An Overview of Methodological Issues in Data Envelopment Analysis: a Primer for Applied Researchers

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Abstract

Data envelopment analysis (DEA) as a method of measuring the efficiency of decision-making units (DMUs) has become an attractive tool used in managerial decision-making in many real-world applications. However, like other sophisticated methods, the application of DEA to a selected decision problem is not straightforward and requires to be carried out in a sequence of successive phases. Moreover, the practical application of DEA presents a range of procedural issues to be examined and resolved. The paper provides a concise guidance through individual steps of DEA analysis process focusing on applied researchers. It includes comprehensive overview of possible issues together with recommendations and hints of possible solutions supported by references to relevant literature presenting more details and alternative viewpoints.

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DEA, methodology	*C60*

INTRODUCTION

Data Envelopment Analysis (DEA), initially presented in (Charnes et al., 1978) and built on the earlier work of Farrell (1957), provides a non-parametric linear programming methodology for assessing the relative efficiency of a set of Decision Making Units (DMUs). The last four decades witness the appearance of a set of articles and books on the theory of DEA and its application to various practical settings, e.g., see (Cook and Seiford, 2009). Since 1999 the number of applied papers exceeds the number of those devoted to theory of the method (see Liu et al., 2013). In other words, DEA became a theoretically well-founded methodology with a proven record of successful applications. All that makes it attractive for decision makers as potential addition in their methodology toolbox for supporting decision making in real world applications. However, for someone not specialized in DEA and thus with limited theoretical

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knowledge of the vast DEA methodology, the application of DEA methods is not as straightforward as it seems to be, based on available software tools and seemingly simple requirements put on data. Everyone who intends to use DEA as a part of his/her decision-making toolbox correctly should have some core knowledge of the underlying theory behind various DEA models and/or be able to consult a DEA expert. It mimics the situation in applications of statistics where availability of easy-to-use software enables powerful statistical methods to be misused due to inadequate statistical knowledge of those applying them. In general, an application of DEA to measure the efficiency of DMUs of interest is carried out in a sequence of successive phases (see Golany and Roll, 1989; Ramanathan, 2003; Ozbek et al., 2009): (1) definition and selection of DMUs, (2) definition, selection, and measurement of input and output variables, (3) selection and formulation of the DEA model, (4) application of the DEA model, (5) post-analysis procedures, and (6) presentation and analysis of results.

In each of these phases, there are some key assumptions and procedures to be followed when applying DEA as certain problems may arise and need to be addressed accordingly in order to achieve credible results and avoid "garbage in-garbage out" issues. It is important to help applied researchers in the process of developing necessary skills to detect possible issues when applying the DEA methods and even guide them directly how to proceed through the above-mentioned individual steps and avoid/resolve problems they encounter during the process. In that regard, methodologically oriented papers summarizing important aspects of the methodology and providing directions how to proceed when applying it can play an important role by speeding up the process and make it leaner. The aim of the paper is to provide a concise and comprehensive guidance on how to tackle individual steps of the DEA analysis process, including minimalist and comprehensive overview of possible issues and hints of solutions with references to relevant literature presenting more details how to address them properly. The paper is structured as follows. In Section 1, we discuss how to identify whether DEA is an appropriate method to tackle a research problem of interest. In Section 2, we give an overview how to define and select DMUs and their inputs and outputs. In Section 3, we focus on selection of an appropriate DEA model. In Section 4, we list some software alternatives which can be used to actually perform DEA analyses. In Section 5, we present a compact overview of sensitivity analysis procedures related to the DEA analyses. Then, in Section 6, we show how results of the DEA analyses should be presented and further analysed. Finally, our last section details our conclusions and we discuss possible future lines of research.

1 IS DEA APPLICABLE TO THE PROBLEM AT HAND?

Having in your toolbox a tool like DEA supports "if I have a hammer everything looks like a nail" approach, so it is important to know how to identify if DEA is the way to go when trying to resolve a problem you are facing. As DEA produces non-parametric estimations of a production function, problem we are dealing with, e.g., identification of most efficient companies/branches/departments, identification of possible ways how to improve a relative efficiency of the company/country, allocation of funds etc., should involve a production process. This process could be either a real one where both inputs and outputs as well as mechanism of transformation of inputs to outputs are well known, or an artificial one, where we have just some insights which variables might play the role of inputs and outputs, but we do not have a clear picture of the production process involved. Although the DEA methods might be used in both cases, we should keep it in mind that the nature of the process sets limits for interpretation or application of the results. Results produced by DEA applied when a real well known production process is in place could have more straightforward and clearer interpretation and could be more easily transformable into insights what key sources of identified inefficiencies of DMUs are and what modifications of the production process are needed, i.e., what actions of DMUs are desirable to improve their relative efficiencies. Once we have established that our problem does indeed involve some kind of production process, we identify, guided by a problem we are trying to resolve, all objects of interest such that insight about their relative efficiencies with respect to an identified production process related to a problem at hand could be essential to resolve the problem, i.e., we identify all potential DMUs. With a set of potential DMUs in place, we identify all relevant variables related to the potential DMUs. From the technical point of view, it is often possible to get results from the DEA specialized or general linear programming software when your data consists of several objects you intend to relatively compare to each other (DMUs) and at least two variables measured for each of the DMUs (possible input and output). To ensure that all relevant pieces of information needed to find a solution to a posed problem are in place, it is essential to check here if our data consists of all relevant DMUs and the initial list of variables includes all variables relevant for efficiency evaluation process, i.e., they cover every dimension, the changes in which may affect the DMUs (see Golany and Roll, 1989; Ozbek et al., 2009), or at least it is as wide as possible. Very often we are not at liberty to freely collect all variables of interest with respect to the production process but rather restricted to selection of appropriate input and output variables from variables available in already collected data which may or may not be an appropriate input-output representation of the production process of interest. If the production process takes place, it usually manifests itself by existence of correlations between some of the variables measured on DMUs. If there is a clear picture of the production process involved, presence of significant correlations between variables identified as inputs and variables identified as outputs is expected. At the same time, weak or non-existent correlations are expected to be identified among input and output variables. On the other hand, if the production process is an artificial one, the presence or lack of correlations can be used to guide identification and selection of appropriate inputs and outputs. However, absence or presence of pairwise correlations in data should not be used as a tool to decide if DEA should be applied at problem at hand or not. The role of correlations is more a supportive and an indicative one. When meaningful correlations coherent with our understanding of the production process are present in data, feasible results of application of the DEA methodology, including meaningful interpretation, are more likely.

2 DEFINITION AND SELECTION OF DMUS AND DEFINITION, SELECTION AND MEASUREMENT OF INPUT AND OUTPUT VARIABLES

In the previous step, we did some preliminary evaluation of feasibility of applying DEA to studied problem, created the first iteration of data containing relevant pieces of information and gathered some initial insight if data seems to be promising with respect to DEA. We proceed with refinement of the data based on assumptions imposed on them by DEA methodology. The next two phases, definition and selection of DMUs, and identification and selection of input and output variables are from the practical point of view so intertwined that we will present them simultaneously. On one hand, the number and definition of DMUs determines the set of potential input and outputs, on the other hand, input and output variables represent properties of DMUs some of them being essential to get meaningful results from application of DEA. Therefore, practically these two phases merge in a multistep nonlinear iterative process which, under ideal circumstances, results in the set of DMUs and set of inputs and outputs satisfying various kinds of theoretical assumptions posed by theory of DEA and ready for analysis. In particular, the following assumptions should be met:

(1) the numbers of DMUs and variables are balanced,
(2) variables are clearly separated into inputs and outputs,
(3) DMUs are mutually independent,
(4) DMUs are homogeneous, i.e., comparable.

2.1 Balance between the number of DMUs and variables, reduction of the number of variables

Balanced numbers of DMUs and variables mean that in order to get meaningful results we should apply DEA on data only if the number of DMUs is sufficient, i.e., not too small in comparison to the number of variables (inputs and outputs). The use of too many variables in DEA tends to shift the compared DMUs towards the efficiency frontier which can result in a relatively large number of DMUs with high efficiency scores, and thus, in practice, to reduce, discriminatory power of the DEA. It has been consistently suggested in the DEA literature that there should be a sufficient number of observations (n) in comparison with the numbers of variables, inputs (m) and outputs (s). We recommend using a rule of thumb presented by Cooper et al. (2011) that n should be greater than $\max(m \times s, 3(m + s))$ to determine if the number of DMUs is sufficient. If that check indicates that we need to think about reduction of the number of variables, we can use one of the following main approaches or combination of them:

(1) selection ex-ante only a subset of the original variables,
(2) the approaches based on using the aggregate measures, and
(3) the most recent approaches based on the inclusion of additional cardinality constraints directly into the DEA program in order to select the relevant inputs and outputs automatically.

Approaches from the first group select ex-ante a subset of the original variables either based on heuristic decision or value judgement about which variables are the most relevant for the given problem (see Allen et al., 1997; Golany and Roll, 1989); or on statistical techniques as proposed by Banker (1996), Simar and Wilson (2001), Pastor et al. (2002), Fanchon (2003), Jenkins and Anderson (2003), Ruggiero (2005), Wagner and Shimshak (2007). We prefer here a balanced approach where an expert judgment coming from knowledge about the production process is emphasized and results of statistical analyses play the secondary role. Our recommendation is supported by Harrell (2015) claiming that statistical criteria should not be the only driver determining if variables are or are not included in the models. Application of hypothesis testing could be especially dangerous here as our potential DMUs are usually not selected utilizing statistical sampling techniques and, therefore, applicability of statistical tests in this context is debatable. Moreover, as Peyrache et al. (2020) state, the use of statistical testing brings many additional problems:

(1) the result about which variables are relevant strongly depends on the order of the testing,
(2) we can encounter a severe problem of multitesting when testing for every possible combination between inputs and outputs and thus severely underestimating probability of error in the whole process,
(3) if the number of potential inputs and outputs is very large, the whole procedure of variable selection can become also very tedious and time consuming.

If we identify multiple variables as candidates to be dropped from our data, we can use their measurement character as an additional criterion for final decision. The conventional DEA models assumes that all the data are quantitative variables taking positive or semipositive (i.e., each DMU has at least one positive input and one positive output) real values (see Charnes et al., 1978, 1991). Consequently, if some of the preselected variables are irregular from this point of view, i.e., are qualitative, ordinal or negative, we should prefer their elimination to that of standard quantitative data. The ratio data can too be seen as good candidates for dropping because they violate requirement of convexity (Emrouznejad and Amin, 2009). Approaches from the second group propose to reduce the dimensionality of the problem by substituting some of the original variables by an aggregate measure. This is usually a two-step procedure where in the first step weights of individual variables are determined and in the second step an aggregation technique is selected, and, finally, new aggregated variables created. We face two main problems here. First, there exists a vast number of techniques to determine weights of variables ranging from completely subjective expert selection of ad-hoc weights, through preference-based techniques to some objective techniques utilizing statistical properties of variables to determine the weights, e.g., correlations, variability, information entropy etc. (e.g., Diakoulaki et al., 1995; Wang and Lee, 2009). Regardless the process we used to determine the weights, for the aggregation itself we can use various aggregation functions. We simple have too many options to be able effectively find an optimal combination of weights and aggregation function and to establish arguments defending our selection against the alternatives. Moreover, we do not have a clear picture of what effect various techniques would have on properties of DEA models. A simple workaround for this decision problem is to restrict ourselves to use some well established statistical technique like Principal Component Analysis (PCA) (see Jolliffe and Cadima, 2016) where selection of the variables in based on some statistical criteria, in the case of PCA on preserved variability, and which was extensively studied in the context of its use in DEA modelling. Based on the above, if we see the use of aggregations techniques as necessity, we recommend the most popular approach, DEA-PCA, proposed by Ueda and Hoshiai (1997), and Adler and Golany (2001) which substitutes inputs and outputs with principal components. This technique was studied extensively in comparison to other variable reduction techniques (see Adler and Golany, 2001; Nataraja and Johnson, 2011; Wilson, 2018) and the published results indicate that PCA-DEA consistently provides more accurate results and performs better especially with highly correlated inputs, even for small data sets. More detailed guidelines for the application of PCA-DEA based on the concept of a rule-of-thumb which considers the trade-off between the two types of erroneous classification, namely efficient decision-making units defined as inefficient (under-estimation) and inefficient DMUs defined as efficient (over-estimation) can be found in Adler and Yazhemsky (2010). The second problem related to the use of aggregation techniques to reduce the number of variables is that of interpretability. The new aggregated variable is not generally easily interpretable which could significantly limit our ability to transform results of the DEA model to managerial decisions regarding modifications of inputs and outputs. The third, latest group of approaches is based on the inclusion of additional conditions directly in the DEA model program in order to select the most appropriate inputs and outputs. Majority of these approaches (see Limleamthong and Guillén-Gosálbez, 2018; Benítez-Peña et al., 2020) is based on the introduction of binary (zero-one) variables into the program, so the model is transformed into a Mixed Integer Linear Programming (MILP) formulation. The relevant inputs and outputs are here selected without any previous statistical analysis, heuristic decision making or expert judgement, but the resulting models differ from the most popular models and, consequently, are more demanding on researcher with respect to theoretical knowledge. As these techniques are quite recent, the corresponding models might not be yet implemented in DEA software tools preferable by the researcher.

2.2 Classification of variables into inputs and outputs

Selection of the appropriate number of variables relevant for efficiency evaluation process of DMUs is closely related to classification of these variables into inputs and outputs. If a well-known real production process is in place, classification should be more or less straightforward. Resources utilized by the DMUs or conditions affecting their operation constitute typical inputs, while measurable benefits generated by the DMUs represent the outputs. However, in some cases, especially if the underlying production process is not fully understood or it is an artificial one, some variables may be interpreted in both ways and play either input or output roles. These variables are usually referred to as flexible measures (Cook and Zhu, 2007) or dual-role factors (Cook and Zhu, 2006). Suppose, for example, that one wishes to evaluate the efficiency of bank branch operations. In this case, a factor such as deposits, could serve as either an input or an output. In some of the previous studies devoted to efficiency of banks, such a factor is regarded as an output, as it is a source of revenue for the branch. At the same time, however, it can be argued that the deposits, in particular, the time spent by employees in processing of customers who are making deposits or opening deposit accounts, is an input since it could be used to better advantage to sell more profitable products to the customer. For more examples of flexible measures see e.g. (Cook and Zhu, 2007). The DEA literature offers several methodological suggestions for deciding the status of such flexible measures. Golany and Roll (1989) propose to carry out a series of regression analyses, of such factors, one at a time, on the factors known to be inputs and outputs. A weak relation to inputs and strong relation to outputs indicates a preference towards classifying the factor as an input, and vice versa a weak relation to all the factors may indicate a need to re-examine the factor and possibly remove it from the analysis. Alternatively, strong relationships may indicate that the information contained in that factor is already represented by other factors and, again, its removal should be considered. Golany and Roll (1989) also pointed out one of the basic assumptions for the DEA models, the condition of "isotonicity", i.e., that an increase in any input should not result in a decrease in any output, may also be helpful. Besides statistical properties of flexible measures, additional information might be used to determine their appropriate status. Bala and Cook (2003) investigated the situation where bank branch consultants provided additional "classification data" specifying quality of each branch (good versus poor branch). Their intention was to assign a status to each flexible measure such as to provide efficiency scores that are in the best agreement with expert opinion. Many other approaches how to deal with the flexible measures were published (see e.g. Cook and Zhu, 2007; Amirteimoori and Emrouznejad, 2011; Amirteimoori et al., 2013; Toloo et al., 2018; Kordrostami et al., 2019; Sedighi Hassan Kiyadeh et al., 2019; Boďa, 2020). Detailed description of these approaches is beyond the scope of the paper. Our recommendation is to start with the classical approach where the status of a flexible measure is determined by its statistical properties reinforced by available additional information and move towards more elaborate techniques only if the results of the modelling are not satisfactory to solve the problem.

2.3 Independence of DMUs

It is commonly assumed that the DMUs are independent of one another, i.e., each DMU has its own values of inputs and outputs not connected in any way to values of the inputs and outputs of other DMUs. In other words, if the assumption of independence of DMUs holds, decreasing the inputs or increasing the outputs of one DMU will not affect the inputs or outputs of other DMUs. However, this natural assumption is not always fulfilled in real conditions. The condition of mutual independence of DMUs is not fully met in the following two cases. First, if the DMUs have some input in common. The efficiency measurement difficulty created by this "shared" resource phenomenon is that in attempting to move an inefficient DMU towards the frontier by reducing that shared resource, the other DMUs in the same set of DMUs will be equally penalised. Second, if some internal or linking activities between DMUs are present. For example, many companies are comprised of several divisions that are linked by some intermediate products or activities. Some DMUs in this multi-stage structure consume resources produced by other DMUs and some of which produce resources consumed by other DMUs. Another example is the supply chain system where supply chain members (suppliers, manufacturers, distributors, and retailers) are interconnected through intermediates, i.e., some measures linked to related supply chain members that cannot be simply classified as outputs or inputs of the supply chain. For example, the supplier's revenue is an output for the supplier, and it is in the supplier's interest to maximize it, and, at the same time, it is an input to the manufacturer who wishes to minimize it. These possible conflicts between supply chain members may result in that one member's inefficiency may be caused by another's efficient operations. Measuring efficiency of the interconnected DMUs becomes a difficult and challenging task because of the need to deal with the multiple intermediate products, and to integrate and coordinate the efficiency of those DMUs. If, based on the presented examples, we identify interdependencies among DMUs, we must address it with selection of an appropriate DEA model as the conventional DEA models will fail to address this important issue. Within the context of DEA, several methods have the potential to be used in these situations. To capture interdependence caused by shared inputs, Avilés-Sacoto et al. (2019) developed a new DEA-like methodology and illustrated it using the problem of evaluating a set of departments in a university setting, where the departments are grouped under various faculties. To deal with interdependence caused by linking of DMUs, Zhu (1996, 2009) presented a DEA-based supply chain model to both define and measure the efficiency of a supply chain and its members and yield a set of optimal values of the intermediate performance measures that establish an efficient supply chain. Liang et al. (2006) developed two types of DEA-based models for supply chain efficiency evaluation, using a seller-buyer supply chain as an example.

2.4 Homogeneity of DMUs

A homogenous set of DMUs is a key assumption within DEA and can be obtained by considering the following criteria (see Golany and Roll, 1989; Dyson et al., 2001; Ozbek et al., 2009):

(1) DMUs should perform the same activities and undertake the same processes with similar objectives,

(2) the input-output variables characterizing the activity of DMUs in the data set should be identical except for the differences in their intensity or magnitude, and

(3) DMUs should operate under the same market and environmental conditions.

While the first two criteria represent the criteria of homogeneity of the DMUs themselves, the last criterion is the criterion of the homogeneous environment of DMUs. However, in practice, homogeneity of DMUs or environment is seldom present. In the cases when the condition of homogeneity across all of the DMUs is not met, it is necessary to use one of the special techniques proposed for the given cases as application of the conventional DEA models may result in efficiency scores reflecting the underlying differences in the conditions of DMUs rather than any inefficiencies. Examples of applied problems where some of the above-mentioned conditions are violated are the following:

- When efficiency of bank branches is examined, differences in activities may occur as large branches usually carry out most banking activities, whilst smaller branches may only engage in some of them.
- Differences in input-output variables may be encountered when assessing the efficiency of faculties at one university. Usually, not all faculties shared the same inputs, i.e., the science faculties required laboratories and equipment, while the humanities faculty did not.
- Non-homogeneous environment can be illustrated on efficiency comparison of individual urban public transport (UPT) lines in a city. Efficiency of UPT lines is highly dependent on the conditions in which transport takes place, i.e., on environmental factors. For example, lines running in the city centre are exposed to a high level of competition because many lines connecting the city centre share the same routes and it is not important for passengers which particular line they choose, in effect of which the numbers of passengers are highly influenced by the possibility of substitution lines.

In general, environmental factors express the influence of the environment on a DMU and characterize the environment and its characteristics in which the DMU operates. These factors are not traditional inputs and are assumed not being under the control of a manager. Environmental factors include (i) external factors such as the current political and macroeconomic situation, the purchasing power of the population, the demographic structure of the population or other location characteristics, or (ii) internal factors such as the ownership differences, e.g., public/private or corporate/non-corporate, and organizational structure differences of DMUs.

2.4.1 Non-homogeneity induced by differences in activities, inputs and outputs

When dealing with non-homogeneity coming from (1) and (2), there are two basic options on how to proceed, either (i) restrict the analysis to a limited number of activities / inputs and outputs shared by all DMUs; or (ii) to cluster DMUs into homogeneous groups and thus study only a limited number of DMUs engaged in identical activities/having the same inputs and outputs (see Athanassopoulos and Thanassoulis, 1995; Molinero, 2008). Unfortunately, these two basic options are in many cases neither possible (e.g., due to unavailable data or due to the low number of DMUs in relation to the number of inputs and outputs) nor desirable (if we wish to evaluate the overall relative efficiency capturing all activities of DMUs in the whole group of evaluated DMUs). In cases when these basic options are not applicable, one of the more advanced approaches must be applied. If non-homogeneity of DMUs is caused by non-homogeneous output sets and we can assume that the input set was common across all DMUs, the approach of Cook et al. (2012, 2013) can be followed. Cook et al. (2013), by extending the earlier research of Cook et al. (2012), developed a general setting encompassing the lack of homogeneity on the output side where some DMUs may produce a certain set of products, but not all products are produced in all DMUs. If non-homogeneity of DMUs is caused by non-homogeneous input sets, the approach of Li et al. (2016) extending earlier research of Cook et al. (2012, 2013) can be employed. Li et al. (2016) examined a three-step procedure to evaluate efficiencies of DMUs in the presence of non-homogeneous input sets. In each DMU, the process of inputs generating outputs is divided into the separate processes. In the first step, for each DMU in each process (in which that DMU is involved), an appropriate split of the inputs and outputs is determined. In the second step, the process scores for each DMU are computed, and in the third step, the aggregate scores computed as a weighted average of the process scores for any given DMU are calculated. In the most general case, two types of non-homogeneity mentioned above are present simultaneously, i.e. some DMUs do not carry out the same activities as others, and they do not share the same set of inputs/outputs with the remaining DMUs, and the model proposed by Molinero (2008) could be applied. The proposed model is presented on the example of three types of university institutions: those, such as standard universities, that engage in both teaching (T) and research (R) activities; those that engage in the T activity but not in the R activity; and those, such as research institutes, that engage in the R activity but not in the T activity. Under the proposed joint efficiency model, it is assumed that some inputs/outputs are related only with the T activity, some inputs/outputs are shared between/reflect the effort devoted to the T and R activities, and some inputs are allocated/depend only to the R activities. The DMU under observation must decide how to allocate shared inputs to the R or to the T activities, and how much effort should be devoted to produce outputs from the T or the R activities. It does that by taking into account the importance attached to the T and R activities (captured by the weights determined outside the model and reflecting the priorities of the decision maker), and the desire to be operating as efficiently as possible under both activities when compared with other DMUs. The rationale of the joint efficiency DEA algorithm is based on the same philosophy as the standard DEA model in the ratio formulation: once the DMU under observation has decided how to allocate shared inputs, and how to attribute shared outputs, this split is applied to all other DMUs and efficiency calculations take place as usual.

2.4.2 Non-homogeneity induced by non-homogeneous environment

There are numerous ways in which non-homogeneous environment can be accommodated in a DEA analysis. They differ mainly based on permissible character of environmental factors, i.e., ordinal, categorical or continuous. There are four traditional approaches (Coelli et al., 2005) to tackle this issue. If the values of the environmental factor can be ordered from the least to the most detrimental effect upon efficiency, then the first approach of Banker and Morey (1986a) can be followed. In this approach, the DMU is compared with those DMUs in the sample that have a value of the environmental factor which is less than or equal to that of the given DMU. This would ensure that no DMU is compared with another DMU that has a more favourable environment. Example of application of this approach can be found in (Roháčová, 2015). The second approach proposed by Charnes et al. (1981) can be employed in the cases when the environmental factor is of a categorical character (e.g., public versus private ownership). This approach involves three stages: (i) divide the DMUs into relatively homogeneous sub-groups and solve a DEA model for each sub-group, (ii) project all observed data points onto their respective frontiers, and (iii) solve a single DEA model using the projected points and assess any difference in the mean efficiency of the two sub-groups. Example of application of this approach can be found in (Soteriou and Zenios, 1999; Fandel et al., 2019). Note that there are two main limitations of the previous two approaches: (i) a high number of DMUs is necessary (subdivision of DMUs into sub-groups may significantly reduce the number of DMUs compared, resulting in many DMUs being found to be efficient and thus reducing the discriminatory power of the analysis), and (ii) the possibility of considering only one environmental factor. The second approach has the additional limitation that it requires that the environmental variable be a categorical variable, while the first approach suffers from the problem that it requires that the direction of the influence of the environmental variable (upon efficiency) be known a priori. The third possible approach is to include the environmental factor(s) directly into the optimization task of a DEA model, either in the form of a non-controllable input variable, a non-controllable output variable, or a non-controllable neutral variable (a non-discretionary variable), (see e.g., Coelli et al., 2005; Cooper et al., 2007). In this approach, it is necessary to determine the direction of influence of the environmental factor first, i.e., whether higher values of the environmental factor will contribute to improving or deteriorating efficiency. If the environmental factor is likely to have a favourable (detrimental) effect upon efficiency, then the environmental factor can be included in the optimization task of a DEA model in the form of non-controllable input (output) variable. In this way, the DMU is compared with a theoretical DMU that has an environment that is no better than that of the given DMU. If the direction of influence of the environmental factor is uncertain, then it can be included in the DEA model as a non-controllable neutral variable. This will ensure that the assessed DMU is only compared with a (theoretical) frontier DMU that has the same environment (neither better nor worse). Although this approach does not require a predetermined direction of the environmental variable influence, it can significantly reduce the reference set for each DMU and hence inflate the obtained efficiency scores. Finally, it should be noted that this third approach also has the disadvantage that the environmental factors have to be continuous variables (i.e., they cannot be categorical or ordinal variables). If there are categorical variables considered as environmental factors, then the more complicated mixed-integer linear programming models, suggested by Banker and Morey (1986b), can be used. For some extensions of this model, the reader is referred to Kamakura (1988), and Rousseau and Semple (1993). The last, and relatively most widely used, approach to include environmental factors consists of a two-stage procedure. In the first stage, the efficiency of the DMUs is measured without including environmental factors, e.g., using traditional inputs and outputs, and then in the second stage, the efficiency scores from the first stage are regressed upon the environmental factors. The environmental factors can be of both continuous and categorical nature, i.e., the effects upon efficiency of the age, experience, education and training of the manager(s) can be estimated. Using this approach, it is possible to examine the direction and intensity of the influence, as well as to correct the efficiency scores for environmental factors by using the estimated regression coefficients to adjust all efficiency scores to correspond to a common level of environment (e.g., the sample means). The two-stage procedure approach has several significant advantages over previous approaches, e.g., it can accommodate more than one variable, it works for both continuous and categorical variables, it does not make prior assumptions regarding the direction of the influence of the environmental factor and one can determine significance and effect size of factor influence upon efficiencies. Moreover, it is easy to calculate, and the approach is simple and therefore transparent. See e.g. (Ray, 1991; Fizel and Nunnikhoven, 1992, 1993; Oum and Yu, 1994; Sexton et al., 1994; Nolan, 1996; Mancebon and Molinero, 2000; Mendelová and Kanderová, 2016) for application of this approach and (Haas and Murphy, 2003; Banker and Natarajan, 2008; Simar and Wilson, 2007, 2011 and Banker et al., 2019) for comparisons and some extensions of this approach.

3 SELECTION AND FORMULATION OF THE DEA MODEL

The right choice of the DEA model used for tackling our problem is a very important and definitely non-trivial issue. To date, a huge variety of the DEA models have been developed so it would be unattainable to carry out systemisation of all of them including some guidelines when they should be selected in a single paper. For this reason, in this section the main attention will be paid to the basic considerations that could guide the analyst in the right direction in choosing an appropriate DEA model in relation to the problem being solved and various selected DEA model alternatives connected to these directions will be presented. In the process of selecting a suitable DEA model, the critical areas are in particular:

(1) the choice of the DEA model orientation,
(2) the choice of a radial or a non-radial approach,
(3) the choice of a returns-to-scale assumption, and
(4) violation of basic assumptions, i.e., non-homogeneity and/or interdependence of DMUs, presence of data irregularities.

3.1 DEA model orientation

The following considerations (see Golany and Roll, 1989; Ramanathan, 2003) may be helpful in selecting the DEA model orientation. In applications where inputs are rather inflexible (e.g., determined to a certain extend by higher managerial levels and thus not fully under control of DMUs), output-oriented DEA model would be more appropriate. On the other hand, in applications that involve inflexible outputs (e.g., matched closely with goals set by management or restricted by environmental conditions), input-oriented DEA model may be more appropriate.

3.2 Radial vs non-radial models

There are two types of approaches in DEA: radial and non-radial. The radial approach, represented by the CCR (Charnes et al., 1978) and BCC models (Banker et al., 1984), neglects the non-radial input/output slacks and provides a radial measure of efficiency. This measure is referred as Farrell efficiency (for the CCR model) or pure technical efficiency (for the BCC model) and, in the case of input-oriented models, represents the maximum equiproportionate, i.e., radial reduction in all inputs that is feasible with given technology and outputs. On the other hand, the non-radial approach, represented by the additive or the SBM models (Tone, 2001), deals with slacks directly and measures a non-radial efficiency that is referred to as Pareto-Koopmans efficiency or strong efficiency. Both approaches have some benefits but also drawbacks. While, the radial approaches neglect the non-radial input/output slacks, the non-radial approaches neglect the radial characteristics of inputs and/or outputs. It is therefore necessary to consider in detail which of these approaches to use in relation to the specifics of the problem addressed. The final selection should be based primarily on the consideration of potential differences in the characterization of the inputs and outputs. The general rule can be formulated as follows. If the non-radial slacks have an important role in evaluating managerial efficiency, the non-radial approaches should be preferred. And, on the other hand, if the loss of the original proportionality of inputs and/or outputs is inappropriate for the analysis, the radial approaches are more suitable. In other words, if all the inputs and/or outputs (depending on the model orientation) are non-radial (substitutional), i.e., they do not have to change proportionally, the non-radial approach should be selected. And, if, all the inputs and/or outputs are radial, i.e., they have to change proportionally, the radial approach should be preferred.

3.3 Returns-to-scale assumption

One of the most critical problems for setting up a DEA model is the identification of suitable Returns To Scale (RTS) for the data. In principle, either the Constant Returns to Scale (CRS), Increasing (non-decreasing) Returns to Scale (IRS), Decreasing (non-increasing) Returns to Scale (DRS) or Variable Returns to Scale (VRS) can be assumed. The identification of the RTS is one of the most discussed areas in DEA. At the outset, it should be emphasized that it is done at two levels: at the technology that is referred as the Technological Returns to Scale (TRTS), and at the DMU level. For identifying the RTS at the DMU level the basic approaches were developed by Färe et al. (1983), Banker et al. (1984), and Banker and Thrall (1992). In the process of selecting the appropriate DEA model for analysis, the proper identification of the TRTS is crucial. Unlike the RTS at the DMU level, the RTS at the technology level has been investigated by only a few researchers. In the DEA literature, there are two main approaches for the determination of the TRTS, subjective, where the TRTS is determined by expert opinions and, objective, where the TRTS is identified through a mathematical model. In the subjective approach, an analyst has to take into consideration that the DEA models under CRS assumption do not allow for the potential existence of economies or diseconomies of scale. Thus, when the performances of DMUs are not normally expected to depend on the scale of operation (e.g., comparisons several large monopolies), the assumption of CRS seem appropriate. However, a unit may be too small to operate with optimal efficiency or so large that it becomes difficult to manage. The DEA models under VRS assumption have been developed specifically to accommodate scale effects in analysis. It is well known, that the VRS model will always envelop the data more closely than the CRS model, irrespective of whether VRS exist. If the VRS model is used, where there are no inherent scale effects, the efficiency of small and large units will tend to be over-rated. Alternatively, if the CRS model is used, where there are evident inherent scale effects, the small and large units can be greatly discriminated. It is therefore advisable, when it is not known a priori if the production technology exhibits CRS or VRS, to use some of the objective approaches that test the data separately for the scale effects. The TRTS investigation is still an active area of scientific interest. The objective approaches for the identification of the TRTS are based on either the statistical methods (see Banker, 1993, 1996; Read and Thanassoulis, 2000; Simar and Wilson, 2002; Banker and Natarajan, 2011), or the non-statistical methods (Alirezaee et al., 2018). Banker (1993, 1996), and Banker and Natarajan (2011) explored the statistical properties of the production frontiers generated by DEA models and developed DEA-based hypothesis tests for addressing a wide range of issues including the TRTS. Read and Thanassoulis (2000), inspired by the approach of Banker (1996), suggested a measure of cross-mix scale size for single-output, multiple-input cases where the production function can be approximated by a homothetic function. Their method can be used to help to decide which of the DEA models to use even in small data sets. Simar and Wilson (2002) used non-parametric statistical tests for recognizing the TRTS. However, since applying a statistical method usually requires some background assumptions, Alirezaee et al. (2018), to overcome this deficiency of the statistical methods, focused on identifying the TRTS using data mining. They designed a novel method, the Angles method, for mining the dataset and discovering its TRTS characteristics.

3.4 Model selection if basic assumptions are violated

In this section, we assume that we determined the final set of DMUs and their corresponding inputs and outputs. Further, we assume that one of the following violation of basic assumptions occurs:

(1) non-homogeneity and/or interdependence of DMUs;

(2) presence of data irregularities: ordinal variables, negative data, ratio data.

The issues of non-homogeneity and interdependence of DMUs were already discussed in Sections 2.3 and 2.4 and some recommendations regarding suitable models were presented. Therefore, we restrict ourselves here to the case when non-homogeneity and interdependence occurs simultaneously. Castelli et al. (2001) dealt with nonhomogeneous subunits by identifying sensibly compared groups of them evaluated through three new-introduced efficiency concepts. Imanirad et al. (2013) extended the conventional DEA models to situations where only partial input-to-output impacts exist. They considered the DMU as a business unit consisting of a set of mutually exclusive subunits, each of which can be evaluated in the conventional DEA sense. Du et al. (2015) pointed out two main deficiencies of the Imanirad et al. (2013) approach: splitting inputs may not be applicable when non-separable inputs exist, and the intermediate measures that link subunits are not considered. Du et al. (2015) expanded the presence of non-homogeneity to a network setting and proposed DEA models that tackle the problem of the parallel network in non-homogeneous situations, where subunits operate with intermediate products but are not restricted to identical input/output variables. Barat et al. (2019) followed the approach of Du et al. (2015) and generalised the idea of nonhomogeneity in parallel network structures to the case where DMUs have various subunits. In the presence of ordinal data, we need to resolve the issue that direct incorporation of ordinal data into the standard linear DEA model leads to DEA model which is a non-linear and non-convex program. Such a DEA model is called Imprecise DEA (IDEA) in the literature (Cooper et al., 1999) as in DEA terminology ordinal data, together with interval or bounded data, and ratio bounded data, is referred to as imprecise data. The modified DEA structure incorporating ordinal data was first presented in Cook et al. (1993, 1996). In general, there are two approaches how to deal with ordinal variables in DEA evaluation. One uses scale transformations and variable alternations to convert the non-linear IDEA model into a linear program (Cooper et al., 1999; Park, 2007). The other identifies a set of exact data from the ordinal (imprecise) inputs and outputs and then uses the standard linear DEA model (Zhu, 2003a, 2004; Chen, 2007). We recommend using the second approach. A comprehensive survey of the IDEA approach that uses the standard DEA model is presented in Chen (2007). In the presence of negative data, there are three ways how to proceed:

(1) to interchange roles between inputs and outputs,

(2) to transform the negative data to be positive and then use a translation invariant DEA model,

(3) to use the modified DEA models capable of handling negative data without data translation.

In the input-output exchange approach suggested by Scheel (2001), the absolute values of negative outputs are treated as inputs and the absolute values of negative inputs are treated as outputs. The rationale is that more negative values, as an output, indicate worse performance, while larger positive values, as an input, also indicate worse performance. It is a very simplistic approach not having influence on selection of DEA model, but as pointed out by Kao (2017), this approach has also its drawbacks, namely:

(1) restriction of this approach to cases when the values of all DMUs are negative and

(2) the fact that it does not reflect the true production process.

We do not recommend the second approach, based on transformation of positive values into negative, as it poses a serious restriction on models which can be used. Essentially, we are restricted to the use of the additive model or the BCC model. However, both models have their drawbacks. The additive model provides the "furthest" target on the production frontier for inefficient DMUs (Cheng et al., 2013) and cannot provide any measure of efficiency (Portela et al., 2004); the BCC model is just restricted translation invariant. In the third approach, the use of modified DEA models, we recommend one of the following models. Cooper et al. (1999), based on the additive model, developed the Range Adjusted Measure (RAM). Portela et al. (2004), to deal with negative data, utilized the directional distance function and develop two variants of a Range Directional Measure (RDM), labelled RDM+ and RDM−, respectively. The proposed RDM model provides an efficiency score that results from the comparison of the unit under assessment with the so-called ideal point by using the corresponding directional distance function. Through combining the RDM+ model with the SBM model, Sharp et al. (2007) proposed a Modified SBM (MSBM) model where both negative outputs and negative inputs could be handled. Tone and Tsutsui (2009) propose the approach how to deal with non-positive data in the SBM and Network DEA model. Emrouznejad et al. (2010) introduced a Semi-Oriented Radial Measure (SORM) model by breaking down each variable into two non-negative variables. Kazemi Matin and Azizi (2011) proposed a new additive based approach to provide a target with non-negative value associated with negative components for each observed unit. Kerstens and Van de Woestyne (2011) recommended a generalized Farrell proportional distance function that handles negative data and maintains a proportional interpretation under mild conditions. Cheng et al. (2013) developed a variant of the traditional radial model, where original values of inputs or outputs are replaced with their absolute values as the basis to quantify the proportion of improvements to reach the efficiency frontier. Some imprecisions of the Cheng et al. (2013) model were corrected by Kerstens and Van de Woestyne (2014). Finally, it should be added that if the researcher is interested in finding out the full ranking of the evaluated units, i.e., to further discriminate efficient units, it is possible to apply one of the most recent approaches that address the issue of negative data for super efficiency evaluation. Hadi-Vencheh and Esmaeilzadeh (2013), Lin and Chen (2017), Wei et al. (2019), and Lin et al. (2019) proposed alternative super-efficiency DEA models, which can fully rank all DMUs, including the efficient DMUs, and can deal with negative data. In the presence of ratios in our variables, standard DEA models methods cannot be directly used as it may lead to incorrect results caused by violation of convexity (see Bogetoft, 1996; Bogetoft et al., 2000; Cooper et al., 2007; Emrouznejad and Amin, 2009; Olesen and Petersen, 2009; Cook and Zhu, 2014; Olesen et al., 2015). This problem can be resolved if we know the original numerator and denominator by placing them accordingly as additional inputs and outputs in the model (Emrouznejad and Amin, 2009). However, if there are too many ratio variables present in the data this might not be feasible as it can lead to the significantly increased number of variables and thus destroy the balance established between the number of DMUs and the number of variables. Alternatively, the ratio variable could be replaced by volume variable (Thanassoulis et al., 1995, 2008; Dyson et al., 2001). An alternative approach can be found in (Hatami-Marbini and Toloo, 2019).

4 APPLICATION OF DEA MODELS

There are relatively many software packages on the market that allow straightforward calculations of DEA models without any particular knowledge of DEA methodologies. Some commercial specialized DEA programs are presented in Table 1 and Table 2 provides a list of some non-commercial alternatives.

Since DEA is a method based on solving linear and non-linear optimization tasks, besides specialized DEA software or macros, we can use general programming languages widely adopted for scientific computing and providing means for solving both linear and non-linear optimization problems.

Table 1 Commercial DEA software packages

Software	Authors/developers	Available from
DEA-Solver-Pro™	K. Tone (Saitech, Inc.)	http://www.saitech-inc.com/Products/Prod-DSP.asp
Frontier Analyst®	Banxia Software Ltd.	https://banxia.com/frontier
PIM-DEAsoft	A. Emrouznejad, E. Thanassoulis	http://www.deasoftware.co.uk
xlDEA	Productivity Tools	https://xldea.windows10compatible.com
LIMDEP	Econometric Software, Inc.	http://www.limdep.com
DEAFrontierTM	J. Zhu	http://www.deafrontier.net
DEA Online Software (DEAOS)	DssBridge Decision Group Inc.	https://www.deaos.com
MaxDEA Ultra	Beijing Realworld Software Company Ltd.	http://maxdea.com/MaxDEA.htm

Source: Own construction

Table 2 Non-commercial DEA software packages

Software	Authors/developers	Available from
DEAP	T. Coelli	https://economics.uq.edu.au/cepa/software
EMS: Efficiency Measurement System	H. Scheel	http://www.holger-scheel.de/ems
OSDEA	H. Virtos	https://opensourcedea.org
MaxDEA Basic	Beijing Realworld Software Company Ltd.	http://maxdea.com/MaxDEA.htm
DEAOS Lite	DssBridge Decision Group Inc.	https://www.deaos.com
DEAFrontierTM Free Version	J. Zhu	http://www.deafrontier.net/deafree.html

Table 2		(continuation)
Software	**Authors/developers**	**Available from**
R packages		
Benchmarking	P. Bogetoft, L. Otto	https://CRAN.R-project.org/package=Benchmarking
nonparaeff	D. Oh, D. Suh	https://CRAN.R-project.org/package=nonparaeff
FEAR	P. W. Wilson	http://media.clemson.edu/economics/faculty/wilson/Software/FEAR/FEAR-1.15/FEAR-manual.pdf
npsf	O. Badunenko et al.	https://CRAN.R-project.org/package=npsf
MultiplierDEA	A. K. Puthanpura	https://CRAN.R-project.org/package=MultiplierDEA
Python Library pyDEA	A. Raith et al.	https://pypi.org/project/pyDEA

Source: Own construction

This option is of particular importance if the DEA model contains several special settings (e.g., extending the model to include uncontrollable variables, non-discretionary variables, bounded variables, undesirable variables, etc.) and if we plan to adopt more advanced sensitivity analyses, post-hoc statistical analyses and visualisations. The drawback is a steeper learning curve. Alternatively, the versatile Microsoft Excel spreadsheet editor can also be used using the Solver Add-in. For instructions on how the Excel Solver can be used to solve various DEA models see e.g. (Zhu, 2003b).

5 POST-ANALYSIS PROCEDURES (SENSITIVITY ANALYSIS OF DEA RESULTS)

The results of DEA as a non-parametric and non-statistical method of measuring efficiency are generally relatively sensitive to both the selection of the compared DMUs and the selection of inputs and outputs considered. There is no guarantee that the initial selection of such DMUs and inputs and outputs are correct and that this serves the best purpose of the analysis. DEA is an extreme point technique where the efficiency frontier is formed by the actual performance of the best-performing DMUs. A direct consequence of this aspect is that errors in measurement can affect DEA results significantly. Hence, it is expected that the robustness of the DEA results be verified using some form of sensitivity analysis. This problem can be mitigated also by using a specially developed robust DEA models, e.g., a model presented in (Hladík, 2019).

5.1 Basic sensitivity analysis approaches

The sensitivity (stability or robustness) analysis has been one of the widely studied topics in the DEA literature. Some basic sensitivity analysis procedures (Smith and Mayston, 1987; Ahn and Seiford, 1993; Ramanathan, 2003) are:

(1) peer analysis,
(2) analysis based on diminution and augmentation in the number of inputs and outputs,
(3) analysis based on diminution and augmentation in the number of DMUs, and
(4) analysis based on the use of different DEA models.

Smith and Mayston (1987) pointed out that according to the DEA technique, it is possible for a DMU to become efficient if it achieves remarkably better results in terms of one output but performs below average in terms of other outputs. These kinds of efficient DMUs can be tested by identifying the peers for inefficient DMUs. Thus, if a DMU is initially identified as efficient by DEA, a supplementary sensitivity analysis should be conducted by checking the number of inefficient DMUs for which it is a peer. If the number is high, then the DMU is genuinely efficient. However, if the number is low, then its best performance is questionable and other evidence for establishing the superiority of its performance is necessary. An alternative way of testing the robustness of the DEA results is to verify whether the efficiency score of a DMU is affected appreciably if either only one input or output is omitted from the DEA analysis (Smith and Mayston, 1987), or different input/output combinations are considered in the analysis (Banker, 1996). An efficient DMU that is ranked inefficient due to these input/output changes should be viewed with caution. Another part of the literature studies responses with given data when DMUs are deleted or added to the set being considered. Smith and Mayston (1987) and Ramanathan (2003) proposed a sensitivity analysis based on omitting efficient DMUs from the analysis. The purpose of this analysis is to assess the sensitivity of the DEA results for this change. Another part of the literature extends sensitivity analyses to eliminating some DMUs entirely and/or introducing additional DMUs. Wilson (1995) studied the effects of removing DMUs in order to determine "influential observations" (i.e., DMUs). A method based on a similar basis is "window analysis", originally proposed by Klopp (1985) for studying trends. Here, however, it is also treated as a method for studying the stability of DEA results because such window analyses involve the removal of entire sets of observations and their replacement by other previously not considered observations. Another topic of interest revolves around effects that might be associated with choices of different DEA models. Ahn and Seiford (1993) focused on whether different DEA models would result in different conclusions in a hypothesis test setting. Nevertheless, there are systematic differences between the different DEA models because of the differences in efficiency characterizations, the models should generate the same hypothesis test results. If DEA results are proved to be sensitive to the model selected for an analysis, the credibility of the results obtained would be seriously weakened.

5.2 Advanced sensitivity analysis approaches

Building on previous basic approaches to assessing the credibility of DEA results, analytically formulated (mathematical) methods for examining stability and sensitivity of results to data variations with given variables and given models have been developed in recent years. These methodological approaches to sensitivity analysis are associated with the development of tools and concepts that can be used to determine the degree of sensitivity to data variations in any particular application of DEA. There has been a great progress in the methodological approaches. The early work in this area, which focused on a single input or output analyses for a single DMU, has now moved to sensitivity analyses aimed at the evaluation of the stability of DEA results when all inputs and outputs are varied simultaneously in all DMUs. According Cooper et al. (2001, 2011), we can distinguish:

(1) algorithmic approach,
(2) metric approach,
(3) multiplier model approach,
(4) a two-stage alternative,
(5) envelopment approach.

Algorithmic approaches are extensions of algorithmic approaches used in linear programming and they are discussed by Charnes and Neralic (1992), and Neralic (1997). The metric approaches, see Charned et al. (1992), use the concept of distance to determine for a DMU its radius of stability, i.e., the range of data variations not causing alteration in DMU being efficient or vice versa. Both algorithmic and metric approaches, work with one DMU at a time. This drawback can be overcome by using a multiplier model approach of Thompson et al. (1994, 1996) where a special multiplier model is present solution of which provides as with information about stability of all DMUs simultaneously. Unfortunately, it involves the use of special algorithms like the interior point methods developed in Thompson et al. (1996). To avoid such difficulties Cooper et al. (2011) proposed the use of a two-stage alternative, where special algorithms as the one mentioned above are used only if standard methods fail. As an alternative to multiplier model approach, Seiford and Zhu (1998), and Zhu (2009) have developed the envelopment approach, where envelopment boundaries are determined for situations where (unequal) changes in all DMUs (both efficient and inefficient DMUs) are considered. Alternatively, we can focus on analysis of inefficient DMUs and define, for each inefficient DMU, so called "NecessaryChangeRegion" (Jahanshahloo et al., 2011) which encompasses ways how this DMU can reach some predetermined level $\alpha < 1$ of efficiency. Besides techniques developed specifically for DEA, we can use also bootstrap to estimate variance of efficiency and determine the related confidence intervals (see Bogetoft and Otto, 2011).

6 PRESENTATION AND ANALYSIS OF RESULTS

When discussing presentation and analysis of the results coming from applications of DEA, two main questions need to be answered. First, what should be presented and/or analysed and second, how it should be presented and/or analysed. We are well aware that detailed answers to these questions depend on the intended audience and character of the problem we are dealing with. Therefore, we restrict ourselves here to some general recommendations.

The direct results of DEA models always need to be presented in such a way that they can be used as a basis for decision making in an economic environment. In the presentation, all previous phases of DEA modelling should be sufficiently covered. The key pieces of information which should be shared with our audience are the following:

(1) goal of the analyses, i.e., the problem we are trying to solve or get insight into,
(2) description of production process, definition and properties of DMUs, selected inputs and output, their properties and their connection to the problem at hand,
(3) what model were used, assumptions of these models and why we opted for the selected models,
(4) software tools used for analysis,
(5) identification of efficient and inefficient DMUs, and pears for inefficient DMUs,
(6) efficiency scores and the impact of efficient DMUs on those identified as inefficient,
(7) results of sensitivity analyses and post-hoc statistical analyses,
(8) recommendation for DMUs regarding ways how to improve their performance.

Points (1)–(4) mean presentation of crucial parts of phases discussed in Sections 1–4 of the paper. From the point of view of presentation these are not particularly demanding and could be satisfactory covered by standard tabulations and series of simple graphs like e.g., boxplots, scatterplots and web charts. If dimension reduction technique, like PCA, was used when identifying appropriate inputs and outputs of analyses, barplots and biplots related to this analysis could be beneficial. The most appropriate approach to points (5)–(6) depends on the number of DMUs. When the analysed dataset comprises of small amount of DMUs and, consequently, of quite few inputs and outputs, it is again satisfactory to present all elements of the analysis via standard tabulation techniques and simple graphing techniques. However, with an increase in the number of DMUs and related inputs and outputs, more advance techniques are required to be implemented in order to quickly identify key pieces of information and avoid information overload. In (Porembski et al., 2005), Sammon's Mapping (Sammon, 1969) is applied to DEA results of efficiency of 140 bank branches in Germany and the paper illustrates using this specific example that this approach allows us to identify simultaneously efficient DMUs, severity of inefficiency of inefficient DMUs, pears for inefficient DMUs, outliers and influential DMUs. An alternative approach called Co-plots can be found in (Adler et al., 2007) applied to 19 Finnish Forestry Boards. Co-plots are a visualization technique based on multidimensional scaling which simplifies multivariate results of DEA modelling into two dimensions where DMUs are represented as points and inputs and outputs are represented as arrows. One of the key advantages of Co-plots is that they preserve the relative distance between DMUs in multidimensional space while transforming them to plain, i.e., proximity of DMUs in Coplot indicates how similar those DMUs are. Similar presentation techniques could be used also for analyses connected to point (7). Finally, point (8) is closely related to the previous points and it is context dependent, so we just point out here that we should use insight from presentation of the previous parts to guide us what actions/recommendations are feasible. Besides the aforementioned, we also fully support a reproducible approach to research, so we recommend making the data used for the modelling available, if not restricted by confidentiality agreements and licensing policies, so the presented results could be independently verified by their recipient.

Before concluding our paper, let us briefly discuss, for completeness, also two problems that may be of interest when applying DEA, ranking of DMUs and analysis of effectiveness over time. When we, in points (5) and (6), identify efficient and inefficient DMUs and compute their efficiency scores, we may not be interested in identifying how their performance could be improved, but our goal could be a ranking of the DMUs with respect to their efficiency. The problem is that more often than not multiple DMUs are seen as efficient, i.e., they attain the best possible efficiency score and thus are incomparable. To resolve this problem, we can use one of so-called super-efficiency methods which allows us to achieve a complete ranking. An example of such method can be found in (Andersen and Petersen, 1993). Regardless if we are interested in ranking of DMUs or in some recommendations how to improve their efficiency, we may be interested how technical efficiency of the DMUs changes and evolves over time with changing economic or technological conditions. In such cases dynamic DEA can be applied. The traditional tools that allow evaluation of the development of technical efficiency over time are window analysis (Klopp, 1985) and the Malmquist Index (MI). While the window analysis is based on the assumption of a constant level of technology and applies so called moving averages, MI includes in the concept of measuring changes in efficiency also a technological progress. The concept of MI was first introduced by Malmquist (1953) and has been further developed in the non-parametric framework by Caves et al. (1982), Färe et al. (1992), Thrall (2000), and Zofio (2007). MI is an index representing Total Factor Productivity (TFP) change of a DMU and in that it reflects both a change in technical efficiency as well as a change of the frontier technology between two periods of time.

CONCLUSION

We present here a guideline researcher can use to navigate through individual steps of applications of the DEA models. For each modelling phase, some general principles and issues as well as main approaches are outlined. We also stressed some of the problems which can encountered during the analysis alongside with some recommendations how to proceed when resolving them. Although we aimed for being as concise as possible, all argumentation is supported with quite exhaustive list of references which can be used as a starting point if further details regarding a particular modelling phase, model or issue need to be retrieved. Nevertheless, we plan to provide a more detailed treatise of some important issues connected to applications of DEA in the future and also provide some software solution which can help researcher in selection of the appropriate models in the context of principles and issues outlined in the paper.

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