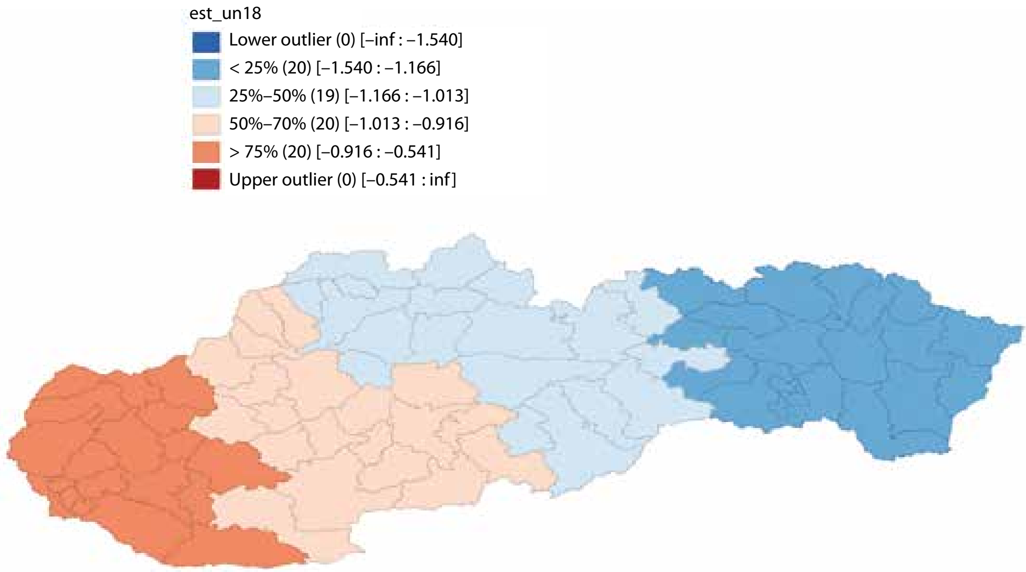


Figure 4 (continuation)



Source: Author's calculations in GWR4 and GeoDa

Comparing the estimation results of the global model (4) and those of the local model (5) based on values of *adjusted* R^2 0.5794 and 0.6332, respectively (Table 1), indicates some improvement in the model performance. Taking into account the AICc values of 461.313 and 453.578 for the OLS and GWR fit, respectively (Table 1), suggests some considerable improvement in the GWR model fit as well (for more information see e.g. Burnham and Anderson, 2002). Standardised residuals from the GWR fit, as documented in Figure 5, did not show a particular spatial pattern (Gutiérrez, Sánchez, and Giorguli, 2011). Moran's I values of –0.005 clearly confirm no evidence of the statistically significant spatial autocorrelation. The statistically significant improvement in the GWR model performance over the global OLS model was confirmed by the GWR ANOVA test with the test statistic F = 3.305.

Figure 5 Moran's scatterplot of the standardised residuals from the GWR fit

Note: Figure available in the online version of Statistika: Statistics and Economy Journal No. 2/2020.
Source: Author's calculations in GeoDa

4 DISCUSSION

The empirical results of this paper are in accordance with those of following studies confirming the significant spatial variation of the regression parameters by analysing of the educational performance. Fotheringham, Charlton and Brunsdon (2001) examined the relationship between the school performance in Britain (measured by Maths scores in 1997) and the socio-economic indicators of school catchment areas revealing some spatial variations in the results, i.e. that "some attributes of school catchment areas have an effect on school performance in some areas and not in others and such variations are masked in global results" (Fotheringham, Charlton and Brunsdon, 2001, p. 2). Spatial heterogeneity in educational outcomes in Mexico in 2000, based on the GWR technique, was confirmed by the study presented by Gutiérrez, Sánchez, and Giorguli (2011). The paper of Qiu and Wu (2011) deals with the geographic variations in the impact of various student characteristics, teacher characteristics and school characteristics onto the American College Test scores for the 447 public high schools in Missouri. Their GWR analysis showed "that some local areas have weak variable relationships or even opposite variable effects from their corresponding global effects at certain local regression neighbourhoods" (Qiu and Wu, 2011, p. 81). The study of Naidoo, van Eeden and Munch (2014) was aimed at identification of spatial patterns among the 2010 matric pass rates of secondary schools in Cape Town as well as at investigation of spatial relationships between matric pass rates and selected socio-economic variables. Regarding the GWR results, the significant spatial variation in the spatial distribution of all parameters was confirmed. Chocholatá (2019) analysed the spatial variation in the relationship between the Slovak districts' school performance and various socio-economic variables. The local GWR approach enabled to confirm the statistically significant spatial variation in the modelled relationship and to reveal quite a high amount of districts with locally different impacts of analysed socio-economic indicators.

CONCLUSION

This paper deals with the spatial relationship among the educational performance and the selected socio-economic indicators at the district level. The results of the spatial analysis revealed the huge differences in educational performance (measured as Testing 9 average percentage Maths scores) as well as in values of both the socio-economic variables (average nominal monthly wage and unemployment rate) across analysed districts depending on their location. One more variable, the dummy 0/1 variable, indicating the impact of the socially disadvantaged background, was also taken into account. The global relationship among the average percentage Maths scores, the socio-economic variables and a dummy variable implies the positive impact of the average nominal monthly wage and the negative impact of the unemployment rate and socially disadvantaged background onto the test results. In order to consider that the location does matter in the analysis of the educational performance and to capture the considerable spatial heterogeneity, the local spatial approach based on the GWR was used as well. Although the corresponding global and local parameter estimates have the same signs, there is a significant spatial variation in the analysed relationship. Furthermore, mapping of the local parameter estimates enables to provide a more detailed view of the modelled relationship in each district.

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