The Impact of the Price Index Formula on the Consumer Price Index Measurement

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Abstract

The Consumer Price Index (CPI) is a common measure of inflation. Similarly to the Harmonised Index of Consumer Prices (HICP), it is determined using the Laspeyres index, thus data on the consumption of the basket of goods do not have to be current. The Laspeyres index, using weights only from the base period, may not reflect changes in consumer preferences that occurred in the studied year. In the ideal case, the CPI should be measured by one of the so called superlative price indices, such as the Fisher, Törnqvist or Walsh index formulas. The main problem with such indices is that they need expenditure data from the current period. The aim of the article is to assess the impact of the choice of the price index formula on the CPI measurement. We verify differences among known index formulas at the lowest and some higher data aggregation levels. We use known bilateral unweighted and weighted formulas together with their chained versions.

Keywords	***JEL code***
Inflation measurement, Consumer Price Index (CPI), price indices, elementary price indices, chain indices, formula bias, scanner data	*C43, C38, E31*

INTRODUCTION

The consumer price index (CPI) measures changes in the price level of market basket of consumer goods and services purchased by households and it is a common measure of inflation. The CPI is a statistical estimate constructed using the prices of a sample of representative items whose prices are collected periodically, and it approximates changes in the costs of household consumption assuming the constant utility (COLI, *Cost of Living Index*). Similarly to the Harmonised Index of Consumer Prices (HICP), the CPI is determined using the Laspeyres index, thus data on the consumption of the basket of goods do not have to be current (White, 1999; Clements and Izan, 1987). The Laspeyres index, using weights only from the base period, may not reflect changes in consumer preferences that occurred in the studied year

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(Hałka and Leszczyńska, 2011). It leads to the conclusion that the Laspeyres index can be biased due to the commodity substitution. Many economists and statisticians treat *superlative indices* (such as the Fisher index or the Törnqvist index) as the best approximation of COLI (Von der Lippe, 2007). The difference between the Laspeyres index and the superlative index should approximate the value of the commodity substitution bias (White, 1999; Białek, 2016). The Fisher index is the most popular among superlative indices and it is called "ideal" since it satisfies most of tests derived from the axiomatic price index theory (Balk, 1995), including *time reversibility*. Nevertheless, the Fisher price index, similarly to other superlative price index formulas, makes use of current-period expenditure data, and thus its usefulness in the CPI measurement is limited. Admittedly, it is possible to approximate the Fisher index by means of indices using only consumption data from the base period (Lloyd, 1975; Moulton, 1996; Shapiro and Wilcox, 1997; Lent and Dorfman, 2009; Białek, 2017a, 2017b), nonetheless most countries in the world continue to use the Laspeyres index to measure the CPI (White, 1999).

Scanner data, i.e. transaction data that specify turnover and numbers of items sold by GTIN (a barcode, formerly known as the EAN code), provide a new opportunity of calculating price indices, since they give information about prices and quantities even at the lowest data aggregation level. The methodology for the CPI (or HICP) construction using scanner data has strongly evolved for the last year (see for instance: Ivancic et al., 2011; Krsinich, 2014; Griffioen and Ten Bosch, 2016; de Haan et al., 2016; Chessa and Griffioen, 2016; Chessa, 2017; Diewert and Fox, 2017). Probably, in the nearest future, statistical agencies will be able to use any price index formula for CPI calculations if only they use daily or weekly updated scanner data. Having scanner data sets, we may calculate superlative price indices at the lowest level of data aggregation (even lower than COICOP 5).

The aim of the article is to assess the impact of the choice of the price index formula on the CPI measurement. We verify differences among known index formulas both at the lowest and some higher data aggregation levels. We focus on known unweighted and weighted formulas together with their chained versions but we do not consider the so called multilateral methods and indices which are strictly dedicated to scanner data cases (Chessa, 2016). Our paper is organised as follows: Sections 2 and 3 discuss elementary and weighted price indices respectively, Section 4 presents the idea of chain indices, Sections 5 and 6 present results from our empirical and simulation studies, while Section 7 provides some final conclusions and remarks.

1 ELEMENTARY PRICE INDICES

A recommendation of the European Commission concerning the choice of the elementary formula at the lowest level of data aggregation can be found on website: <*http://www.ilo.org/public/english/bureau/stat/download/cpi/corrections/annex1.pdf*> and it is as follows: *"For the HICPs the ratio of geometric mean prices or the ratio of arithmetic mean prices are the two formulae which should be used within elementary aggregates. The arithmetic mean of price relatives may only be applied in exceptional cases and where it can be shown that it is comparable"*. In other words, if expenditure information is not available, the European Commission recommends the Jevons (1865) price index (see also: Diewert, 2012; Levell, 2015), which can be written for the base period and the current period as follows:

$$P_J^{0,t} = \prod_{i \in G_{0,t}} \left(\frac{p_i^t}{p_i^0}\right)^{\frac{1}{N_{0,t}}}, \tag{1}$$

where p_i^τ denotes the price of the i-th product at time $\tau \in \{0,t\}$, $G_{0,t}$ denotes the set of matched products in both moments 0 and t and $N_{0,t} = card\ G_{0,t}$. On the other hand, the same recommendation takes also into consideration ("in exceptional cases") the Carli (1804) price index, which can be written as follows:

$$P_C^{0,t} = \frac{1}{N_{0,t}} \sum_{i \in G_{0,t}} \frac{p_i^t}{p_i^0}, \tag{2}$$

In the literature, we can find also some other elementary price indices. One of the oldest propositions of elementary indices is the Dutot price index, i.e.

$$P_D^{0,t} = \frac{\frac{1}{N_{0,t}} \sum_{i \in G_{0,t}} p_i^t}{\frac{1}{N_{0,t}} \sum_{i \in G_{0,t}} p_i^0}. \tag{3}$$

There are many papers that compare the above-mentioned unweighted price index numbers. Early contributions of Eichhorn and Voeller (1976), Dalen (1992) and Diewert (1995) provide studies of properties of elementary indices from an axiomatic point of view. The differences between elementary indices, in terms of changes in the price variances, have been considered for sample indices by using the Taylor approximations (see e.g.: Dalen, 1992; Diewert, 1995; Balk, 2005 for details). The earlier literature, using the actual data underlying the consumer price index, has shown that the differences at the elementary aggregate level between the Dutot, Carli and Jevons indices can be quite substantial (see Carruthers et al., 1980; Dalen, 1994; Schultz, 1995; Moulton and Smedley, 1995).

1.1 Weighted price index formulas

As it was mentioned in the *Introduction*, in practice, the Laspeyres price index is used to measure the CPI (White, 1999; Clements and Izan, 1987). The Laspeyres price index (1871) can be expressed as follows:

$$P_{La}^{0,t} = \frac{\sum_{i \in G_{0,t}} p_i^t q_i^0}{\sum_{i \in G_{0,t}} p_i^0 q_i^0}, \tag{4}$$

where q_i^τ denotes the price of the i-th product at time $\tau \in \{0,t\}$.The Paasche price index (1874) uses quantities from the current period in its body and it can be written as follows:

$$P_{Pa}^{0,t} = \frac{\sum_{i \in G_{0,t}} p_i^t q_i^t}{\sum_{i \in G_{0,t}} p_i^0 q_i^t}. \tag{5}$$

In the so called economical approach (in the price index theory), it is assumed that the real value of the COLI should belong to the interval whose lower and upper limits are determined by values of the Laspeyres and Paasche indices. The most recommended index formulas for the CPI measurement are *superlative* price indices, firstly proposed by (Diewert, 1976). In this paper, we consider the Fisher (1922), Törnqvist (1936) and Walsh (1901) superlative price index formulas which can be defined respectively:

$$P_F^{0,t} = \sqrt{P_{La}^{0,t} \cdot P_{Pa}^{0,t}}, \tag{6}$$

$$P_T^{0,t} = \prod_{i \in G_{0,t}} \left(\frac{p_i^t}{p_i^0}\right)^{\frac{s_i^0 + s_i^0}{2}}, \tag{7}$$

$$P_W^{0,t} = \frac{\sum_{i \in G_{0,t}} p_i^t \cdot \sqrt{q_i^0 q_i^t}}{\sum_{i \in G_{0,t}} p_i^0 \cdot \sqrt{q_i^0 q_i^t}} . \tag{8}$$

In the paper, we consider two additional and well-known weighted price index formulas, namely the Marshall-Edgeworth index (1887) and the Geary Khamis (GK) index (Geary, 1958; Khamis, 1972), i.e.

$$P_{ME}^{0,t} = \frac{\sum_{i \in G_{0,t}} p_i^t \cdot (\frac{q_i^0 q_i^t}{2})}{\sum_{i \in G_{0,t}} p_i^0 \cdot (\frac{q_i^0 q_i^t}{2})} , \tag{9}$$

$$P_{GK}^{0,t} = \frac{\sum_{i \in G_{0,t}} p_i^t \cdot (\frac{q_i^0 q_i^t}{q_i^0 + q_i^t})}{\sum_{i \in G_{0,t}} p_i^0 \cdot (\frac{q_i^0 q_i^t}{q_i^0 + q_i^t})} . \tag{10}$$

Formulas (9) and (10) are not superlative but they are symmetrical, i.e. they remain the same upon interchanging of quantity vectors. These formulas have good axiomatic properties (Von der Lippe, 2007). Let us note that the Marshall-Edgeworthand the Geary-Khamis price indices differ from the Walsh formula only with respect to the used type of mean of quantities.

1.2 Chain indices

In the previously presented so called "direct approach", the results of price index formula $P^{0,t}$ are not influenced by what happens to prices and quantities in the intermediate points in time. A chain index $P_{CH}^{0,t}$ is essentially a specific type of temporal aggregation (over time) and it provides a measure of the cumulated effect of successive price steps from time moment 0 to 1, from 1 to 2, …, and from $t-1$ to t. In other words, the chain index takes into account all intermediate periods (months as a rule): 1, 2, …, $t-1$ and it can be written in a general form as follows:

$$P_{CH}^{0,t} = P^{0,1} \cdot P^{1,2} \cdot \ldots \cdot P^{t-1,t} = \prod_{\tau=1}^{t-1} P^{t,t+1} . \tag{11}$$

Please note that any price index formula can play a role of the base price index P. For instance, taking the Laspeyres index as the base index, we obtain the following Laspeyres chain index:

$$P_{CH-La}^{0,t} = P_{La}^{0,1} \cdot P_{La}^{1,2} \cdot \ldots \cdot P_{La}^{t-1,t} = \prod_{\tau=1}^{t-1} P_{La}^{t,t+1} . \tag{12}$$

2 EMPIRICAL STUDY

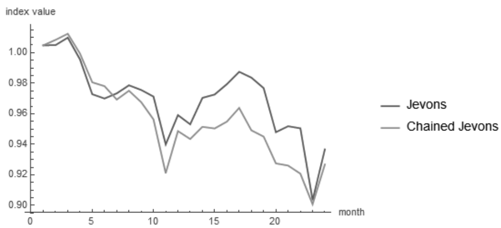
Case 1

The first data set was obtained from *allegro.pl*, which is one of the biggest e-commerce platform in Poland. We collected monthly transaction data on a homogeneous group of 33 different child safety seats. The time interval for observations was Dec. 2016–Dec. 2018 and the reference month was Dec. 2016 ($\tau = 0$).

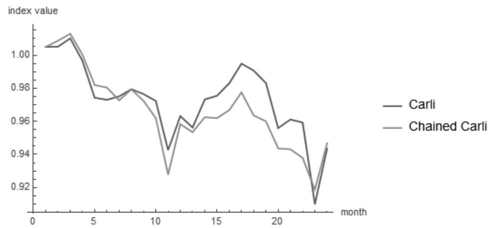
We collected data on average monthly prices of sold child safety seats, numbers of monthly transactions, numbers of items sold and corresponding expenditures. We matched observed products for each pair of subsequent months by using EAN codes and, having their descriptions, by using also some text mining methods. We ruled out from the sample poorly available products and products with relatively small expenditures to reduce the sample to the most typical and popular models of child safety seats (17 models). As a consequence, we took into consideration 17 378 transactions. As mentioned above, scanner data allow us to apply both unweighted and weighted indices for their analysis, and that is just what we did. Our results are presented in Figures 1–3 and in Table 1 which present differences between considered price indices.

Figure 1 Comparison of unweighted indices together with their chained versions (a homogeneous group: child safety seats, time interval: Dec. 2016–Dec. 2018)

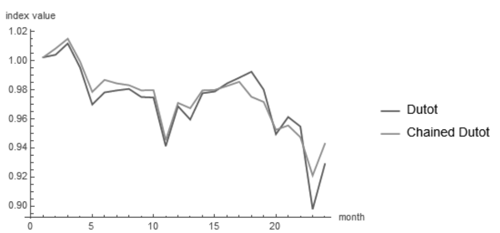
a) The Jevons index vs the Chained Jevons index



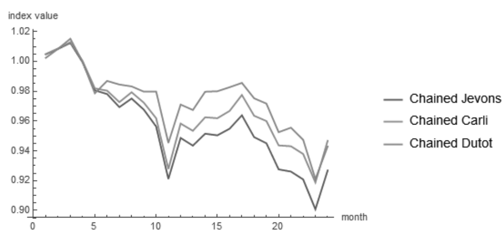
b) The Carli index vs the chained Carli index



c) The Dutot index vs the Chained Dutot index



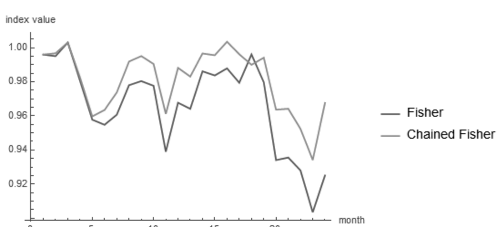
d) Comparison of chained elementary indices



Source: Own elaboration based on data from: <allegro.pl>

Figure 2 Comparison of weighted indices together with their chained versions (a homogeneous group: child safety seats, time interval: Dec. 2016–Dec. 2018)

a) The Fisher index vs the Chained Fisher index



b) Comparison of superlative indices

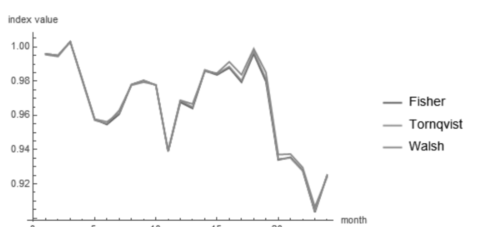
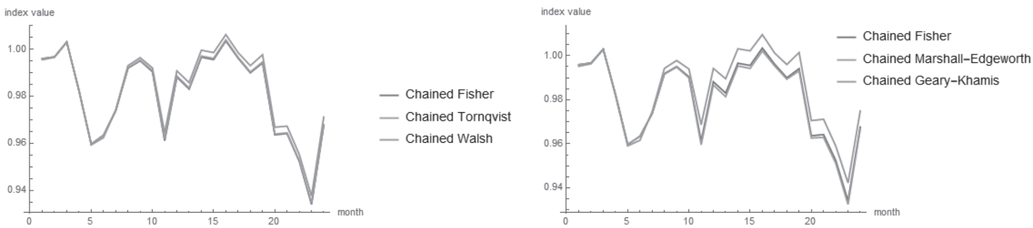


Figure 2 (continuation)

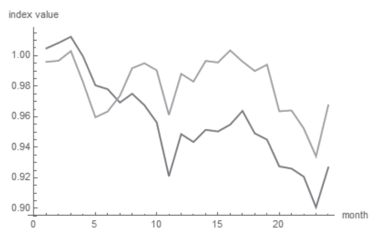
c) Comparison of chained superlative indices

d) Comparison of chained symmetrical indices



Source: Own elaboration based on data from: <allegro.pl>

Figure 3 Comparison of chained Jevons and Fisher indices (a homogeneous group: child safety seats, time interval: Dec. 2016–Dec. 2018)



Source: Own elaboration based on data from: <allegro.pl>

Table 1 Comparison of all discussed price indices for different time intervals

Price index	**Time interval**			
	[0,6]	[0,12]	[0,18]	[0,24]
Unweighted formulas				
Jevons	0.9700	0.9591	0.9837	0.9367
Chained Jevons	0.9781	0.9486	0.9492	0.9268
Carli	0.9730	0.9631	0.9906	0.9435
Chained Carli	0.9804	0.9583	0.9635	0.9467
Dutot	0.9783	0.9688	0.9925	0.9289
Chained Dutot	0.9869	0.9711	0.9752	0.9429
Weighted formulas				
Fisher	0.9547	0.9677	0.9960	0.9250

Table 1 (continuation)

Price index	Time interval			
	[0,6]	[0,12]	[0,18]	[0,24]
Weighted formulas				
Chained Fisher	0.9634	0.9881	0.9899	0.9676
Törnqvist	0.9563	0.9685	0.9971	0.9252
Chained Törnqvist	0.9635	0.9885	0.9903	0.9680
Walsh	0.9552	0.9688	0.9989	0.9241
Chained Walsh	0.9624	0.9907	0.9929	0.9712
Marshall-Edgeworth	0.9539	0.9665	0.9936	0.9125
Chained ME	0.9634	0.9867	0.9893	0.9668
Geary-Khamis	0.9568	0.9708	1.0033	0.9356
Chained GK	0.9616	0.9942	0.9960	0.9749

Source: Own elaboration based on data from: <allegro.pl>

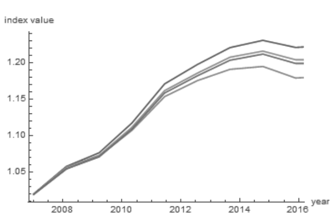
Case 2

At the beginning of the following case study, we used all discussed indices together with their chain versions for the inflation analysis in the United Kingdom and Bulgaria in the years: 2007–2017. Currently there are no differences between the CPI and the HICP (Harmonized Index of Consumer Prices) in the case of these countries. We decided to consider also the Czech Republic using the corresponding HICP data from Eurostat. We collected year-to-year data on CPI/HICP levels and weights for each group of goods from the COICOP-4 digit level of data aggregation. We calculated all the above-mentioned weighted price indices including the yearly chained Laspeyres and Fisher price indices (see Figures 4–6). Some detailed results (concerning the yearly inflation rate in considered countries measured by using different index formulas) for sample years (2011, 2014 and 2017) and additionally for the COICOP-3 digit level are presented in Tables 2–4.

Figure 4 Comparison of values of weighted price index formulas (CPI data from the United Kingdom, ECOICOP-4, 2007–2017)

Substitution bias effect

a. The reference year = 2006



b. Annual changes (year-to-year)

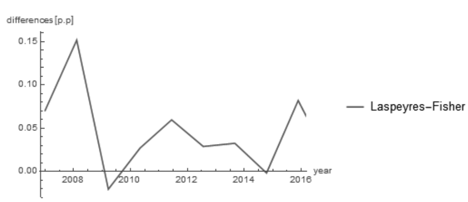
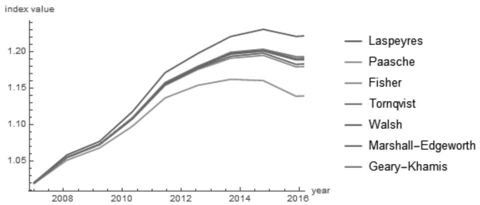


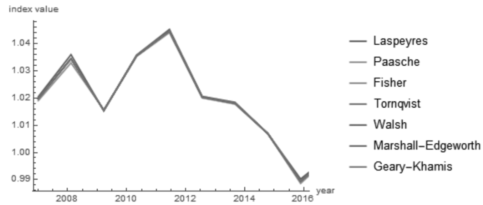
Figure 4 (continuation)

Comparison of all discussed price indices

c. The reference year = 2006



d. Annual changes (year-to-year)

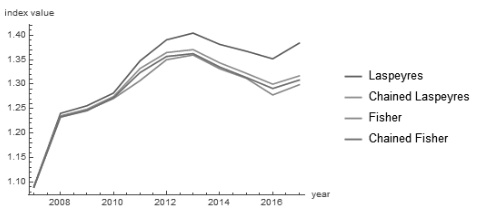


Source: Own elaboration based on data from Eurostat

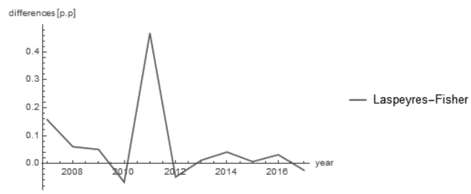
Figure 5 Comparison of values of weighted price index formulas (CPI data from Bulgaria, ECOICOP-4, 2007–2017)

Substitution bias effect

a. The reference year = 2006

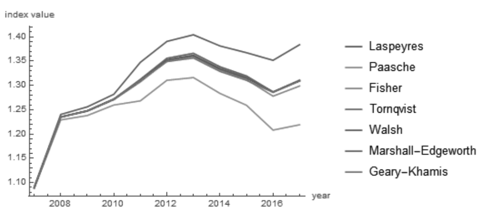


b. Annual changes (year-to-year)

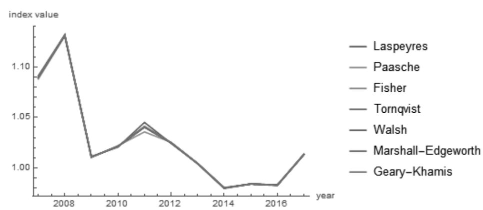


Comparison of all discussed price indices

c. The reference year = 2006



d. Annual changes (year-to-year)

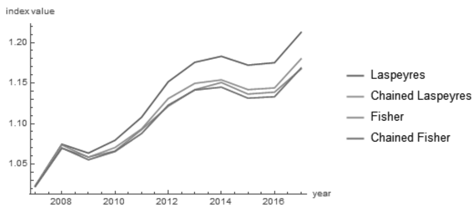


Source: Own elaboration based on data from Eurostat

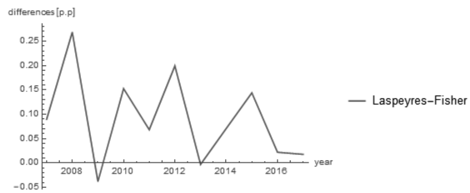
Figure 6 Comparison of values of weighted price index formulas (HICP data from the Czech Republic, ECOICOP-4, 2007–2017)

Substitution bias effect

a. The reference year = 2006

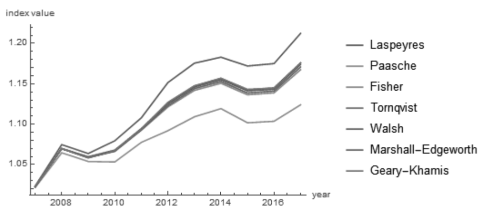


b. Annual changes (year-to-year)

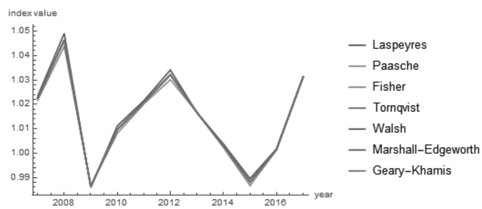


Comparison of all discussed price indices

c. The reference year = 2006



d. Annual changes (year-to-year)



Source: Own elaboration based on data from Eurostat

Table 2 Yearly inflation rate [%] measured by different index formulas in the United Kingdom (years: 2011, 2014, 2017)

Used index formula	**COICOP 3**			**COICOP 4**		
	2011	2014	2017	2011	2014	2017
Laspeyres	4.416	1.092	2.572	4.518	1.689	2.772
Paasche	4.224	1.144	2.539	4.399	1.693	2.739
Fisher	4.320	1.118	2.556	4.459	1.691	2.755
Walsh	4.323	1.126	2.556	4.461	1.696	2.756
Marshall-Edgeworth	4.322	1.118	2.556	4.460	1.691	2.754
Geary-Khamis	4.324	1.132	2.556	4.463	1.699	2.755

Source: Own elaboration based on data from Eurostat

Table 3 Yearly inflation rate [%] measured by different index formulas in Bulgaria (years: 2011, 2014, 2017)

Used index formula	COICOP 3			COICOP 4		
	2011	2014	2017	2011	2014	2017
Laspeyres	3.643	–1.730	1.111	4.502	–1.963	1.309
Paasche	3.163	–1.708	1.140	3.567	–2.046	1.357
Fisher	3.403	–1.719	1.126	4.033	–2.004	1.333
Walsh	3.421	–1.719	1.126	4.083	–2.002	1.333
Marshall-Edgeworth	3.407	–1.719	1.126	4.043	–2.005	1.333
Geary-Khamis	3.435	–1.720	1.127	4.119	–1.999	1.334

Source: Own elaboration based on data from Eurostat

Table 4 Yearly inflation rate [%] measured by different index formulas in the Czech Republic (years: 2011, 2014, 2017)

Used index formula	COICOP 3			COICOP 4		
	2011	2014	2017	2011	2014	2017
Laspeyres	2.059	0.512	2.446	2.152	0.375	3.138
Paasche	2.051	0.477	2.461	2.015	0.2333	3.102
Fisher	2.055	0.495	2.453	2.083	0.304	3.120
Walsh	2.056	0.495	2.454	2.083	0.305	3.120
Marshall-Edgeworth	2.055	0.495	2.453	2.084	0.304	3.121
Geary-Khamis	2.057	0.495	2.455	2.081	0.306	3.121

Source: Own elaboration based on data from Eurostat

CONCLUSIONS – RESULTS

Case 1 in our empirical study (see Empirical Study section) concerns the elementary aggregation. Our first results are not surprising, i.e. after using scanner data on child safety seats, we observe substantial differences between direct and chained elementary indices (see Figure 1a, 1b and 1c), in particular, the smallest differences are observed in the case of the Dutot formula (Figure 1c) and the biggest differences rise in the case of the Jevons index. The relations between chained elementary indices seem to be adequate to their known relations in the fixed basket approach, i.e. the chained Jevons index provides the smallest values (see Figure 1d). Figure 2 compares the superlative Fisher, Törnqvist and Walsh indices together with their chained versions and it considers also two well-known, symmetrical price indices, namely the Marshall-Edgeworth and the Geary-Khamis formulas. In the fixed basket approach, superlative indices approximate each other (Diewert, 1976), and in our case, they behave in the same way, i.e. there are no substantial differences between superlative indices and between their chained versions (see Figure 2b and 2c, Table 1) in our study. Similarly, the chained Marshall-Edgeworth and the chained Geary-Khamis indices do not differ strongly from the chained Fisher index in the considered case (Figure 2d).

Nevertheless, due to the dynamic structure of the used scanner data set, the choice between the direct method and the chained one does matter (see Figure 1a). For instance, the two-yearly price dynamics in the considered group of products measured by the chained Fisher index is bigger over 4 p.p. than the analogous price change measured by the direct Fisher index (see Table 1). Case 1 of our empirical study allows us to also note that differences between the chained Jevons and the chained Fisher indices are large (see Figure 3, Table 1) and in the case of measurement of yearly price dynamics the difference may exceed 3.9 p.p. (Table 1).

In **Case 2**, we compare CPIs of three countries (the United Kingdom, Bulgaria, the Czech Republic) calculated by using several weighted price index formulas and for two levels of data aggregation (COICOP-3 and COICOP-4). Firstly let us note that the CPI substitution bias is rather small in the case of the year-to-year inflation measurement (the highest bias level is observed in Bulgaria in 2011 – see Figure 5b) and as a rule the differences between the Laspeyres and the Fisher price indices do not exceed 0.15 p.p. (see Figure 4b, 5b and 6b). Nevertheless, calculating inflation rates for longer time intervals, we observe much bigger differences between the Laspeyres and Fisher formulas (together with their chained versions), so the annual updating of weights in the CPI measurement is very important for the CPI substitution bias reduction (see Figure 4a, 5a and 6a). Although all considered weighted formulas provide quite similar values for year-to-year CPI calculations (see Figure 4d, 5d and 6d), we should be aware of the fact that even small underestimation or overestimation of the real value of inflation may be very dangerous for national economies. Table 2–4 provide some detailed information about differences between yearly inflation rates measured by different price index formulas in considered countries. For instance, the above-mentioned differences in 2011 after using the Laspayres and the Fisher formulas are 0.096 p.p. and 0.24 p.p. in the case of the United Kingdom and Bulgaria respectively (see Table 2 and Table 3). The choice between the superlative formulas and the Marshall-Edgeworth or the Geary-Khamis indices is not so important in our study as we thought it would be. Moreover, the used data aggregation level seems to strongly influence yearly inflation rate calculations (see Table 2–4). For instance, the yearly inflation rate in 2017 in the Czech Republic measured by the Laspeyres formula equals 2.446 % and 3.138% for the COICOP-3 digit and COICOP-4 digit data aggregation levels respectively.

DISCUSSION AND GENERAL REMARKS

In the traditional CPI measurement, we use elementary price indices for calculations of price dynamics at the lowest level of data aggregation and the most recommended (by Eurostat) elementary price index formula is the Jevons index. This recommendation is based on economical, statistical and axiomatic approaches in the price index theory (Levell, 2015). For instance, from the axiomatic point of view, the Jevons index satisfies desirable axioms: the time reversal test and the circular test. The same property can be observed in the case of Dutot formula, and thus, when the basket of goods is fixed in compared time moments, we do not observe differences between values of these indices and values of their chained versions. Nevertheless, it may change in the case of using scanner data, mainly due to the fact that scanner data sets have a very dynamic structure, i.e. they include many cases of new and disappearing goods, strongly seasonal goods or temporary unavailable goods. In the case when the basket of goods observed during the year is not fixed, we should not expect that the direct and chained Jevons (or Dutot) indices will have the same values. Although most countries that use scanner data in their CPI calculations still apply the chained Jevons index for this purpose, many statisticians recommend using the so called multilateral methods for scanner data sets (Chessa, 2017). Multilateral methods are not investigated in this paper, nevertheless in Case 1 we consider additionally superlative indices which play an important role in constructing some multilateral methods (such as the GEKS or CCDI). Please note that scanner data provide information about prices and also quantities, so it is possible to use weighted price index formulas in their case. Taking into consideration the dynamic structure of scanner data sets, we should prefer the chained superlative price index formulas rather than the direct ones or rather, as it is suggested in the literature, the multilateral methods should be applied (the multilateral indices do not suffer from the chain drift). To sum up please note, that the choice between the elementary formulas and weighted price indices will have a substantial impact on final results in the CPI measurement. The general remark is also the fact that not only the choice of the price index formula but also the level of data aggregation used in calculations is really important in CPI calculations. Similarly, the level of the CPI substation bias seems to be sensitive to changing the COICOP level.

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