Economic Growth, Business Cycles and Okun's Law: Unobserved Components Approach

Andrea Čížků [1] | *University of Economics, Prague, Czech Republic*

Abstract

Clark's (1989) bivariate unobserved components model is applied in order to estimate and analyse the trend and cycle of GDP and the unemployment rate as well as to quantify and discuss the relationship known as Okun's law. Empirical analysis is performed for 28 European countries for a time period including the current economic crisis – the end period is 2018 Q4 for all economies and the beginning period ranges from 1983 Q1 to 2000 Q1 according to data availability. Important results indicate that in virtually all European countries: (1) the growth of the trend component of GDP decreased systematically after the crisis; (2) the output gap improved in the last five years – this finding proved to be quite robust as it was also confirmed by Hodrick-Prescott estimates of the output gap for different smoothing parameter values; (3) the trend component of the unemployment rate turned out to be constant over time, indicating that possible hysteresis effects have not played an important role in European labour markets; (4) the output gap and cyclical unemployment rate are highly negatively correlated, confirming the strength and validity of Okun's law across European countries.

Keywords	***JEL code***
Unobserved components, Clark's bivariate model, Okun's law, Kalman filter, HP filter	*C32, E24, E32*

INTRODUCTION

Economists concern themselves with unobserved features of an economy such as potential (trend) output or output gap (cycle) in order to separate long-run and short-run components of economic variables. This is usually motivated by the goal of dampening economic fluctuations by keeping output at its potential level. Trend-cycle decomposition also enables the assessment of long-term effects of the current economic crisis, which is a widely discussed issue in economic literature these days. Ball (2014) found empirical evidence supporting the hypothesis of permanent effects of deep recessions on output. Similar findings are reported by Barro (2001), Cerra and Saxena (2005, 2008) and Haltmaier (2012).

[1] University of Economics, Prague, Faculty of Informatics and Statistics, Department of Econometrics, W. Churchill Sq. 4, 130 67 Prague 3, Czech Republic. E-mail: andrea.cizku@vse.cz, phone: (+420)224095443. ORCID: 0000-0003-3053-3222.

Trend-cycle decomposition is usually performed in empirical literature for aggregated European data, such as by Azevedo et al. (2003), Berger and Everaert (2008), Berger (2011), Bernhofer et al. (2014), Chen and Mills (2012), Galati et al. (2016), Lemoine et al. (2010), Orlandi and Pichelmann (2000) and Proietti (2004).

It is also common for authors to decompose variables for only one specific economy or a small group of countries, e.g. Hájek and Bezděk (2000), Hájková and Hurník (2007), Kloudová (2013), and Beneš and N'Diaye (2004) for the Czech economy; Bernardi and Di Ruggiero (2014) for Italy; Boďa et al. (2015), and Boďa and Považanová (2019) for Visegrad group countries; Jemec (2012) for Slovenia; Volos and Hadjixenophontos (2015) for Cyprus; and Ochotnický (2008), and Zimková and Barochovský (2007) for Slovakia.

This paper contributes to existing literature by providing an extensive empirical investigation and comparison of 28 European economies taking into account their heterogeneity. The focus is placed on: (1) characterizing changes in the trend and cycle of GDP and the unemployment rate during the current crisis, which enables the assessment of the long-run and transient influence of the crisis on these two variables; (2) estimation and discussion of the relationship between cyclical components of GDP and the unemployment rate, known as Okun's law; (3) making a comparison of these features across individual European countries.

Primarily, the trend-cycle decomposition is performed by applying the unobserved components (UC) methodology. However, Hodrick and Prescott's (1997) (HP) filter with different smoothing parameter values is applied as a robustness check as well.

Such a vast empirical study has only been performed by Ball (2014), Ball et al. (2017), and Brůha and Polanský (2015). Nonetheless, these papers are methodologically based on simple methods such as the production function approach, HP filter or analysis of correlations.

There are various methods for decomposing economic variables. A survey of alternative methodologies was performed by Dupasquier et al. (1999). This paper applies structural time series modelling as advocated by Harvey (1989), which uses the state-space form and Kalman filtering in order to estimate unobserved components of time series. Specifically, Clark's (1989) unobserved components (UC) model is employed in order to explicitly model the trend and cycle of GDP and the unemployment rate. Harvey and Jaeger (1993) showed that this technique of detrending economic time series is superior to ARIMA modelling and to mechanical statistical tools like the HP filter.

The structure of the paper is as follows. The model is formulated in Section 1 and data are described in Section 2. Econometric estimation of parameters is discussed in Section 3, and the unobserved trend and cycle of GDP and the unemployment rate are presented in Section 4. Subsequently, Section 5 compares the calculated output gap with that obtained using the HP filter. The final section summarizes the main findings and concludes.

1 MODEL

A minor modification of Clark's (1989) bivariate UC model as specified by Kim and Nelson (1999) is applied in this paper in order to study GDP and unemployment in European countries. This model extends the univariate model of GDP decomposition formulated and estimated for the US economy by Clark (1987), which is given as follows:

$$y_t = n_t + x_t, \tag{1}$$

$$n_t = g_{t-1} + n_{t-1} + v_t, \; v_t \sim i.i.d.\; N\left(0, \sigma_v^2\right), \tag{2}$$

$$g_t = g_{t-1} + w_t, \; w_t \sim i.i.d.\; N\left(0, \sigma_w^2\right), \tag{3}$$

$$x_t = \phi_1 \cdot x_{t-1} + \phi_2 \cdot x_{t-2} + e_t,\ e_t \sim i.i.d.\ N\left(0, \sigma_e^2\right), \tag{4}$$

where y_t is the log of real GDP, n_t represents the stochastic trend component evolving according to a random walk process with drift g_t, which is itself the random walk, and x_t is a stationary cyclical component (output gap). Second-order autoregression for x_t allows the output gap to exhibit cyclical movements. It is also possible to derive AR(2) specification for the gap from a standard model of cyclical components based on sine and cosine functions (Orlandi and Pichelmann, 2000).

Clark's (1987) formulated univariate UC model given by Formulas (1)–(4) was extended by Clark (1989) and later also by Kim and Nelson (1999) by including the unemployment rate U_t in the following fashion:[2]

$$U_t = L_t + C_t, \tag{5}$$

$$L_t = L_{t-1} + \varepsilon_t,\ \varepsilon_t \sim i.i.d.\ N\left(0, \sigma_\varepsilon^2\right), \tag{6}$$

$$C_t = \alpha_0 \cdot x_t + \alpha_1 \cdot x_{t-1} + \eta_t,\ \eta_t \sim i.i.d.\ N\left(0, \sigma_\eta^2\right), \tag{7}$$

where L_t is the trend of the unemployment rate, C_t represents the cyclical stationary component of the unemployment rate, and ε_t and η_t are white noise processes. Random errors v_t, w_t, e_t, ε_t and η_t in the formulated model (1)–(7) are all not only assumed serially but also mutually independent.

Formula (7) is interpreted as Okun's law postulating a negative relation between the output gap and the unemployment rate gap. It corresponds to Clark's (1989) formulation in that it is assumed that only the current output gap x_t and the output gap lagged one period x_{t-1} have an influence on the cyclical unemployment rate C_t. The same specification was also applied by Berger (2011). Kim and Nelson (1999) also included the output gap lagged two periods x_{t-2} in Okun's law reflecting that the unemployment rate is a lagging indicator of the business cycle. Nonetheless, this specification turned out to be unsuitable for many European countries as it caused higher standard errors of many estimated parameters (not only relating to Okun's law).

Firstly, it is worth mentioning that output and unemployment rate are modelled simultaneously, not separately. The output decomposition model (1)-(4) is interconnected with the unemployment rate model (5)-(7) by Okun's law in the form of Formula (7). Estimation of unobserved cyclical components of the output and unemployment rate therefore utilizes this structural relationship between these two variables. Secondly, the applied specification of Okun's law (7) postulates that the output gap influences the cyclical unemployment rate so that assumed causality goes from output to unemployment, which is a lagging indicator. This is a slight modification of Okun's (1962) empirical investigation where he never postulated this one-way causality.

2 DATA

Seasonally adjusted quarterly GDP and unemployment rate data for 28 European economies were obtained from Eurostat (2018). GDP is measured in chain-linked volumes (reference year 2010, million euro) and the name of the series in the Eurostat database is "GDP and main components (output, expenditure and income) [namq_10_gdp]". Unemployment rate includes all ages of unemployed workers and relates

[2] Clark's (1989) extension slightly differs from that performed by Kim and Nelson (1999). Clark (1989) specifies the GDP trend component as a random walk without a drift term . Kim and Nelson (1999) follow Clark's (1987) original specification (1)-(4) in their extension to the bivariate model, which is an approach taken in this paper as well.

the number of unemployed persons to the active population (expressed as a number between 0 and 1, not in per cent), and the title of the source data in Eurostat is "Unemployment by sex and age – quarterly average [une_rt_q]". The observable variable y_t is represented by the natural logarithm of the GDP, and the variable U_t corresponds directly to the unemployment rate.

The last observation in the data set is 2018 Q4 for all 28 countries. However, the first observation differs in individual European economies due to data availability and starting period of the data for countries ranges from 1983 Q1 to 2000 Q1. The starting date for each economy is indicated in the following Table 1, which contains estimation results (placed in Section 3) in the first column below the country name.

3 ECONOMETRIC ESTIMATION AND DISCUSSION

The bivariate UC model (1)–(7) was expressed in state-space form and econometrically estimated by maximizing the likelihood function, which was performed numerically in MATLAB. The likelihood function was constructed by applying the Kalman filter algorithm, which was implemented in its square root version in order to achieve better numerical precision. Kalman filtering was initialized by standard method – unconditional mean and variance were used as initial values for stationary cyclical components and a diffuse prior was applied for non-stationary trend components. Parameter transformations ensuring non-negativity of standard deviations of random errors and stationarity of autoregressive process (4) were also performed.

Results for 28 European countries are summarized in the following Table 1. Standard errors of estimated coefficients are shown in parentheses below estimated parameters. For readers' convenience, the symbols ***, ** and * are used as well in order to indicate statistical significance at the 1%, 5% and 10% level, respectively. Standard deviations of random shocks are multiplied by 100 in order to obtain more readable results. Alternatively, data for the log of GDP and unemployment rate could be multiplied by 100 in order to increase estimated standard errors of random shocks 100 times.

Table 1 Estimation results of the bivariate UC model (1)–(7) for European countries

Country (start date)[3]	GDP					Unemployment rate			
	$\hat{\phi}_1$	$\hat{\phi}_2$	$100\hat{\sigma}_e$	$100\hat{\sigma}_w$	$100\hat{\sigma}_v$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$100\hat{\sigma}_\varepsilon$	$100\hat{\sigma}_\eta$
Austria	1.53***	−0.58***	0.37***	0.04	0.31***	0.05	−0.34***	0.16***	0.09***
(1996 Q1)	(0.13)	(0.13)	(0.09)	(0.03)	(0.09)	(0.09)	(0.12)	(0.04)	(0.03)
Belgium	1.46***	−0.54***	0.39***	0.03	0.00	−0.05	−0.22**	0.29***	0.18***
(1995 Q1)	(0.14)	(0.14)	(0.09)	(0.02)	(5.30)	(0.10)	(0.10)	(0.04)	(0.04)
Bulgaria	1.77***	−0.78***	0.21**	0.13**	0.76***	−0.11	−1.11	0.34**	0.01
(2000 Q1)	(0.09)	(0.09)	(0.08)	(0.06)	(0.07)	(0.78)	(0.88)	(1.16)	(0.16)
Croatia	1.79***	−0.80***	0.24**	0.11	1.13***	−0.52	−0.42	0.02	0.09
(2000 Q1)	(0.08)	(0.09)	(0.10)	(0.07)	(0.10)	(0.59)	(0.52)	(0.90)	(0.07)
Cyprus	1.81***	−0.82***	0.35***	0.09*	0.67***	−0.32	−0.38	0.34***	0.15
(2000 Q1)	(0.09)	(0.09)	(0.11)	(0.05)	(0.08)	(0.32)	(0.31)	(0.12)	(0.10)
Czech Rep.	1.76***	−0.77***	0.38***	0.28***	0.30***	−0.18*	−0.36***	0.09	0.07
(1996 Q1)	(0.09)	(0.09)	(0.08)	(0.06)	(0.06)	(0.09)	(0.12)	(0.11)	(0.05)
Denmark	1.68***	−0.71***	0.31***	0.00	0.76***	0.17	−0.68**	0.00	0.07
(1995 Q1)	(0.11)	(0.11)	(0.08)	(0.11)	(0.07)	(0.24)	(0.29)	(2.91)	(0.05)
Estonia	1.66***	−0.69***	0.94***	0.09	1.51***	−0.13	−0.42**	0.42	0.37***
(2000 Q1)	(0.12)	(0.12)	(0.25)	(0.08)	(0.19)	(0.18)	(0.19)	(0.28)	(0.13)
Finland	1.82***	−0.83***	0.40***	0.05	1.04***	0.03	−0.62***	0.05	0.00
(1990 Q1)	(0.06)	(0.06)	(0.08)	(0.04)	(0.08)	(0.18)	(0.22)	(0.25)	(0.63)
France	1.78***	−0.79***	0.20***	0.04**	0.29***	−0.01	−0.45***	0.10***	0.00
(1983 Q1)	(0.06)	(0.06)	(0.03)	(0.02)	(0.02)	(0.10)	(0.12)	(0.02)	(0.16)
Germany	1.70***	−0.71***	0.33***	0.09**	0.64***	−0.18**	−0.25***	0.06	0.00
(1991 Q1)	(0.09)	(0.09)	(0.06)	(0.04)	(0.05)	(0.08)	(0.09)	(0.06)	(0.11)

Table 1 (continuation)

Country (start date)[3]	GDP					Unemployment rate			
	$\hat{\phi}_1$	$\hat{\phi}_2$	$100\hat{\sigma}_e$	$100\hat{\sigma}_w$	$100\hat{\sigma}_v$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$100\hat{\sigma}_\varepsilon$	$100\hat{\sigma}_\eta$
Greece	1.85***	−0.86***	0.37***	0.12*	1.11***	−0.87*	0.07	0.01	0.11
(1998 Q2)	(0.07)	(0.07)	(0.10)	(0.07)	(0.10)	(0.47)	(0.39)	(2.41)	(0.09
Hungary	1.78***	−0.80***	0.11	0.22***	0.67***	−0.41	−1.08	0.20**	0.01
(1996 Q1)	(0.12)	(0.12)	(0.08)	(0.06)	(0.06)	(1.62)	(1.97)	(0.10)	(0.41)
Ireland	1.80***	−0.81***	0.19	0.00	2.82***	−1.34	−0.01	0.00	0.15**
(1995 Q1)	(0.09)	(0.09)	(0.17)	(0.57)	(0.21)	(3.40)	(2.83)	(10.87)	(0.08)
Italy	1.78***	−0.80***	0.31***	0.00	0.45***	−0.08	−0.28**	0.20***	0.05
(1995 Q1)	(0.09)	(0.09)	(0.07)	(0.11)	(0.05)	(0.14)	(0.14)	(0.04)	(0.06)
Latvia	1.69***	−0.71***	0.83***	0.21**	1.19***	−0.38**	−0.23	0.19	0.37***
(1998 Q2)	(0.10)	(0.10)	(0.20)	(0.09)	(0.15)	(0.17)	(0.14)	(0.46)	(0.09)
Lithuania	1.74***	−0.75***	0.73***	0.07	1.61***	−0.41*	−0.23	0.01	0.28***
(1998 Q1)	(0.09)	(0.09)	(0.17)	(0.08)	(0.15)	(0.22)	(0.19)	(8.00)	(0.09)
Luxembourg	1.64***	−0.66***	0.21	0.05	1.61***	−0.65	0.03	0.01	0.03
(1995 Q1)	(0.19)	(0.19)	(0.15)	(0.07)	(0.12)	(0.88)	(0.58)	(0.79)	(0.11)
Malta	0.45	0.53*	0.82*	0.13*	1.05***	−0.03	−0.07	0.33***	0.09
(2000 Q1)	(0.32)	(0.29)	(0.49)	(0.08)	(0.37)	(0.10)	(0.11)	(0.05)	(0.08)
Netherlands	1.86***	−0.86***	0.19***	0.03	0.50***	0.17	−0.69***	0.00	0.03
(1996 Q1)	(0.06)	(0.06)	(0.04)	(0.03)	(0.04)	(0.23)	(0.26)	(0.97)	(0.04)
Norway	1.67***	−0.69***	0.13	0.03	1.16***	0.39	−1.20	0.07	0.00
(1989 Q1)	(0.14)	(0.15)	(0.09)	(0.04)	(0.08)	(0.65)	(1.05)	(0.14)	(0.48)
Poland	1.78***	−0.79***	0.11*	0.00	0.95***	−3.40	0.57	0.00	0.00
(1997 Q1)	(0.10)	(0.10)	(0.08)	(0.18)	(0.07)	(4.58)	(3.32)	(0.20)	(0.02)
Portugal	1.76***	−0.77***	0.31***	0.07**	0.52***	−0.23	−0.54**	0.17	0.10
(1995 Q1)	(0.09)	(0.09)	(0.07)	(0.03)	(0.05)	(0.19)	(0.21)	(0.11)	(0.06)
Romania	0.90**	0.06	0.30	0.25**	1.57***	−0.96	−0.36	0.00	0.00
(1997 Q1)	(0.37)	(0.32)	(0.36)	(0.11)	(0.15)	(2.26)	(0.79)	(31.19)	(3.29)
Slovenia	1.75***	−0.76***	0.52***	0.07	0.71***	0.04	−0.31**	0.25***	0.14**
(1996 Q1)	(0.10)	(0.11)	(0.13)	(0.05)	(0.10)	(0.12)	(0.13)	(0.06)	(0.05)
Spain	1.88***	−0.88***	0.21***	0.09***	0.13***	−1.12***	0.01	0.28***	0.00
(1995 Q1)	(0.05)	(0.05)	(0.03)	(0.03)	(0.03)	(0.30)	(0.23)	(0.07)	(1.06)
Sweden	1.61***	−0.65***	0.34***	0.02	0.72***	0.05	−0.55***	0.00	0.08**
(1993 Q1)	(0.07)	(0.08)	(0.07)	(0.03)	(0.06)	(0.12)	(0.16)	(1.86)	(0.04)
UK	1.82***	−0.83***	0.24***	0.05**	0.44***	−0.11	−0.43***	0.10***	0.00
(1983 Q1)	(0.06)	(0.06)	(0.04)	(0.02	(0.03)	(0.11)	(0.13)	(0.04)	(0.21)

Source: Authors own calculations based on data obtained from Eurostat (2018)

Table 1 contains detailed estimation results for all 28 studied economies, which enables differences between individual European countries to be assessed. Nonetheless, some sort of aggregation by using median values to characterize a typical European economy will often be utilized in subsequent discussion for two reasons. Firstly, it facilitates the reader's basic orientation in these extensive results. Secondly, it enables a comparison with findings of other empirical studies analysing aggregated European data.

3.1 Parameters relating to GDP decomposition

3.1.1 Autoregressive parameters ϕ_1 and ϕ_2

Parameters ϕ_1 and ϕ_2 are statistically significant at the 1% level of significance with the only exceptions being Malta and Romania. The estimated parameters $\hat{\phi}_1 = 0.90$ and $\hat{\phi}_2 = 0.06$ for Romania resemble

[3] The last observation in the sample is 2018 Q4 for every country.

closely a stationary AR(1) process. Estimates for Malta are quite atypical, which applies not only for parameters ϕ_1 and ϕ_2.

Another substantially robust finding is that the sum $\hat{\phi}_1 + \hat{\phi}_2$ is close to one (except for Malta and Romania). This means that the output gap x_t is highly persistent. Clark (1989) reports the following results:

a) $\hat{\phi}_1 = 1.47$, $\hat{\phi}_2 = -0.59$ for the US economy (1947 Q1–1986 Q2),

b) $\hat{\phi}_1 = 1.59$, $\hat{\phi}_2 = -0.63$ for Canada (1955 Q1–1986 Q2),

c) $\hat{\phi}_1 = 1.73$, $\hat{\phi}_2 = -0.73$ for West Germany (1960 Q1–1986 Q2),

d) $\hat{\phi}_1 = 1.82$, $\hat{\phi}_2 = -0.82$ for the United Kingdom (1960 Q1–1986 Q2),

e) $\hat{\phi}_1 = 0.54$, $\hat{\phi}_2 = 0.20$ for Japan (1960 Q1–1986 Q2).

The sum of these parameters estimated for the US economy, $\hat{\phi}_1 + \hat{\phi}_2 = 0.88$, is not as close to 1 as in most European countries. This is also confirmed by Kim and Nelson's (1999) estimates for the US economy (1952 Q1–1995 Q3): $\hat{\phi}_1 = 1.44$, $\hat{\phi}_2 = -0.52$. When excluding Malta and Romania, the median value of $\hat{\phi}_1 + \hat{\phi}_2$ from Table 1 is 0.99. The output gap in virtually all European countries is thus remarkably more persistent than the corresponding gap in the US economy. It is also confirmed that estimates presented in Table 1 for Germany ($\hat{\phi}_1 = 1.70$, $\hat{\phi}_2 = -0.71$) and for the United Kingdom ($\hat{\phi}_1 = 1.82$, $\hat{\phi}_2 = -0.83$) are very close to those reported by Clark (1989).

Similar results regarding the persistence of Visegrad group countries were found by Boďa and Považanová (2019). Berger's (2011) estimates for aggregated European data from 1970 to 2005 are also in line with these conclusions as he estimated these parameters as $\hat{\phi}_1 = 1.88$ and $\hat{\phi}_2 = -0.91$ (with standard errors of 0.05 and 0.04, respectively).

These findings regarding persistence are also supported by Chen and Mills (2012), who analysed aggregate euro area data ranging from 1970 Q1 to 2009 Q2. Nonetheless, some comments are needed before we report their results as they did not model the cyclical component of output directly as the AR(2) process. Instead, they modelled the cyclical component (ψ_t in their notation) using sine and cosine functions as follows:

$$\begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix}, \tag{8}$$

where κ_t and κ_t^* are mutually and serially independent random errors with $Var(\kappa_t) = Var(\kappa_t^*) = \sigma_\kappa^2$, $0 \le \rho < 1$ is the damping parameter and λ_c is the cycle frequency (in radians). Orlandi and Pichelmann (2000) showed that specification (8) is equivalent to the following AR(2) model:

$$\psi_t = (2\rho\cos\lambda_c)\psi_{t-1} - \rho^2\psi_{t-1} + \zeta_t, \tag{9}$$

where random error ζ_t is given by:

$$\zeta_t = (1 - \rho\cos\lambda_c L)\kappa_t - (\rho\sin\lambda_c L)\kappa_t^*, \tag{10}$$

where L is the lag operator.

Formula (9) is already comparable to specification (4). Chen and Mills (2012) estimate parameter ρ and the period of the cycle $2\pi / \lambda_c$ for aggregate euro area data as $\hat{\rho} = 0.82$ and $2\pi / \hat{\lambda}_c = 35.34$. This implies that $\hat{\lambda}_c = 0.18$, $\hat{\phi}_1 = 2\hat{\rho}\cos\hat{\lambda}_c = 1.61$ and $\hat{\phi}_2 = -\hat{\rho}^2 = -0.67$. This result is somewhere between Clark's (1989) estimate for the US economy and Berger's (2011) estimate for Europe.

3.1.2 Volatility of output gap σ_e

The random error e_t in Formula (4) describing the development of the output gap plays an important role as the coefficient σ_e is statistically significant in the majority of studied countries. This parameter failed to be statistically significant only for Hungary, Ireland, Luxembourg, Norway and Romania. The output gap is thus stable in these economies.

Chen and Mills (2012) estimated that $100\hat{\sigma}_\kappa = 0.27$ and it follows from that:

$$\begin{aligned} Var(\zeta_t) &= \sigma_\kappa^2 + \rho^2 \cos^2 \lambda_c \sigma_\kappa^2 + \rho^2 \sin^2 \lambda_c \sigma_\kappa^2, \\ &= \sigma_\kappa^2 \left(1 + \rho^2\right). \end{aligned}$$

Correspondence between (9) and (4) then yields $\sigma_e^2 = Var(\zeta_t)$. Therefore, $100\hat{\sigma}_e = 100\hat{\sigma}_\kappa \cdot \sqrt{1 + \hat{\rho}^2} = = 0.27 \cdot \sqrt{1 + 0.82^2} = 0.35$. This value corresponds very closely with the results presented in Table 1 where the median of estimates $100\hat{\sigma}_e$ for individual economies is 0.31. Nonetheless, Table 1 also shows that there are considerable differences across individual countries as the estimated standard error $100\hat{\sigma}_e$ varies from 0.11 in Poland to 0.94 in Estonia.

Berger's (2011) estimate of volatility of the GDP cyclical component is $100\hat{\sigma}_e = 0.09$ (with a standard error of this estimate of 0.02). This is a slightly lower value than those presented in Table 1, which is probably caused by the fact that Berger worked with pre-crisis data.

Clark (1989) estimated the standard deviation of the random error in the equation for the output gap for the US economy as $100\hat{\sigma}_e = 0.73$. His estimate is more than two times higher than the median of the corresponding estimates for European countries presented in Table 1 and also than the value calculated above using the results reported by Chen and Mills (2012). Nonetheless, this does not mean that the variability of the output gap in the US economy is significantly higher than in Europe as the output gap in the USA is less persistent, as discussed above.

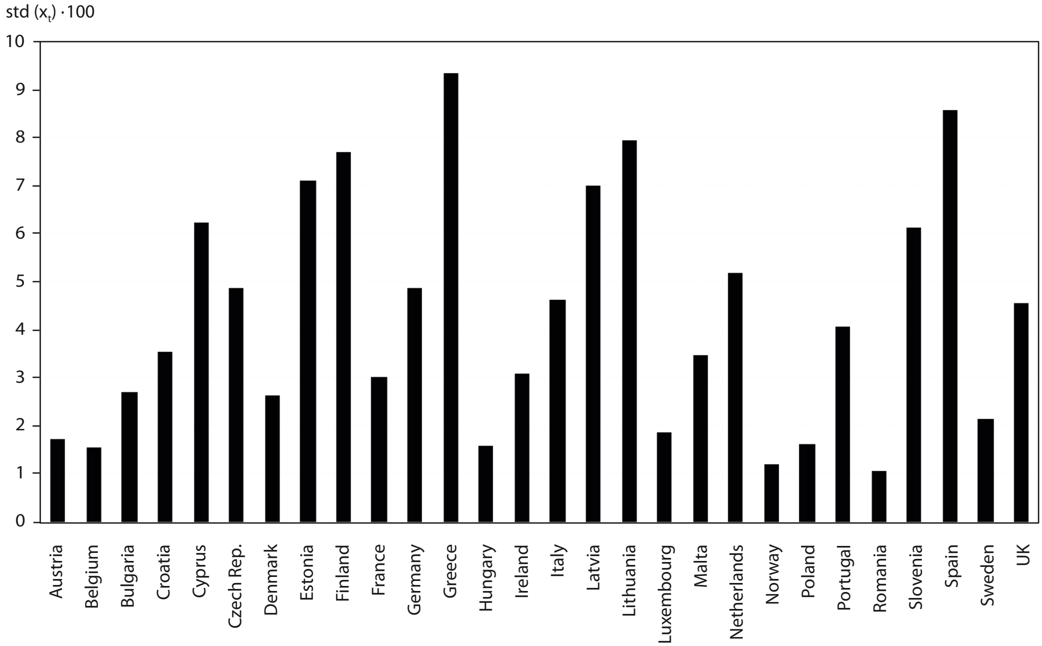
Output gap volatility might be measured by unconditional standard deviation of the AR(2) process (4), which is given by:

$$std(x_t) = \sqrt{\frac{(1 - \phi_2) \cdot \sigma_e^2}{(1 + \phi_2) \cdot \left[(1 - \phi_2)^2 - \phi_1^2\right]}}. \tag{11}$$

Estimate of $std(x_t)$ is obtained by substituting the corresponding estimates $\hat{\phi}_1$, $\hat{\phi}_2$ and $\hat{\sigma}_e^2$ into the relation (11). Results are depicted in Figure 1. Unconditional standard deviation is multiplied by 100 in order to obtain more readable results.

The three countries with the highest output gap volatility are Greece, Spain and Lithuania. A surprising finding is that a substantially high variability of the gap is also detected for Finland. Low output gap volatility is observed in Austria, Belgium, Hungary, Luxembourg, Norway, Poland, Romania and Sweden. This finding is especially surprising for Romania, which has the lowest output gap volatility among all 28 studied economies. The case of Romania is substantially specific. As will be seen in subsequent paragraphs, Romania also has one of the highest volatilities of the growth rate g_t of the GDP trend component as measured by $100\sigma_w$ and also an extremely high volatility of the trend component n_t as measured by $100\sigma_v$. Movements in real GDP are thus mostly permanent as they influence primarily the trend components of real output.

Figure 1 Output gap volatility as measured by *std* (x_t) · 100



Source: Authors own calculations based on data obtained from Eurostat (2018)

Output gap volatility in the USA might be obtained by plugging Clark's (1989) estimates $\hat{\phi}_1 = 1.47$, $\hat{\phi}_2 = -0.59$ and $100\hat{\sigma}_e = 0.73$ into relation , which yields $std\left(x_t\right) \cdot 100 = 2.37$. The volatility of the cyclical component of output in the US economy is thus substantially low, even though $100\hat{\sigma}_e = 0.73$ is rather high. As already mentioned, this result is a direct consequence of low output gap persistency in the USA.

3.1.3 Volatility of the growth rate g_t of the GDP trend component (measured by $100\sigma_w$)

In contrast to the previous results obtained for parameter σ_e, the coefficient σ_w is statistically significant (at least at the 10% level of significance) only for approximately half of the studied countries – Bulgaria, Cyprus, the Czech Republic, France, Germany, Greece, Hungary, Latvia, Malta, Portugal, Romania, Spain and the United Kingdom. The growth rate of the trend component g_t is not constant over time in these countries as the standard deviation σ_w is significantly different from zero. Mostly, these are less developed European countries except for France, Germany and the United Kingdom, where $100\sigma_w$ is statistically different from zero, but attains considerably low values – France (0.04), Germany (0.09) and the United Kingdom (0.05). The highest values of $100\hat{\sigma}_w$ are detected for the Czech Republic (0.28), Hungary (0.22), Latvia (0.21) and Romania (0.25).

Literally a stable growth rate g_t of the GDP trend component (measured by $100\sigma_w$) is detected for Denmark (0.00), Ireland (0.00), Italy (0.00) and Poland (0.00). One might be tempted to interpret from these results that almost nothing will affect the growth rate of these economies in the long run – not even the economic crisis of 2008. Such an interpretation would of course be misleading as the long-run trend component of output is determined not only by $100\sigma_w$ but also by $100\sigma_v$ (to be discussed systematically later on). The most striking contrast between these two measures is observed for Ireland ($100\hat{\sigma}_w = 0$ is the lowest value among the studied economies and at the same time $100\hat{\sigma}_v = 2.82$ is the highest value among all 28 countries). A similar situation is also detected for Denmark and Poland.

Very low estimates of $100\hat{\sigma}_w$ are also obtained for Belgium (0.03), the Netherlands (0.03), Norway (0.03) and Sweden (0.02). These estimates correspond closely to the result reported by Kim and Nelson (1999) for the US economy ($100\hat{\sigma}_w = 0.03$, 0.02 being the standard error of this estimate) and by Chen and Mills (2012) for aggregated euro area data ($100\hat{\sigma}_w = 0.02$, with a standard error of 0.01). Their estimates are slightly lower than the results presented in this paper as the median of $100\hat{\sigma}_w$ for the 28 analysed European economies is 0.07.

3.1.4 Volatility of GDP trend component n_t (measured by $100\sigma_v$)

The coefficient σ_v measuring the volatility of the trend component is statistically significant at the 1% level in all countries except Belgium. The random error v_t influencing the GDP trend component n_t (without changing the growth rate g_t of this trend component) thus proved to have a highly significant effect in virtually all studied economies. Belgium is the only country where both standard deviations $100\hat{\sigma}_w = 0.03$ and $100\hat{\sigma}_v = 0.00$ describing the variability of the trend component of GDP are statistically insignificant and very low. The trend component of GDP is thus extremely stable in this country.

Berger (2011) estimated a similar UC model for aggregated European data from 1970 to 2005 and his estimate of $100\sigma_v$ is 0.45 (with a standard error of this estimate of 0.03). Kim and Nelson (1999) report their estimate for the US economy as $100\hat{\sigma}_v = 0.49$ (with a standard error of 0.06). These findings are slightly lower than the median 0.74 of estimates for the 28 individual countries presented in Table 1, which is probably caused by the fact that they did not analyse data during the huge economic crisis. The results in Table 1 also show that there are substantial differences in the volatility of the trend component of GDP (σ_v) across individual European countries.

The medians of the coefficients $100\hat{\sigma}_e$, $100\hat{\sigma}_w$ and $100\hat{\sigma}_v$ are 0.31, 0.07 and 0.74, respectively. A lower than median value for all three estimated parameters is observed for France, Italy, the Netherlands, Portugal and the United Kingdom. These countries show a stable development of the GDP trend n_t, and growth of the GDP trend g_t as well as the output gap x_t. The other extreme is economies with higher than median values for all three estimated parameters, which means that components n_t, g_t and x_t are rather volatile. This condition is satisfied for the following less developed countries: Estonia, Greece, Latvia, Lithuania, Malta and Slovenia.

3.2 Trend component of unemployment rate

The volatility of the trend component of the unemployment rate is measured by σ_ε. This parameter turned out to be significantly different from zero at least at the 5% level of significance for only about half of the studied economies: Austria, Belgium, Bulgaria, Cyprus, France, Hungary, Italy, Malta, Slovenia, Spain and the United Kingdom. This suggests that the variability of the unemployment rate trend is not as high as the volatility of the GDP trend component in quite a lot of European countries.

Berger (2011) estimated the coefficient $100\sigma_\varepsilon$ for the whole of Europe as 0.09 (with 0.01 being the standard error of this estimate). Clark (1989) reports the value for the US economy as $100\hat{\sigma}_\varepsilon = 0.17$. Empirical analysis performed in this paper shows that the estimate for a typical European countryis quite close to Berger's result as the median of estimates for 28 economies is equal to 0.10.

3.3 Okun's law

As far as the relation between output gap and cyclical unemployment rate is concerned, estimated parameters $\hat{\alpha}_0$ and $\hat{\alpha}_1$ mostly have a negative sign. In some (rather rare) cases, an estimate of $\hat{\alpha}_0$ or $\hat{\alpha}_1$ turned out to be positive. Nonetheless, for Okun's law to hold, it is sufficient that the sum $\hat{\alpha}_0 + \hat{\alpha}_1$ of the estimated coefficients is negative. This condition is satisfied in all 28 studied economies, thereby confirming the validity of Okun's law.

Statistical insignificance (at the 10% level) of both parameters α_0 and α_1 is observed for 10 economies: Bulgaria, Croatia, Cyprus, Hungary, Ireland, Luxembourg, Malta, Norway, Poland and Romania. The Czech Republic and Germany stand on the other side with both parameters α_0 and α_1 being statistically significant (at least at the 10% level of significance).

These results might be improved if the coefficient α_1 is a priori set to zero. Only the contemporary output gap x_t is then assumed to have an influence on the unemployment rate gap (C_t). This approach is taken, for example, by Ball et al. (2017). Statistical insignificance of the parameter α_0 (at the 10% level) was in this case detected only for Luxembourg, Malta, Norway and Romania. Statistical significance of α_0 at the 10% level was observed for two countries (Croatia and Poland). Okun's law parameter α_0 was significant at the 5% level in the cases of Belgium, Bulgaria, the Czech Republic, Denmark and Hungary. In the remaining 17 economies it turned out that the coefficient α_0 was statistically significant even at the 1% level of significance. This finding confirms the validity and strength of Okun's law across individual European economies. Cyclical properties of the unemployment rate are thus driven to a great extent by the output gap.

The output gap lagged one period (x_{t-1}) has a stronger negative influence on the unemployment rate than the current gap (x_t) in 20 of the monitored economies. Thus, only in eight cases is the reverse true (Greece, Ireland, Latvia, Lithuania, Luxembourg, Poland, Romania and Spain). This confirms that the unemployment rate is a lagging indicator (its current values depend mainly on the lagged output gap).

The estimated parameters of Okun's law are in line with other empirical studies for most countries. Kim and Nelson (1999) estimated Okun's law in a slightly modified form to specification (7) as they assumed that not only the current and one-period lagged output gap (x_t and x_{t-1}) but also the output gap lagged two periods x_{t-2} have an influence on the unemployment rate gap C_t. They report the following estimates (standard errors are in parentheses): $\hat{\alpha}_0 = -0.34\ (0.06)$, $\hat{\alpha}_1 = -0.16\ (0.03)$ and $\hat{\alpha}_2 = -0.07\ (0.01)$. The overall effect of the output gap on the gap of unemployment rate is thus $\hat{\alpha}_0 + \hat{\alpha}_1 + \hat{\alpha}_2 = -0.57$. Clark's (1989) estimates for the US economy are $\hat{\alpha}_0 = -0.33$ and $\hat{\alpha}_1 = -0.18$ with the overall effect $\hat{\alpha}_0 + \hat{\alpha}_1 = -0.51$. The median value for such an overall effect calculated from Table 1 is $median\left(\hat{\alpha}_0 + \hat{\alpha}_1\right) = -0.60$, which is quite close to the estimate obtained by Kim and Nelson (1999).

Mankiw (2012) posited that for the US economy a one per cent deviation of output from potential causes an opposite change in the unemployment rate of half a percentage point. This assertion is practically the same as Clark's (1989) estimate of $\hat{\alpha}_0 + \hat{\alpha}_1 = -0.51$ and also corresponds closely to the median value found by empirical investigation in this paper for 28 European economies.

Ball et al. (2017) estimated Okun's law (by ordinary least squares using annual data from 1980 to 2011) in the form:

$$U_t - U_t^* = \beta \cdot \left(y_t - y_t^*\right) + \varepsilon_t, \tag{12}$$

where U_t is the unemployment rate and y_t is (the log of) GDP, U_t^* and y_t^* are trend estimates obtained using the Hodrick-Prescott filter and ε_t is i.i.d. random error.

In the long run, the coefficient β is equivalent to $\alpha_0 + \alpha_1$ in the presented model (7). Furthermore, Clark's bivariate model (1)-(7) was also estimated here using the a priori assumption $\alpha_1 = 0$ in order to obtain results more comparable with regression (12). Ball et al. (2017) econometrically analysed relationship (12) for the US economy and for 20 advanced OECD countries. Comparison with estimates presented earlier in Table 1 (for countries analysed in this paper as well as by Ball et al., 2017) is summarized in the following Table 2. Standard errors of estimated coefficients are again shown in parentheses below estimated parameters, and the symbols ***, ** and * indicate statistical significance at the 1%, 5% and 10% level, respectively.

Table 2 Comparison of results reported by Ball et al. (2017) ($\hat{\beta}$ in the first row) with findings of this paper ($\hat{\alpha}_0 + \hat{\alpha}_1$ and $\hat{\alpha}_0$ in the second and third row)

Country	Austria	Belgium	Denmark	Finland	France	Germany	Ireland	Italy
$\hat{\beta}$	–0.14*** (0.04)	–0.51*** (0.08)	–0.43*** (0.05)	–0.50*** (0.05)	–0.37*** (0.04)	–0.37*** (0.06)	–0.41*** (0.04)	–0.25*** (0.07)
$\hat{\alpha}_0 + \hat{\alpha}_1$	–0.29*** (0.07)	–0.27*** (0.09)	–0.51*** (0.15)	–0.59*** (0.09)	–0.46*** (0.06)	–0.43*** (0.09)	–1.35 (1.20)	–0.36*** (0.07)
$\hat{\alpha}_0$ $(\alpha_1 = 0)$	–0.26*** (0.09)	–0.19** (0.08)	–0.76** (0.31)	–0.69*** (0.14)	–0.55*** (0.14)	–0.62*** (0.15)	–0.51*** (0.18)	–0.40*** (0.09)

Country	Netherlands	Norway	Portugal	Spain	Sweden	UK
$\hat{\beta}$	–0.51*** (0.07)	–0.29*** (0.04)	–0.27*** (0.04)	–0.85*** (0.05)	–0.52*** (0.07)	–0.34*** (0.05)
$\hat{\alpha}_0 + \hat{\alpha}_1$	–0.52*** (0.09)	–0.81 (0.61)	–0.77*** (0.15)	–1.11*** (0.16)	–0.50*** (0.10)	–0.54*** (0.08)
$\hat{\alpha}_0$ $(\alpha_1 = 0)$	–0.59*** (0.13)	–1.13 (1.10)	–0.78*** (0.18)	–1.39*** (0.26)	–0.75*** (0.18)	–0.61*** (0.12)

Source: Authors own calculations based on data obtained from Eurostat (2018)

Firstly, Table 2 shows that Okun's law estimates $\hat{\alpha}_0 + \hat{\alpha}_1$ are slightly higher in absolute value than the corresponding values $\hat{\beta}$ reported by Ball et al. (2017). The only notable exception is Belgium (and Sweden with very similar results: $\hat{\beta} = -0.52$ and $\hat{\alpha}_0 + \hat{\alpha}_1 = -0.50$). Secondly, even slightly higher (in absolute value) estimates were obtained in most studied cases when the a priori setting was $\alpha_1 = 0$. Exceptions are Austria ($\hat{\alpha}_0 + \hat{\alpha}_1 = -0.29$, $\hat{\alpha}_0 = -0.26$), Belgium ($\hat{\alpha}_0 + \hat{\alpha}_1 = -0.27$, $\hat{\alpha}_0 = -0.19$) and Ireland ($\hat{\alpha}_0 + \hat{\alpha}_1 = -1.35$, $\hat{\alpha}_0 = -0.51$). Thirdly, the results obtained in this paper confirm the validity and strength of Okun's law reported by Ball et al. (2017) as Okun's law parameters are statistically significant even at the 1% level in most studied cases (exceptions are Norway and partly Ireland). Fourthly, standard errors of parameters estimated in this paper ($\alpha_0 + \alpha_1$ and α_0) are generally higher than the corresponding standard deviations of coefficients estimated by Ball (β). This is most clearly seen in the case of Norway and also Ireland. This is caused by the fact that Ball et al. (2017) treat the unobserved output gap and unemployment rate gap ($y_t - y_t^*$, $U_t - U_t^*$) as observable variables.

Ball et al. (2017) argue that reasonable values for β (for the US economy) should lie in the interval $(-1.5;0)$, which is satisfied in all studied cases. The highest (in absolute value) estimate of β is reported by Ball for Spain ($\hat{\beta} = -0.85$). Ball explains this strong negative influence of the output gap on the gap of unemployment rate by the prevalence of temporary employment contracts in Spain. The highest (in absolute value) estimate of $\alpha_0 + \alpha_1$ is detected for Ireland ($\hat{\alpha}_0 + \hat{\alpha}_1 = -1.35$). Nonetheless, this estimate is not statistically significant. The second highest value is observed for Spain ($\hat{\alpha}_0 + \hat{\alpha}_1 = -1.11$), confirming Ball's results. His finding relating to the strength of Okun's law in Spain is also confirmed by the highest (in absolute value) estimate of $\hat{\alpha}_0 = -1.39$.

Boďa et al. (2015) estimate Okun's law for Visegrad group countries by applying autoregressive distributed lag (ARDL) methodology using quarterly data from 1998 Q1 to 2014 Q2. Their long-run multiplier is comparable to the estimate of the overall effect $\hat{\alpha}_0 + \hat{\alpha}_1$ calculated here. Comparison of their findings with results presented in this paper is summarized in Table 3. Parameter standard deviations as well as the symbols *, ** and *** are indicated as before.

Table 3 Comparison of the long-run multiplier reported by Boďa et al. (2015) (the first row in the table) with findings of this paper ($\hat{\alpha}_0 + \hat{\alpha}_1$ in the second and third row)

Country	Czech Republic	Hungary	Poland
Long-run multiplier	−0.12*** (0.03)	−0.06 (0.04)	−0.08 (0.09)
$\hat{\alpha}_0 + \hat{\alpha}_1$	−0.54*** (0.14)	−1.49 (1.25)	−2.83 (1.86)

Source: Authors own calculations based on data obtained from Eurostat (2018)

Table 3 shows that findings presented in this paper imply a stronger negative relationship between the output gap and the gap of unemployment rate for these three countries than results reported by Boďa et al. (2015). Estimates of the long-run multiplier obtained by Boďa are unusually small (in absolute value). This is the case especially of Hungary and Poland, where estimates $\hat{\alpha}_0 + \hat{\alpha}_1$ calculated by the author of this paper are conversely substantially high. One robust finding reported here as well as by Boďa is that Okun's parameters are statistically insignificant in Hungary and Poland. Nonetheless, further empirical investigation of Okun's law for Visegrad group countries will be necessary.

Many empirical studies analyse Europe as a whole and not individual countries. Orlandi and Pichelmann (2000) estimated Okun's law for aggregated European annual data for 1960–1998 in the form:

$$\frac{\left(y_t - y_t^*\right)}{y_t^*} \cdot 100 = -\alpha \cdot \left(U_t - U_t^*\right) + \varepsilon_t, \tag{13}$$

where the variables have a similar interpretation to that in regression (12) except that GDP y_t is not in logs.

Parameter α from (13) relates to parameter β in according to $\beta \approx -1/\alpha$.[4] Orlandi and Pichelmann (2000) report the value $\hat{\alpha} = 1.8$ implying $\hat{\beta} = -1/1.8 = -0.56$, which is a value typically found for most European economies by Ball et al. (2017) as well as by empirical investigation performed in this paper.

Berger and Everaert (2008) formulated a UC model and applied Bayesian econometric techniques for aggregated European and US data for the period 1970 Q1–2003 Q4. Specifically, they estimated an equation for Okun's law in a form similar to (13) with the analogical parameter for α estimated for Europe as 2.15, which yields $\hat{\beta} = -1/2.15 = -0.47$.

Berger (2011) works with a similar UC model to the one applied in this paper with a Phillips curve added to the model specification. Berger's (2011) analysis is based on aggregated European data over the period 1970–2005. His equation for Okun's law is the same as specification (7) and reports the following estimation result: $\hat{\alpha}_0 = -0.49$ and $\hat{\alpha}_1 = -0.10$.

Novák and Darmo (2019) estimate Okun's law in a disaggregated manner for individual European economies by applying panel data econometric methods. Unfortunately, they assume that the parameter relating output with unemployment is the same for all countries. Moreover, they work with Okun's law in a differenced form that is not directly comparable with the specification applied in this paper. Specifically, Novák and Darmo (2019) estimate the following panel data regression:

$$\Delta U_{it} = \alpha_i + \beta \cdot gy_{it} + \gamma \cdot \Delta U_{i,t-1} + \varepsilon_{it}, \tag{14}$$

where ΔU_{it} is the first difference (a year-on-year change) of the unemployment rate in country i, gy_{it} represents real GDP growth in country i and ε_{it} is random error.

[4] The relation $\beta \approx -1/\alpha$ applies only approximately for two reasons. Firstly, there is a slightly different definition of the output gap in Formulas (12) and (13). Secondly, interchanging dependent and independent variables in the regression for Okun's law does not produce algebraically equivalent regression estimates (Plosser and Schwert, 1979).

Formula (12) considered by Ball et al. (2017) might be expressed in a differenced form if we assume constant trend components $U_t^* = U^*$ and $y_t^* = y^*$. Differencing (12) then yields:

$$U_t - U_{t-1} = \beta \cdot \left(y_t - y_{t-1} \right) + \varepsilon_t^* . \tag{15}$$

A comparison of relations (14) and (15) reveals that coefficient β relates GDP to unemployment rate in both cases. Thus, the coefficient β from (14) is in this sense comparable to the parameter β from (12). Novák and Darmo (2019) used data between 2001 and 2014 and report the value $\hat{\beta} = -0.29$, which is also in line with results presented and discussed earlier in this paper.

Novák and Darmo (2019) also found structural breaks in regression (14). Nonetheless, Brůha and Polanský (2015) argue that instability is often found when using a difference form of Okun's law and found evidence that the relationship is stable when cyclical components of output and unemployment are used instead of differences. Similarly, Ball et al. (2017) estimated Okun's law in its gap form and state that Okun's law is a strong and stable relationship in most countries, one that did not change substantially during the Great Recession. Ball also found that coefficient β in Okun's law (12) varies substantially across countries, which is confirmed by the results presented in this paper as well. Therefore, the assumption of constancy of this parameter across individual economies introduced by Novák and Darmo (2019) is not satisfied.

4 TREND-CYCLE DECOMPOSITON

Smoothed estimates of the GDP cycle (x_t), growth of the GDP trend component (g_t) and finally the trend component of the unemployment rate (L_t) together with the observed unemployment rate (U_t) are illustrated for all 28 European economies in Figure 2.

Figure 2 Smoothed estimates of output gap x_t, growth g_t of the GDP trend component and the trend component of the unemployment rate L_t together with the observed unemployment rate

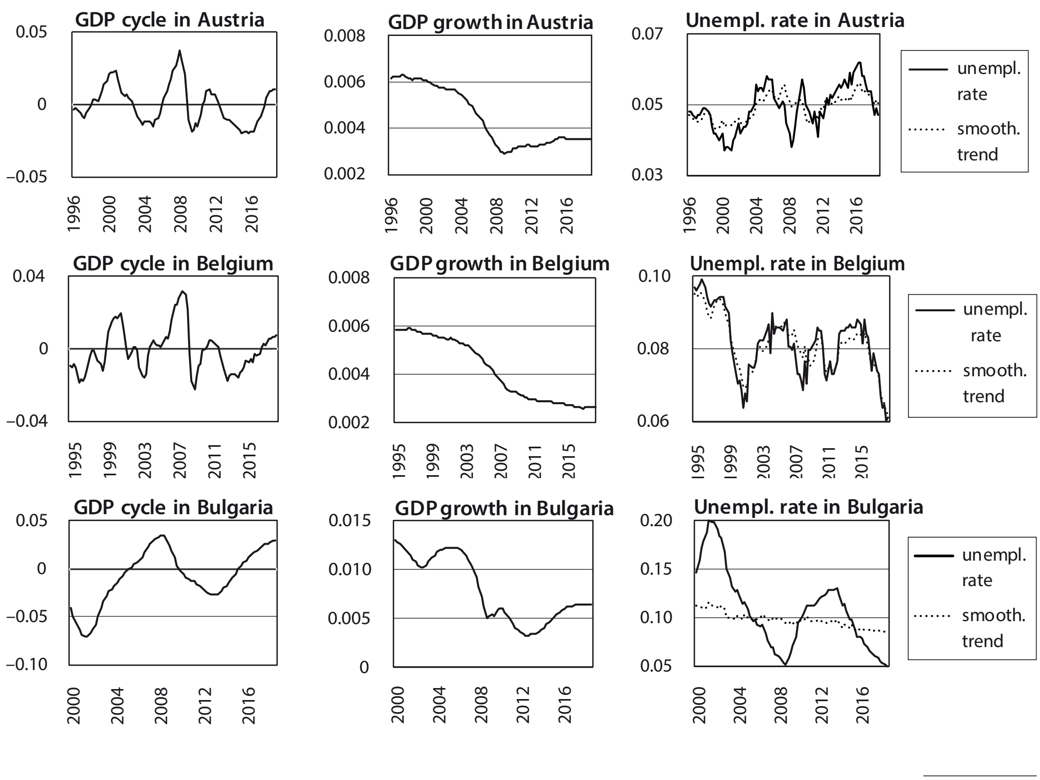


Figure 2 (continuation)

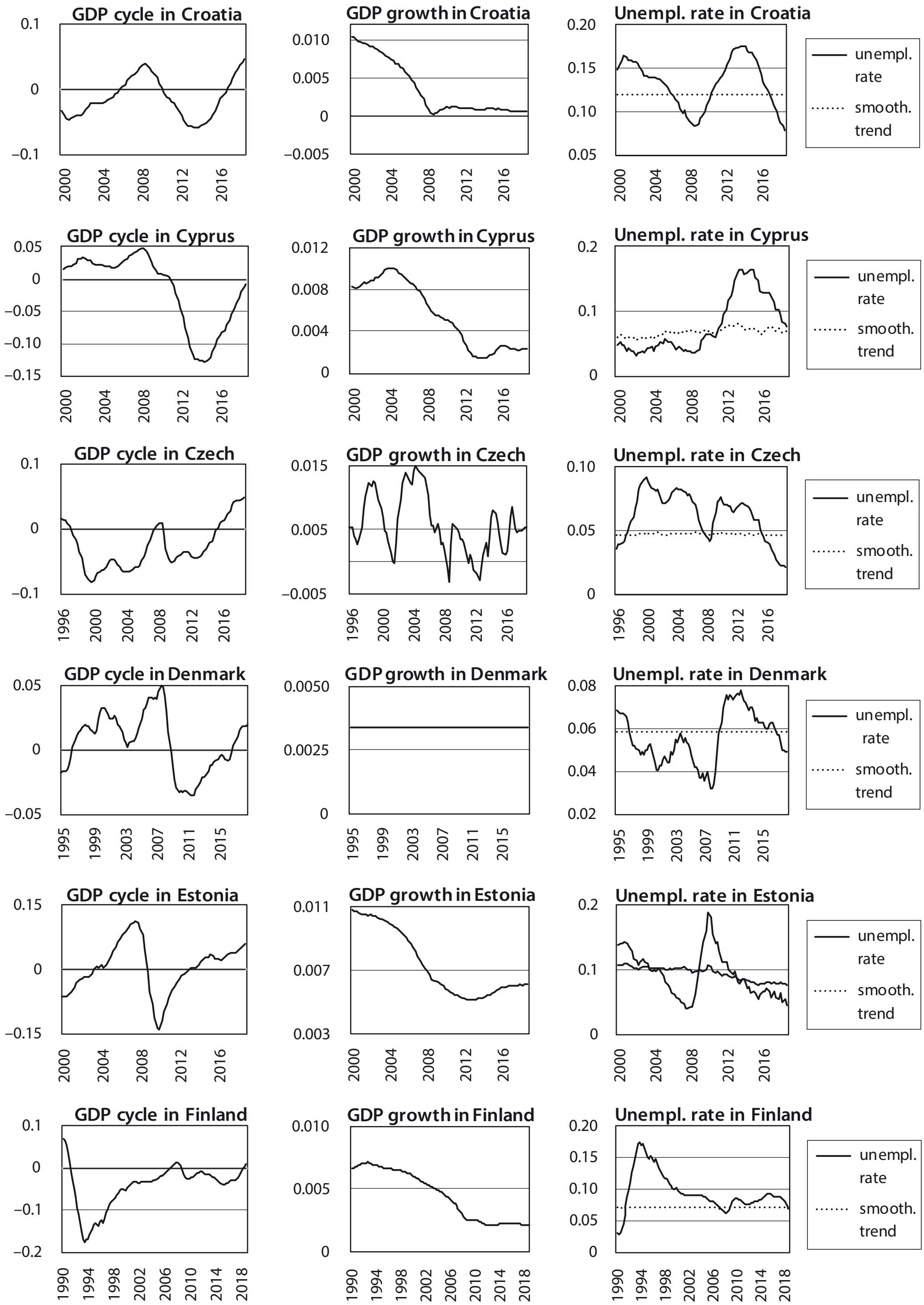


Figure 2 (continuation)

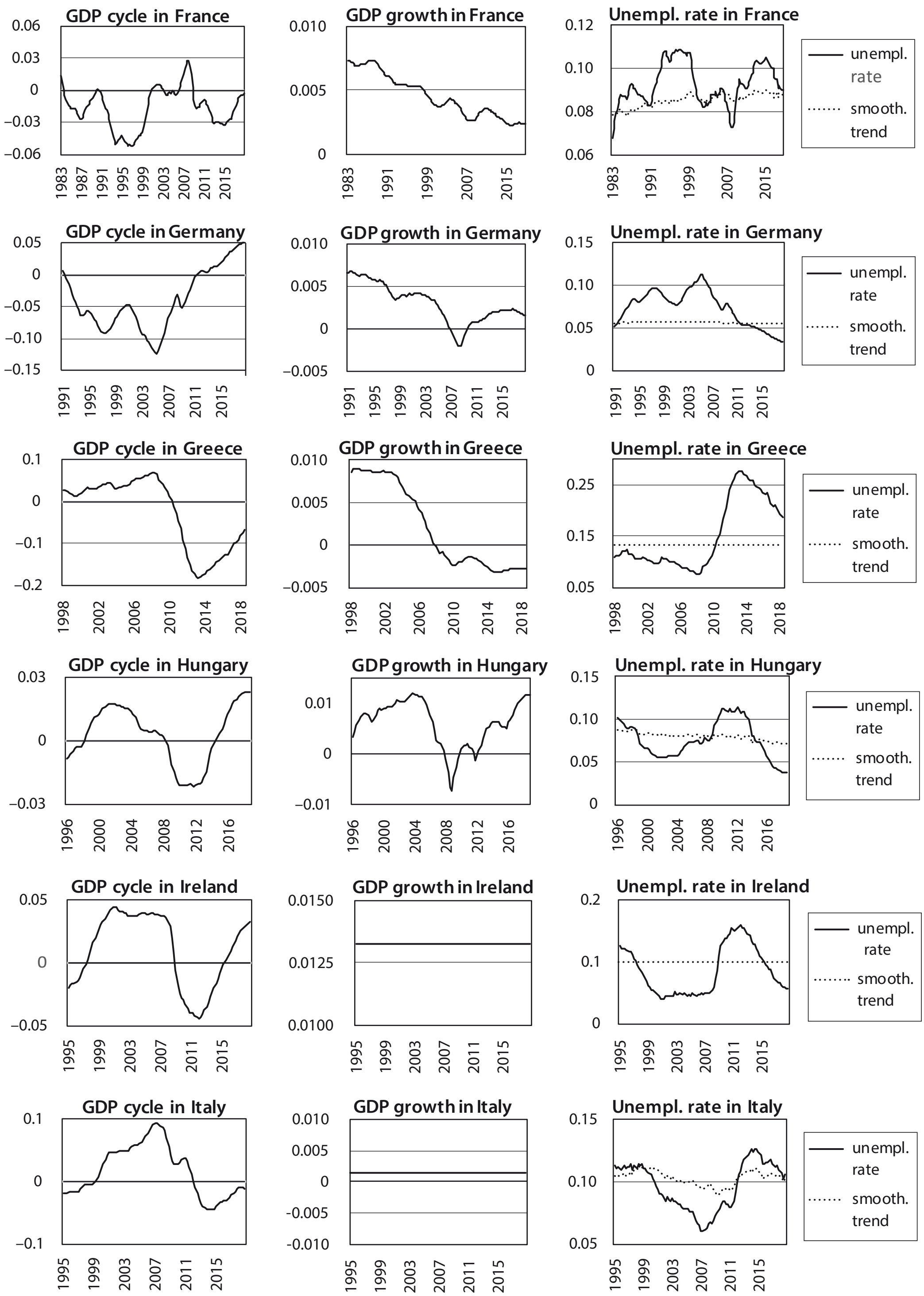


Figure 2 (continuation)

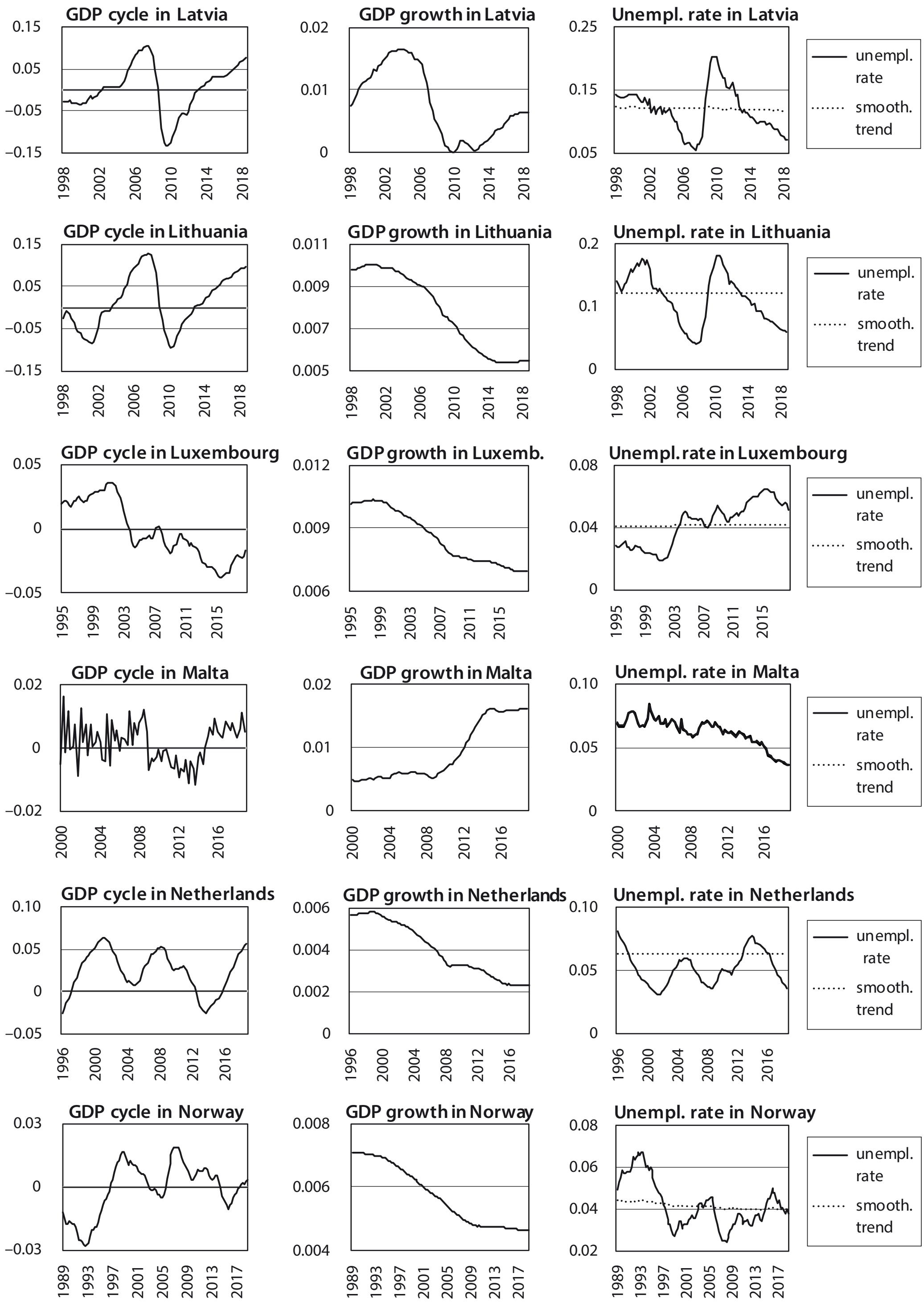


Figure 2 (continuation)

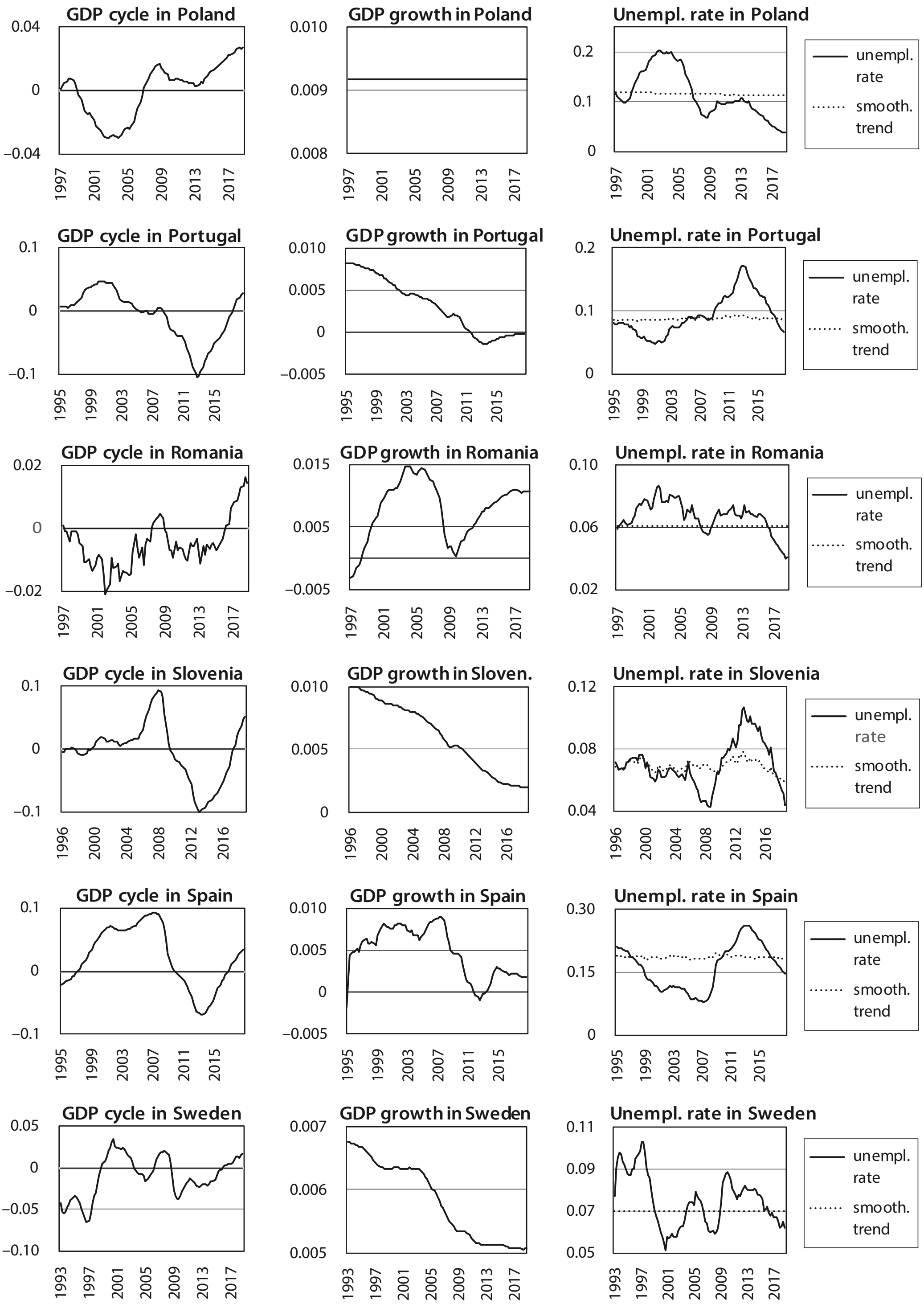


Figure 2 (continuation)

Source: Authors own calculations based on data obtained from Eurostat (2018)

4.1 GDP cycle

The cyclical component of GDP declined after the beginning of the crisis in 2008 in most countries. However, there are a few exceptions. The development of the gap in Finland seems untouched by the crisis, Germany experienced an increase in the output gap after 2008, and only a slight decrease has been observed for Malta, Norway, Poland, Romania and Sweden.

One important finding is that the output gap was very close to zero or even positive in 2018 for all countries except Greece and Luxembourg where GDP is approximately 7% and 2% below the trend value, respectively. Therefore, the huge long-lasting fall in GDP experienced by most countries is not a consequence of a decline in the output gap, but is caused by a decrease in the trend component of GDP.

A more detailed discussion on the output gap can be found in Section 5 where a comparison with HP-filtered estimates of the gap is described.

4.2 Growth of GDP trend component

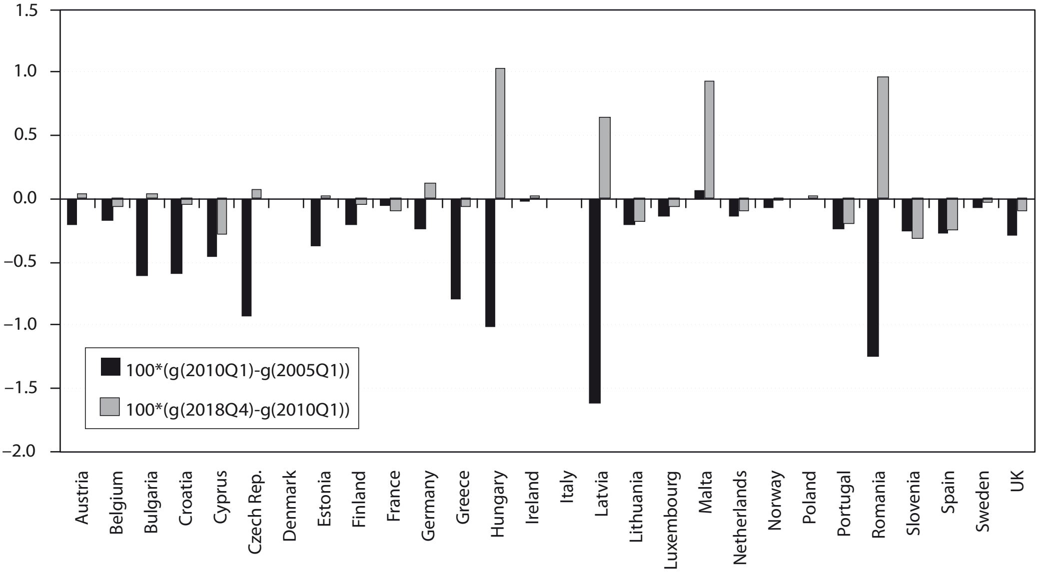
A decline in the growth g_t of the GDP trend component during the current economic crisis has been detected for most countries. Similar results were found by Lemoine et al. (2011), Ball (2014) and Haltmaier (2012). However, there are some exceptions. GDP growth in the Czech Republic was highly volatile with downturn as well as upturn movements. Malta even experienced an increase in the growth g_t of the GDP trend component. This makes the long-run development of GDP in this economy practically unaffected by the current economic crisis. Growth g_t was virtually constant during the studied periods in Denmark, Ireland, Italy and Poland.

These results correspond to the previous findings reported earlier in this paper in Section 3.1 when discussing estimates of $100\sigma_w$. The estimated volatility $100\hat{\sigma}_w$ of the random error w_t in Formula $g_t = g_{t-1} + w_t$ was found to be zero for these aforementioned countries (Denmark, Ireland, Italy and Poland). As already discussed above, zero estimates of $100\sigma_w$ in these countries (leading to the constant growth rate g_t observed in the second column of Figure 2) are compensated by high estimates of $100\sigma_v$, which applies especially for Ireland ($100\hat{\sigma}_v = 2.82$), but also for Poland ($100\hat{\sigma}_v = 0.95$) and Denmark ($100\hat{\sigma}_v = 0.76$).

Figure 3 summarizes the development of the growth g_t of the GDP trend component during the crisis and post-crisis periods in selected European economies. Specifically, a change (in percentage points) of g_t is calculated for the crisis period 2005 Q1–2010 Q1 and the post-crisis period 2010 Q1–2018 Q4 according to the relations $100 \cdot (g_{2010Q1} - g_{2005Q1})$, $100 \cdot (g_{2018Q4} - g_{2010Q1})$.

The most remarkable decline of g_t during the crisis period is detected for Latvia, Romania and Hungary. Fortunately, those countries hit most by the current crisis experienced an increase of g_t during the post-crisis period. A substantial decrease of g_t in the crisis period is also observed for Bulgaria, Croatia, the Czech Republic and Greece. Furthermore, Cyprus, Lithuania, Portugal, Slovenia and Spain suffered from a (moderate) decline of g_t in both crisis and post-crisis periods.

Figure 3 Decline in the growth g_t of the GDP trend component between 2005 Q1–2010 Q1 (crisis period) and 2010 Q1–2018 Q4 (post-crisis period)



Source: Authors own calculations based on data obtained from Eurostat (2018)

These results correspond closely to the findings previously discussed in this paper. Specifically, the growth g_t of the GDP trend component was found to be highly volatile (due to high and statistically significant σ_w) in the following countries: Bulgaria, Cyprus, the Czech Republic, Greece, Hungary, Latvia, Malta, Portugal, Romania and Spain. Comparison with Figure 3 shows that all these countries (except Malta) are economies facing problems with a decline of g_t.

Figure 2 shows that the growth g_t of the GDP trend component was even negative for some time in some countries, namely the Czech Republic, Germany, Greece, Hungary, Portugal and Spain. Nonetheless, it was in positive values in 2018 for all these countries except Greece where a negative growth rate g_t persisted until 2018Q4 (the last date in the data set). But it would be too soon to conclude that the long-run GDP growth g_t has already recovered. The variable g_t in 2018 attained much lower values (compared with the values of the pre-crisis period) for most of the economies.

4.3 Trend of unemployment rate

The smoothed unemployment rate trend is found to be constant for most economies. A slight variation of the trend L_t is observed for Austria, Belgium, Bulgaria, Estonia, France, Hungary, Italy, Malta, Slovenia and the United Kingdom. The case of Malta is specific in that the trend L_t closely corresponds to the observed unemployment rate U_t. A similar result is reported by Clark (1989) for Japan. An interesting finding is that some upward tendencies of L_t might be observed only in France. Downward tendencies of L_t are visible in Estonia, Hungary, Malta and the United Kingdom. These findings support the hypothesis that hysteresis effects have not played an important role.

The constant smoothed unemployment rate trend for most economies is an important and interesting result. It is surprising, especially for countries like Cyprus, Greece, Ireland, Latvia, Lithuania, Poland, Portugal and Spain, because the observed unemployment rate in these economies ranges from 0.05 to 0.25 (or even higher). Huge fluctuations of the observed unemployment rate U_t in these countries are thus caused by large swings in cyclical component C_t of the unemployment rate and not by movements in the trend L_t. This conclusion might seem at odds with economic intuition. The reason for this result is that the unemployment rate U_t has been decreasing systematically and significantly in the last few (5–8) years in these economies.

Similar results regarding the constancy of the trend component L_t were also obtained by Clark (1989) for West Germany and the United Kingdom. Clark reports that the estimated trend component L_t and observed unemployment rate U_t ranged between:

a) $L_t \in (0.045;0.075)$, $U_t \in (0.03;0.11)$ for the US economy (1947 Q1–1986 Q2),

b) $L_t \in (0.05;0.08)$, $U_t \in (0.03;0.13)$ for Canada (1955 Q1–1986 Q2),

c) $L_t = 0.015$, $U_t \in (0.01;0.09)$ for West Germany (1960 Q1–1986 Q2),

d) $L_t = 0.02$, $U_t \in (0.015;0.13)$ for the United Kingdom (1960 Q1–1986 Q2),

e) $L_t = U_t \in (0.01;0.03)$ for Japan (1960 Q1–1986 Q2).

According to Clark's (1989) findings, trend component L_t *L*turned out to be practically constant in West Germany and the United Kingdom despite the fact that the observed unemployment rate U_t spanned a considerably wide interval. Clark also reports that L_t and U_t practically coincide in Japan, which is a result that we detected for Malta. A surprising result is that the development of L_t and U_t, which would resemble Clark's (1989) findings for the USA and Canada (L_t is less volatile than U_t, but is not constant), was detected only for a few European economies. This suggests that individual European labour markets are fundamentally different to labour markets in the USA and Canada. One possible explanation is that the trend component L_t is influenced to a great extent by labour market institutions, which are not as flexible in Europe as in the USA or Canada.

5 COMPARISON WITH HP FILTER

The output gap estimated by Kalman smoother will now be compared with the gap obtained by applying an HP filter as a robustness check. Hodrick and Prescott (1997) proposed setting a smoothing parameter equal to 1 600 for quarterly data. This suggestion is based on their empirical investigation of the US economy. As individual European countries studied in this paper might be fundamentally different to the US economy, other values of the smoothing parameter suggested in literature (Baxter and King, 1999; Backus et al., 1992; Correia et al., 1992; Cooley and Ohanian, 1991) will be taken into account. This robustness check is important as Boďa et al. (2015) argue that an improperly chosen smoothing parameter might cause illusive cycles to appear. Other disadvantages of an HP filter are discussed by Boďa and Považanová (2019) and Plašil (2011).

For annual data, Baxter and King (1999) recommended a value of around 10, and Backus et al. (1992) advised setting a smoothing parameter equal to 100, whereas Correia et al. (1992), and Cooley and Ohanian (1991) proposed a value of 400. According to Ravn and Uhlig (1997), the smoothing parameter for quarterly data relates to its analogue for annual data according to the relation $\lambda_{quarter} = \lambda_{annual} \cdot 4^4$, which yields the following smoothing parameter values for quarterly data: 2 560, 25 600 and 102 400.

The output gap calculated by the HP filter with a smoothing parameter equal to 2 560 was virtually the same as for the most common value of 1 600. For this reason, only the values 1 600, 25 600 and 102 400 are taken into account. Figure 4 compares the output gap calculated by applying the HP filter with that obtained by Kalman smoother.

Figure 4 Estimates of output gap obtained by Kalman smoother and HP filter with smoothing parameter set equal to values of 1 600, 25 600 and 102 400

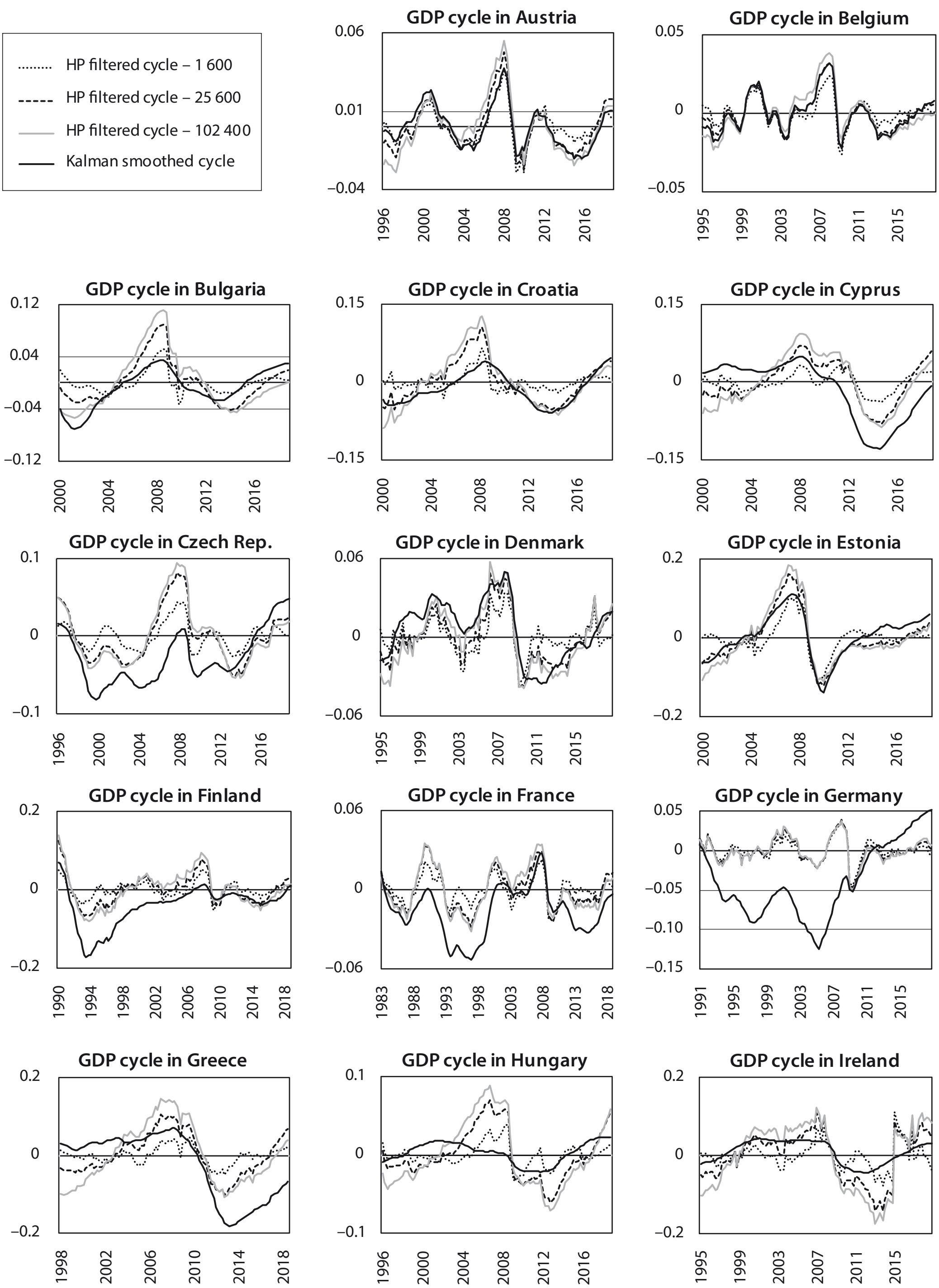
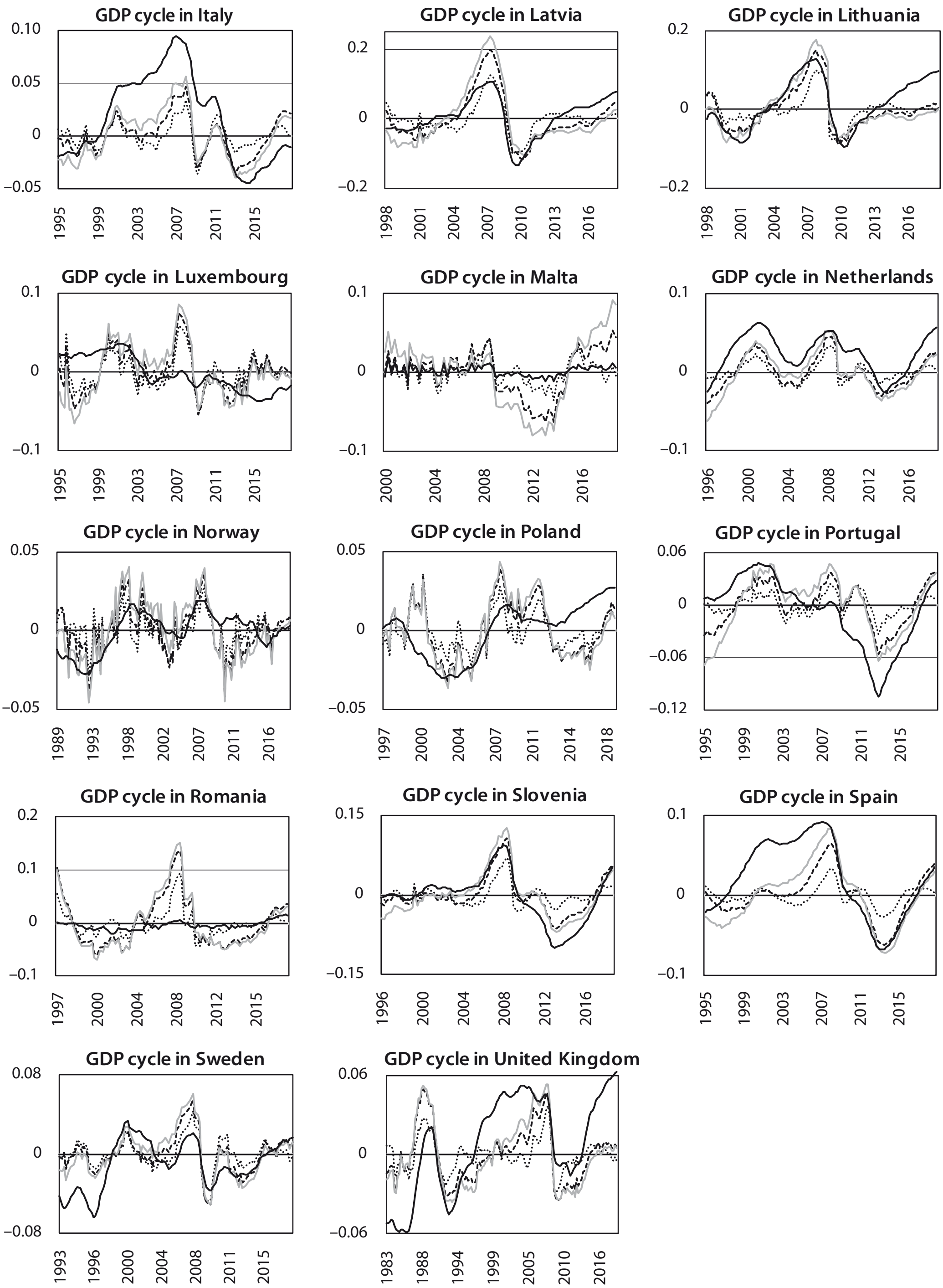


Figure 4 (continuation)



Source: Authors own calculations based on data obtained from Eurostat (2018)

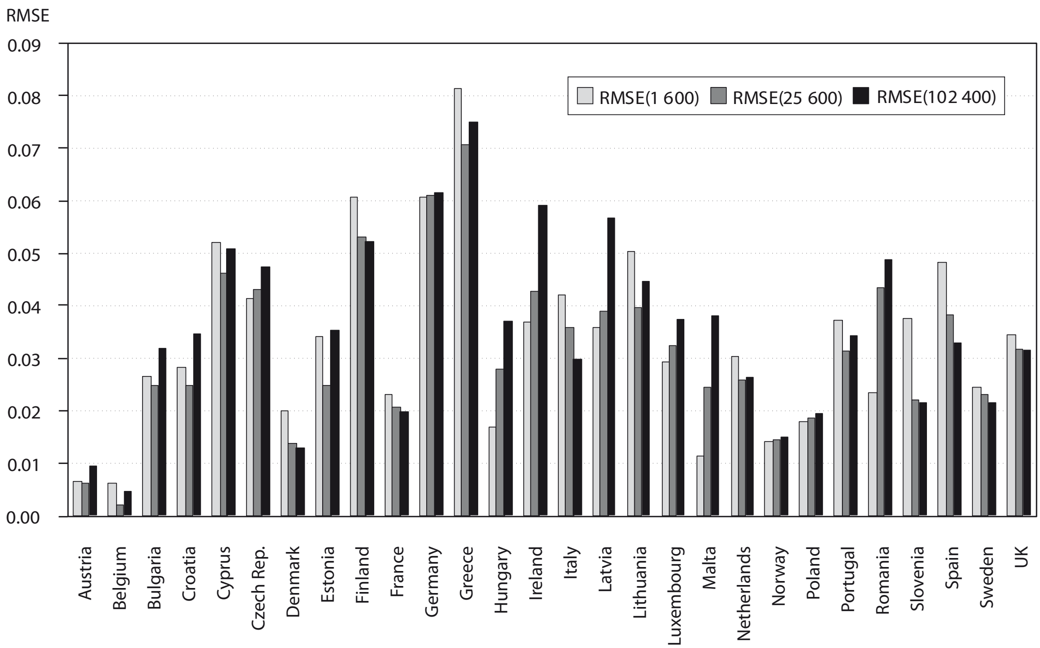
The (dis)similarity of the discussed output gap estimates can be measured by root mean squared error (RMSE) as follows:

$$RMSE(\lambda) = \sqrt{\frac{\sum_{t=1}^{n}\left(x_t^{Kalman} - x_t^{HP,\lambda}\right)^2}{n}},$$

where x_t^{Kalman} is the output gap obtained by Kalman smoother, $x_t^{HP,\lambda}$ represents the output gap calculated by HP filter with smoothing parameter λ, and n is the number of observations for a given economy.

RMSE was calculated for all 28 European economies with results summarized in Figure 5.

Figure 5 Comparison of output gap estimates x_t^{Kalman} and $x_t^{HP,\lambda}$ (λ ´= 1 600, 25 600, 102 400) using root mean squared error (RMSE)



Source: Authors own calculations based on data obtained from Eurostat (2018)

Figure 5 illustrates that different methods produced very similar results, especially for Austria, Belgium, Denmark, France, Norway, Poland and Sweden. The opposite is true mainly for Greece, where x_t^{Kalman} differs significantly from $x_t^{HP,\lambda}$ regardless of the smoothing parameter λ. Considerable differences between Kalman- and HP-filtered output gap measures are also detected for Cyprus, the Czech Republic, Finland, Germany, Ireland, Latvia, Lithuania and Romania.

Nonetheless, it can be seen from Figure 4 that a similar development of x_t^{Kalman} and $x_t^{HP,\lambda}$ is observed even in countries with the highest RMSE (Greece and Romania), i.e. time periods where the output gap is rising (decreasing) are mostly the same for all output gap estimation methods.

Moreover, there are some facts that proved to be robust across all the applied methods of estimation. Firstly, the output gap has already recovered since the crisis and is close to zero or even positive at the end of the sample in 2018 for most studied economies. Exceptions might be Greece, Italy and Luxembourg, where the HP filter suggests positive close-to-zero values of the output gap for all values of the smoothing parameter, but the Kalman smoother indicates a negative value of the gap. Secondly, the output gap experienced a decline after the beginning of the crisis in 2008 in all countries. There are a few exceptions to this when the gap is estimated by Kalman smoother (Germany, Malta, Poland and Romania). Nonetheless, HP-filtered estimates of the gap indicate a slump even in these countries.

Mostly, the Kalman smoother estimates the decrease in the gap after 2008 to be deeper and more prolonged than the HP filter. Nonetheless, this difference is smaller when the smoothing parameter is set to higher values.

The output gap calculated by HP filter with a smoothing parameter equal to 1 600 is usually very close to zero even in times of crisis, which seems unrealistic, especially for economies like Greece. The output gap in this country ranges only from –0.05 to 0.05, was already equal to zero in 2014 and GDP was already above its trend in 2018 according to this estimate. Such a situation is highly unrealistic given the huge problems in this economy. Setting higher values of the smoothing parameter leads to a much wider range within which the HP-filtered output gap in Greece oscillates. This makes the HP-filtered gap much closer to the gap estimated by Kalman smoother. A similar situation can also be seen in Cyprus, Denmark, Italy, Portugal, Slovenia and the United Kingdom.

Another unrealistic feature of the gap calculated by HP filter is that it was estimated to be positive at the beginning of the crisis in 2008 in all 28 studied economies. It is rather unrealistic that all 28 economies with mostly unsynchronized business cycle fluctuations would be in the same phase of the cycle at one specific moment. Output gaps obtained by the Kalman smoother have more realistic features as the (Kalman) smoothed gap had a negative value in Germany in 2008 and was roughly zero in Finland, Hungary, Luxembourg, Malta, Portugal and Romania.

CONCLUSION

An extensive empirical investigation has been carried out in this paper. Parameters of the well-known unobserved components Clark's (1989) model were estimated for 28 European economies. Consequently, the estimated model was applied in order to (1) empirically analyse cyclical and trend components of GDP and the unemployment rate, and (2) investigate the validity and strength of Okun's law.

Econometric estimation revealed that output has a highly persistent stationary cyclical component for all studied economies except Malta and Romania. Both cyclical and trend components in Malta are quite atypical and different to other European economies. Interestingly, there is a notable similarity between this economy and Japan when the results reported in this paper are compared with those obtained by Clark (1989). Comparison with Clark's (1989) estimates also confirms that the persistence of the gap in all European countries (apart from Malta and Romania) is much higher than in the US economy.

The negative impact of the current economic crisis on the long-run growth of the GDP trend turned out to be highest in Bulgaria, Croatia, the Czech Republic, Hungary, Greece, Latvia and Romania. Fortunately, those countries hit most by the current crisis (Hungary, Latvia and Romania) experienced an increase in the growth of the GDP trend component during the post-crisis period after 2010.

These results correspond closely to the findings regarding the variability of the growth of the GDP trend component – high and significant volatility was detected for Bulgaria, Cyprus, the Czech Republic, Greece, Hungary, Latvia, Malta, Portugal, Romania and Spain. The highest standard deviation value was observed for the Czech Republic, so the long-run growth in the Czech Republic is substantially variable.

These findings regarding substantial heterogeneity across individual countries emphasize the importance of analysing disaggregated European data. Such an approach taken in this paper represents a significant contribution to empirical literature as many studies use only aggregated European data (Azevedo et al., 2003; Berger and Everaert, 2008; Berger, 2011; Bernhofer et al., 2014; Chen and Mills, 2012; Galati et al., 2016; Lemoine et al., 2010; Orlandi and Pichelmann, 2000; Proietti, 2004).

Parameter estimation of the equation describing the negative relation between output gap and cyclical unemployment rate confirmed the validity of Okun's law as the total effect of the output gap on the gap of unemployment rate turned out to be negative in all studied economies. When assuming that only the contemporary output gap has an influence on unemployment, then (1) statistical insignificance of Okun's law coefficient at the 10% level was detected only in four economies (Luxembourg, Malta, Norway and Romania), and (2) statistical significance of Okun's law parameter at the 1% level was observed in 17 countries among all 28 studied economies. Cyclical properties of the unemployment rate are thus driven mainly by the cyclical component of GDP. We therefore confirm Ball et al.'s (2017) assertion that Okun's law is a strong and stable relationship in most countries, one that has not changed substantially during the current economic crisis.

Estimated coefficients in Okun's law turned out to be in line with other empirical studies. The median value for the overall effect of the output gap on the unemployment rate for the 28 analysed economies is –0.60, which is quite close to Clark's (1989) estimate of –0.51 as well as to Kim and Nelson's (1999) estimate of –0.57 for the US economy. Other empirical studies for aggregate euro area data usually report values within the interval (–0.6; –0.5) (Berger, 2011; Berger and Everaert, 2008; Orlandi and Pichelmann, 2000).

Another important finding is that estimated parameters in Okun's law vary quite substantially across countries, which is in line with Ball et al.'s (2017) results. Therefore, the assumption of constancy of this parameter across individual economies made in some empirical studies (e.g. Novák and Darmo, 2019) is not satisfied.

The volatility of the trend component of the unemployment rate turned out to be quite low. This was suggested by the fact that the standard deviation of the random error associated with this trend component was statistically significant in only approximately half of the studied economies. It was also supported by the calculated smoothed trend of the unemployment rate, which proved to be constant for most countries. Therefore, possible hysteresis effects in European labour markets have not played an important role yet. This is an interesting result, especially for economies like Cyprus, Estonia, Greece, Ireland, Latvia, Lithuania, Poland, Portugal and Spain. These countries exhibit huge swings in their unemployment rate, which are caused by movements in the cyclical component and not in trend. Similar findings were reported, for example, by Clark (1989) for West Germany and the United Kingdom.

The opposite is true for GDP because the huge long-lasting decline in GDP experienced by most countries is caused by a decline in the trend component of GDP and not in the cycle. The output gap decreased after 2008 in most countries (except Germany, Malta, Poland and Romania) but was very close to zero or positive in 2018 (except Greece). The long-lasting slump of GDP is thus caused by a decline in the trend.

A robustness check was performed for the output gap estimated by Kalman smoother because of the great economic importance of this variable. Specifically, the Hodrick-Prescott filter was applied and multiple values of the smoothing parameter were taken into account due to uncertainty about the correct value of this parameter for different European economies. The results obtained from the HP filter were quite similar to those obtained by the Kalman smoother, especially for Austria, Belgium, Denmark, France, Italy and Norway. The opposite is true mainly for Greece, which is an economy with non-standard economic conditions as this country is currently facing huge economic problems.

Some important facts proved to be robust across different methods – the output gap declined after the beginning of the crisis in 2008 in virtually all countries, but the gap has already recovered and was close to zero or even positive at the end of the sample in 2018. The output gap obtained from the HP filter also had some unrealistic features. This makes the estimates obtained by the Kalman smoother preferable, which supports the view given by Harvey and Jaeger (1993) regarding the superiority of the unobserved components approach to simple detrending methods. In some cases, unrealistic features of the HP-filtered gap were overcome by increasing the smoothing parameter above the most commonly used value of 1 600 (for quarterly data), which demonstrates that this commonly applied value might not be appropriate for all European economies.

The formulated model could be extended mainly in two ways. Firstly, the Phillips curve describing the relation between the output gap and inflation could be added into the model as done by Kloudová (2013), Orlandi and Pichelmann (2000), Beneš and N'Diaye (2004), Berger and Everaert (2008) and Berger (2011). Secondly, structural breaks in some variables as well as correlations between shocks to the unemployment rate and corresponding shocks to output might be considered as in Berger (2011), Chen and Mills (2012) and Proietti (2004).

ACKNOWLEDGEMENT

This work was processed with the contribution of the long-term institutional support of research activities by the Faculty of Informatics and Statistics, University of Economics, Prague (IP400040) and was supported by the Internal Grant Agency of the University of Economics, Prague, under Grant F4/53/2019.

References

AZEVEDO, J., KOOPMAN, S., RUA, A. *Tracking Growth and the Business Cycle: A Stochastic Common Cycle Model for the Euro Area*. Tinbergen Institute Discussion Paper, No. TI 2003-069/4, 2003.

BACKUS, D. K., KEHOE, P. J., KYDLAND, F. E. International Real Business Cycles. *Journal of Political Economy*, 1992, 100(4), pp. 745–775.

BAXTER, M. AND KING, R. Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series. *The Review of Economics and Statistics*, 1999, 81(4), pp. 573–593.

BALL, L. Long-Term Damage from the Great Recession in OECD Countries. *European Journal of Economics and Economic Policies: Intervention*, 2014, 11(2), pp. 149–160.

BALL, L., LEIGH, D., LOUNGANI, P. Okun's Law: Fit at 50? *Journal of Money, Credit and Banking*, 2017, 49(7), pp. 1413–1441.

BARRO, R. J. *Economic Growth in East Asia Before and After the Financial Crisis*. NBER Working Paper, No. W8330, Cambridge, Massachusetts: National Bureau of Economic Research, 2001.

BENEŠ, J. AND N'DIAYE, P. *A Multivariate Filter for Measuring Potential Output and the NAIRU: Application to the Czech Republic*. IMF Working Paper, No. 04/45, International Monetary Fund, 2004.

BERGER, T. AND EVERAERT, G. Unemployment Persistence and the NAIRU: A Bayesian Approach. *Scottish Journal of Political Economy*, 2008, 55(3), pp. 281–299.

BERGER, T. Estimating Europe's Natural Rates. *Empirical Economics*, 2011, 40(2), pp. 521–536.

BERNARDI, M. AND DI RUGGIERO, A. *Extracting the Italian Output Gap: a Bayesian Approach.* arXiv:1411.4898 [Stat.AP], Cornell University Library, 2014.

BERNHOFER, D., AMADOR, O, GÄCHTER, M., SINDERMANN, F. Finance, Potential Output and the Business Cycle: Empirical Evidence from Selected Advanced and CESEE Economies. *Focus on European Economic Integration*, 2014, 2, pp. 52–75.

BOĎA, M., MEDVEĎOVÁ, P., POVAŽANOVÁ, M. (A)symmetry in Okun's Law in the Visegrad Group Countries (in Slovak). *Politická ekonomie*, 2015, 63(6), pp. 741–758.

BOĎA, M. AND POVAŽANOVÁ, M. Okun's Law in the Visegrád Group Countries. *Europe-Asia Studies,* 2019, 71(4), pp. 608–647.

BRŮHA, J. AND POLANSKÝ, J. *Empirical Analysis of Labor Markets over Business Cycles: An International Comparison*. Working Papers 2015/15, Czech National Bank, Research Department, 2015.

CERRA, V. AND SAXENA, S. C. Did Output Recover from the Asian Crisis? *IMF Staff Papers*, 2005, 52(1), pp. 1–23.

CERRA, V. AND SAXENA, S. C. Growth Dynamics: The Myth of Economic Recovery. *American Economic Review*, 2008, 98(1), pp. 439–457.

CHEN, X. AND MILLS, T. C. Measuring the Euro Area Output Gap Using a Multivariate Unobserved Components Model Containing Phase Shifts. *Empirical Economics*, 2012, 43(2), pp. 671–692.

CLARK, P. K. The Cyclical Component of U.S. Economic Activity. *The Quarterly Journal of Economics*, 1987, 102, pp. 197–814.

CLARK, P. K. Trend Reversion in Real Output and Unemployment. *Journal of Econometrics*, 1989, 40, pp. 15–32.

COOLEY, T. J. AND OHANIAN, L. E. The Cyclical Behavior of Prices. *Journal of Monetary Economics*, 1991, 28(1), pp. 25–60.

CORREIA, I. H., NEVES, J. L., REBELO, S. T. Business Cycles from 1850–1950: New Facts about Old Data. *European Economic Review*, 1992, 36(2–3), pp. 459–467.

DUPASQUIER, C., GUAY, A., ST-AMANT, P. A Survey of Alternative Methodologies for Estimating Potential Output and the Output Gap. *Journal of Macroeconomics*, 1999, 21(3), pp. 577–595.
EUROSTAT. *Database–Eurostat* [online.]. [cit. 2.6.2019] <http://ec.europa.eu/eurostat>.
GALATI, G., HINDRAYANTO, I., KOOPMAN, S., VLEKKE, M. *Measuring Financial Cycles with a Model-Based Filter: Empirical Evidence for the United States and the Euro Area*. DNB Working Paper, No. 495, 2016.
HÁJEK, M. AND BEZDĚK, M. *Estimation of Potential Product and Output Gap in the Czech Republic* (in Czech). ČNB Working Paper, No. 26, 2000.
HÁJKOVÁ, D. AND HURNÍK, J. Cobb-Douglas Production Function: The Case of a Converting Economy. *Czech Journal of Economics and Finance*, 2007, 57(9–10), pp. 465–476.
HALTMAIER, J. *Do Recessions Affect Potential Output?* International Finance Discussion Paper 1066, Federal Reserve Board, 2012.
HARVEY, A. C. *Forecasting, Structural Time Series Models and the Kalman Filter*. 1st Ed. Cambridge: Cambridge University Press, 1989.
HARVEY, A. C. AND JAEGER, A. Detrending, Stylized Facts and the Business Cycle. *Journal of Applied Econometrics*, 1993, 8(3), pp. 231–247.
HODRICK, R. J. AND PRESCOTT, E. C. Postwar U.S. business Cycles: An Empirical Investigation. *Journal of Money, Credit and Banking,* 1997, 29(1), pp. 1–16.
JEMEC, N. *Output Gap in Slovenia. What Can We Learn from Different Methods?* Banka Slovenije Working Paper, No. 4/2012, 2012.
KIM, C. AND NELSON, C. R. *State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications*. Cambridge: The MIT Press, 1999.
KLOUDOVÁ, D. Output Gap as Indicator of Inflation – Case for Czech Economy (in Czech). *Politická ekonomie*, 2013, 61(55), pp. 639–652.
LEMOINE, M., DE LA SERVE, M. E., CHETOUANE, M. *Impact of the Crisis on Potential Growth: An Approach Based on Unobserved Components Models* (in French). Banque de France Working Paper, No. 331, 2011.
LEMOINE, M., MAZZI, G. L., MONPERRUS-VERONI, P. A. New Production Function Estimate of the Euro Area Output Gap. *Journal of Forecasting*, 2010, 29(1–2), pp. 29–53.
MANKIW, G. *Principles of macroeconomics.* 6th Ed. Mason, OH: South-Western Cengage Learning, 2012.
NOVÁK, M. AND DARMO, L. Okun's Law over the Business Cycle: Does It Change in the EU Countries after the Financial Crisis? *Prague Economic Papers*, 2019, 28(2), pp. 235–254.
OCHOTNICKÝ, P. Production Function Choice by Potential Output Estimation (in Slovak). *Ekonomický časopis/Journal of Economics*, 2008, 56(8), pp. 800–815.
OKUN, A. M. *Potential GNP: Its Measurement and Significance*. Reprinted as Cowles Foundation Paper 190, 1962.
ORLANDI, F. AND PICHELMANN, K. *Disentangling Trend and Cycle in the EUR-11 Unemployment Series: An Unobserved Component Modelling Approach*. Brussels: European Commision, 2000.
PLAŠIL, M. Potential Product, Output Gap and Uncertainty Rate Associated with Their Determination while Using the Hodrick-Prescott Filter (in Czech). *Politická ekonomie*, 2011, 59(4), pp. 490–507.
PLOSSER, C. AND SCHWERT, W. Potential GNP: Its Measurement and Significance: A Dissenting Opinion. In: BRUNNER, K. AND MELTZER, A. eds. *Three Aspects of Policymaking, Knowledge, Data and Institutions*, Amsterdam: Carnegie-Rochester Conference Series on Public Policy, North-Holland, 1979, pp. 179–186.
PROIETTI, T. State Space Decomposition under the Hypothesis of Non-Zero Correlation between Trend and Cycle with an Application to the Euro-zone. In: MAZZI, L. L. AND SAVIO, H. eds. *Monographs of Official Statistics*, Papers and Proceedings of the Third Colloquium on Modern Tools for Business Cycle Analysis, Research in Official Statistics, Eurostat, Luxembourg, 2004, pp. 292–325.
RAVN, M. AND UHLIG, H. On Adjusting the HP-Filter for the Frequency of Observations. *The Review of Economics and Statistics*, 2002, 84(2), pp. 371–380.
VOLOS, C. AND HADJIXENOPHONTOS, A. *An Unobserved Components Model Approach to the Relationship between Real GDP and Unemployment for Cyprus*. Neapolis University of Paphos Working Paper, No. 5, 2015.
ZIMKOVÁ, E. AND BAROCHOVSKÝ, J. Estimation of Potential Product and Output Gap in Slovak Conditions (in Czech). *Politická ekonomie*, 2007, 55(4), pp. 473–489.