Modelling Structural Relations for Tourism Demand: the Central European Cases

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Abstract

This application study concentrates on causal links, directed from economic parameters to internal Czech and Slovak tourism for both, short- as well as long-term changes in time series. As tourism forms a fundamental part of the service industry with many social and environmental connections, the structural equation modelling (SEM) methodology, i.e. simultaneous equations covering the number of fundamental and derived variables, is used. Taking into account the market specifics with relative close history, the identical models for countries are selected, enabling a precise mutual comparison. The data on a ratio of non-residents and residents, together with nights spent and other parameters, are examined for first as well as seasonal differencing. For instance, the destination living cost, relative wages and salaries, the balance of payments, labour productivity, trade openness and harmonized unemployment as exogenous variables, are introduced. Covering short and long horizons, labour productivity is a fundamental parameter in the Czech Republic. Significant relations proved differently between countries presented by trade openness as a factor of the global economy. The members of destination management or other authorities can appreciate the results, concentrated especially on various accommodation establishments and hospitality.

Keywords

Structural equation modelling, causality, simultaneous equations, tourism

JEL code

C38, C50, Z30

INTRODUCTION

Industrial production and manufacturing still present an important segment in the Czech Republic and Slovakia. However, many direct, indirect, and induced effects of tourism make it a significant contributor as well. Tourism is a very dynamic sector ranking globally third position behind the petrochemical and car industries in terms of volume of sales, employing a vast workforce. It represents a fundamental activity of society, connecting employment, retail, services, and many other fields in a variety of effects. According to the World Tourism Organization, tourism demonstrates resilience to geopolitical and global economic instabilities. The travel and tourism industry are important sectors of the economy, contributing 10.4% of world GDP (WTTC, 2019). It has created 319 million jobs worldwide,

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representing roughly 1 out of every 10 jobs. The prediction for tourism is also positive despite recent global events. In tourism time series data, the econometric approaches of time-varying parameter models, error correction model and autoregressive distributed lag model have been widely adopted (Assaker, Vinzi, O'Connor, 2010; Song and Witt, 2000). Although the application of SEM is appropriate in tourism, with demand related to the determinants of consumer behaviour, such methodology is seldom applied. It is also suitable at an aggregate level due to causal links between a variety of variables related to supply and corresponding demand.

Tourism generally supports foreign exchange earnings, it promotes investment (Krueger, 1980; Balaguer and Cantavella-Jorda, 2002) and introduces an economy of scale that decreases local production costs and reduces unemployment (Brida and Pulina, 2010). There are also differences of opinion for economic-driven tourism growth expressed in many works (see, e.g. Oh, 2005; Payne and Mervar, 2010). They presume that tourism is being developed through current economic steps connected to human capital on the rebound and this forms the basis for subsequent support of tourism activities. Dritsakis (2012) has shown more significant influence of tourism on small developing countries as opposed to developed ones. Using the VAR-based spillover index, the selected studies also demonstrate dynamic bidirectional causality as a tourism-economic relationship that is not stable over time and heavily dependent on economic events such as the Great Recession of 2007 or debt crises (Antonakakis, Dragouni, Filis, 2015). Despite current economic development, tourism is generally linked to negative influences such as overcrowding, increasing demand for goods and services, lower living standards for locals, pollution, and the devastation of environment. Society and tourism as a whole form a complex system with many induced effects.

Tourism and some related economic variables are characterized by extreme high seasonality that encompasses remarkable differences between reported levels in summer and winter months. Seasonal information is subject to strong cyclic variations within the year and should be removed from the data before analysis. However, the magnitude of seasonal fluctuations is seldom constant, together connecting the combination of economic parameters. For this reason, the standard techniques of seasonal adjustment used in relatively more stable systems, cannot be applied. In the case of variables that are non-stationary and not causally connected, the use of original time series can cause a problem especially for random variables with infinite variance. Differencing approaches help to reduce trends and seasonality. Usually the process of differencing continues until I(0) is reached, although the higher-orders lack economic interpretability. It is evident that the first differencing of the 4-quarter moving average is the same as quarterly differencing up to the constant (Robertson, 2003, p. 62). The moving averages smooth out fluctuations and short-term volatility in majority of time-series data. But traditional moving averages have the disadvantage of insinuating a lag effect or generally close processes into computations. Despite the fact that no every econometrician agrees with the differencing approach, because this process causes some loss of information in the original series, the results can be interpreted as relative to the shifts of time series.

SEM is an increasingly popular statistical method used for testing complex models with several endogenous (dependent) and exogenous (independent) variables. This popularity is also increasing in the tourism industry despite debates on the appropriateness of different fit indices, multivariate normality conditions, sample size, and various estimation methods. The advantage of SEM over other theoretically well-equipped regression approaches (Bílková, 2020) is that structural equations are able process complex models covering various real situations. This is despite fact that the requirement of sufficient reliability is seldom fulfilled in social sciences (Bohrnstedt and Carter, 1971). Modelling for unexplained variances in endogenous variables is also an advantage of structural equations. Moreover, for a model with a poor fit, it can be decided if this is due to a lack of correlations between input variables or because of the poor reliability of indicators used. SEM also allows for modelling recursive and non-recursive causal relationships. But opposite to explanatory approaches, all relational patterns in SEM must be fully established on the background theory of the problem. Studies suggest that no unified rule-of-thumb exists in the SEM modelling environment, rather the emphasis should be placed on a decision regarding each type of evidence in a particular study, taking into account the whole model and deviations from input assumptions (Nye and Drasgow, 2011; Williams and O' Boyle, 2011).

The approaches of structural modelling can be divided into i) the confirmatory factor analysis model, ii) simultaneous equations approach (also called path analysis), and iii) a full structural solution. In this study, the econometric technique of simultaneous equations is used for exclusively measurable (directly observed) variables without incorporating latent constructs because the variables do not allow their generation in a differentiated data space. For the number of specific rules, these approaches differ by identification, where necessary and sufficient conditions are often directly available. The estimation methods that can be mentioned include maximum likelihood, generalized least squares, weighted and unweighted least squares, asymptotically distribution-free criterion, and ordinary least squares. Maximum likelihood estimation with the assumption of multivariate normality is the dominant approach for SEM techniques. An alternative estimation approach of generalized least squares does not involve a multivariate normality assumption based on input data (Hayduk, 1987). But the application requirement of such alternate methods often lies in a large sample of data, so the results can be evaluated as valid. Currently, Bayesian structural equation modelling should be mentioned. Partial least squares path modelling can also be considered as a variant approach to SEM that is not based on a covariance matrix, which belongs to the family of soft-modelling techniques.

Since SEM is an only approximation to reality, the number of fit indices measures the goodness of the model hypothesized. The most common fit statistic is chi-square, based on the likelihood ratio of value testing a hypothesized model opposite to the alternative, where the covariance matrix is unconstrained (Bagozzi and Yi, 1988). But there exist many additional indices measuring the appropriateness of the model from different perspectives. Evaluation, reporting and the consequent interpretation of model fit indices are one of the most controversial issues in practice. Covering the effect size, R^2 criterion is essential to evaluate the structural model and it corresponds to a proportion of endogenous variables variance explained by the exogenous ones. But there are known cases of acceptable fit indices that account for 1% of the variance in endogenous variables (see, e.g. Tomarken and Waller, 2003). Hoyle and Panter (1995) also mention the incomplete reporting of R^2 for endogenous variables in the model. Any alternative models almost always exist for the data and the specification of alternative models is often ignored (MacCallum et al., 1993). The models are usually respecified due to relations being weaker than expected, in accordance with the model substantive meaning, and with theoretical and empirical justification (Anderson and Gerbing, 1984; Hoyle and Panter, 1995).

Literature focusing on tourism for Central and Eastern Europe lack specific causal modelling, predominantly included in a panel data analysis targeted to wide areas, without incorporating local results. We concentrate on the Czech Republic and Slovakia for differenced data. Due to the nature of this study and inductive results, satisfying both substantive (economic) and statistical significance (Gunter, Önder, Smeral, 2019), is expected.

1 DATA AND METHODS

1.1 Data used

According to Lim (2006), and Song and Witt (2000), the dependent variables most often used for tourism demand modelling are the number of arrivals, length of stay or overnights, and tourist receipts. On the other hand, independent variables consider many fundamental and derived indicators, e.g. income in the country of origin and discretionary income omitting expenditures for necessities, are the most frequent. It consists of the nominal or per capita disposable or national income and GDP or GNP as proxies for income. The studies also use income divided into wage and non-wage elements. Relative prices should be mentioned where the consumer price index and other factors often serve as proxies adjusted for differences in exchange rates between two destinations. Rather, the nominal exchange rates frequently used in studies are mistakenly perceived by tourists and do not respond to relative inflation rates. Other factors include transportation costs, promotion, direct foreign investment, capital outflows and economic indicators such as unemployment, real assets, government budget forecast, or change in income and income distributions. On the other hand, the study separating business, holiday flows and visits to friends and relatives (Turner and Witt, 2001) use explanatory variables based on destination living cost, airfare, retail sales, new car registration, GDP, trade openness, exports, imports, domestic loans, number of working days lost, and population.

Covering the time range of January 2003 to December 2019, the European Statistical Office database (Eurostat, 2019) has been used to download the information, unless otherwise stated. The scale is month or quarter for consequent use of a disaggregation approach (Rojíček et al., 2009) on neither seasonally nor calendar adjusted data. In the case of incorporating destination living cost, relative wages and salaries, and balance of payments, these are weighted by nights spent in the three most significant source countries in 2018 relative to the target one. The main sources are namely Germany, Slovakia and Poland for the Czech Republic and the Czech Republic, Poland and Germany covering Slovakia. Despite the fact, that relationships should be specified in advance for using causal modelling analysis, we consider all inputs significant in terms of tourism demand. However, the balance of payments parameter, as an example, is used only for seasonally differenced data due to the expectation of its impact on a longer scale. Rather than modelling regression connections between endogenous variables, the covariance relations are used, having a beneficial interpretation in a zero lag. The input variables considered in this study are summarized below, together with their abbreviated expressions.

Adjusted consumer prices correspond to the harmonized index of consumer price divided by Euro/ECU exchange rates in cases of national currency. The harmonized index is the overall index excluding energy and it connects the base in 2015. With wages and salaries at current prices as a part of national accounts in millions of Euros, disaggregation is used according to the wholesale and retail trade indicator. The balance of payments covers both goods and services in millions of Euros and, with a partner, the rest of the world. In this case, few-months forecasting is applied. Labour productivity index is also based on 2015 and covers real labour productivity per person. For labour productivity, disaggregation according to the industrial production indicator is used. In the case of trade openness, total import and export are outside the EU28 (extra EU) for trade. Here, disaggregation is applied for GDP with the industrial production indicator. In this study, GDP and its main components (output, expenditure and income) is the gross domestic product at market prices, current prices, and in millions of Euros. The use of two identical indicators has been tested for the interrelationships of the variables. Using the adjusted test on equality for two correlation coefficients, with the resulting statistic approximated by normal distribution, the null hypothesis is not rejected at the 1% significance level. Unemployment is expressed in thousands of persons. The data for domestic loans are collected by central banks. Here, it is converted to Euros based on averaged monthly series in the case of currency other than the national one. The consumer confidence indicator is used for balance.

Exogenous variables

DLC – destination living cost: adjusted consumer prices for weighted source countries at the ratio to the studied country;

RWS – relative wages and salaries: wages and salaries as a part of the GDP for source countries in the weighted form to the wages and salaries of the target country;

BAP – balance of payments: the balance of payments for source countries in weighted form to the studied one;

ACP – adjusted consumer prices: the harmonized index of consumer price divided by Euro/ECU exchange rates in the case of national currency;
LAP – labour productivity: real labour productivity per person as part of the quarterly national accounts;
TRO – trade openness: the sum of total import and export divided by the GDP of the target country;
HAU – harmonized unemployment: total unemployment considered;
DOL – domestic loans: the liabilities of households and non-profit institutions serving households with loans;
COC – consumer confidence: the balance of consumer confidence indicator.

Internal data for the number of arrivals and nights spent are provided by national statistical offices. They are linked to selected accommodation establishments, i.e. hotels, holiday and other short-stay accommodation, camping grounds, recreational vehicle parks and trailer parks. A forecast for arrivals is used in Slovakia. Forecasts for nights spent are used, however, for both the Czech Republic and Slovakia.

Endogenous variables
NRR – non-resident's and resident's ratio: arrivals of non-residents from the three most significant source countries to residents of the target country;
NTS – nights spent: overnights totalling three source countries and the studied one.

1.2 Methods

We consider trend and seasonality to be stochastic. The quarterly data averaged from monthly observations used as inputs cover further analyses. Such an approach allows for the monthly lag shift a more detailed process of the time series and serves to achieve an accurate prediction. Given the interpretational purposes of the demand elasticities, heteroskedasticity of input variables or potential asymmetric distribution, the log form of data is used, to which differentiation is further applied. Although the dummy variables are often used to model the remaining seasonality, the data are first differenced directly, and then for the 4-quarter moving averages, to smooth out seasonal variations. The Augmented Dickey-Fuller test reveals that the resulting series has no unit root at a 5% significance level, excepting tourism parameters that are stationary up to a significance of 0.1. In analysis, the number of observations varies, covering both differencing approaches and time series processes. The last step in treating the data is min-max normalization applied for overcoming excessively high input variances of some variables and for results interpretation. However, this adjustment modifies the model specified in a certain sense, generating certain goodness-of-fit measures, and it changes the parameter estimates and standard errors differently to using the correlation matrix (Ramlall, 2016). At min-max normalization, the fitting functions are scale-invariant with scale-free structural estimates. Any variable with a steadier distribution and minor distant observations makes a significant entrance into the analysis, which should be considered. Note that the strongly within collinear exogenous variables are rather omitted from analysis, as the modelling of covariance relations is limited. But the outputs for full extent of covariances and the one used herein do not differ in a great extent for the sample examples. Some numerical obstacles occur exceptionally when covering the time lag of selected variables.

Although the general model is more complex, the simultaneous equations system for measurable variables is used herein as a part of multivariate statistical analysis with broad applications (Bollen, 1989). For the presentation of the examined relations, we introduce the equation entry (Jöreskog, 1973; Jöreskog and Sörbom, 1979) as well as path diagrams (Keesing, 1972). Often, expressions for 1st and 2nd derivatives according to their structural parameters are necessary for the realization of numerical methods. The more general model than applied in this study and 1st derivative by using matrix differential calculus in a reduced form, are demonstrated in the Annex. It consists of the alternative derivation using a matrix trace in connection with the differential instead of the usual *vec* operator. Magnus and Neudecker (2019) mention such form in a universal sense. We use predominantly the maximum likelihood estimation for some of its advantages, e.g. it can overcome some degree of violation of the multivariate normality condition set for input data (Bollen, 1989). The number of independent elements in induced covariance matrix with some parameters fixed, make for 19 or 20 estimated parameters the approaches overidentified. Null **B** rule is sufficient for identification in our case. However, the models have correlated error terms and each equation consists of different set of regressors. To distinguish our method from those as seemingly unrelated regression, we use the structural equation methodology for its advantageous representation and perform some improvements on fit function having a direct effect on convergence. The variances of the variables are explicitly set as unknown. The R software environment (R Core Team, 2019) is used to solve the specific problem with the support of various packages: *sem* (Fox, 2017) – especially optimizerSem is used as a modified version of the current standard R optimizers, *tempdisagg* (Sax, 2020) – with special attention paid to the Chow-Lin method and *forecast* (Hyndman, 2020) – the ARIMA forecasting is targeted herein.

The goodness-of-fit statistics most often measure the difference between the observed covariance matrix and the covariance matrix implied by the model. The hypothesis $H:\boldsymbol{\Sigma}=\boldsymbol{\Sigma}(\boldsymbol{\theta})$ forms the main test in this study, approximated by chi-square distribution, opposite to the alternative, where $\boldsymbol{\Sigma}$ is unstructured (Bollen, 1989). The test statistic converges asymptotically to chi-square given by:

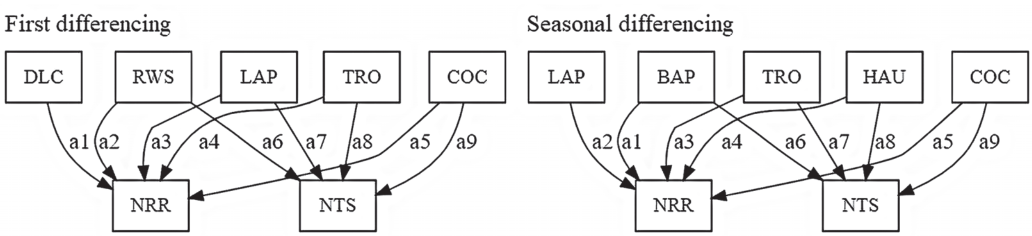
$$\chi^2 \approx (N-1)F\left(\mathbf{S},\boldsymbol{\Sigma}(\boldsymbol{\theta})\right), \tag{1}$$

sensitive to a lack of multivariate normality, and particularly, by kurtosis. The simulation studies confirm that in small samples of $N < 100$, the chi-square test statistic tends to be large (Boomsma, 1983; Anderson and Gerbing, 1984). The chi-square dependencies on correlations within the model are also known. Despite this fact, we consider chi-square as the most significant and applied test overall. Although the decision that the proposed model is not wrong, a p-value greater than 0.05 is selected for an acceptable model fit. The second criterion consisted of Akaike's information criterion (AIC) derived from the chi-square estimator indicated by (1) covering a null hypothesis used for comparison of models without the usual split samples incorporation. For overall model evaluations, the Bayesian criterion (BIC) is also used. Squares of multiple correlation coefficients and t-tests of individual coefficients and covariances are included herein. We use modification indices like Lagrange-multiplier asymptotically distributed as one-df chi-square score test statistics for the fixed and constrained parameters of the structural equation model.

2 RESULTS AND DISCUSSION

The model structures for the Czech Republic and Slovakia, separate in both first and seasonally differenced data, are graphically represented below (Figure 1). In the following, the results are with zero lags deep

Figure 1 Path diagrams



Source: Authors

interpreted. Negative values of BIC in such points demonstrate an exquisite fit for all models. The explicitly defined variances set as unknown, are not introduced in the following text, together with the modelled covariances demonstrated only in equational format.

In both countries, the proper model for first differenced data is achieved after a few modifications of the initial idea for adopting the additional covariances (cov). The only variable excluded by the modified version is DOL, due to the great degree of collinearity. The ACP parameter is omitted directly before the analysis. Expressed by equations, the first differenced system is as follows:

$$\text{NRR} = \text{a1 DLC} + \text{a2 RWS} + \text{a3 LAP} + \text{a4 TRO} + \text{a5 COC},$$

$$\text{NTS} = \text{a6 RWS} + \text{a7 LAP} + \text{a8 TRO} + \text{a9 COC},$$

Czech Republic

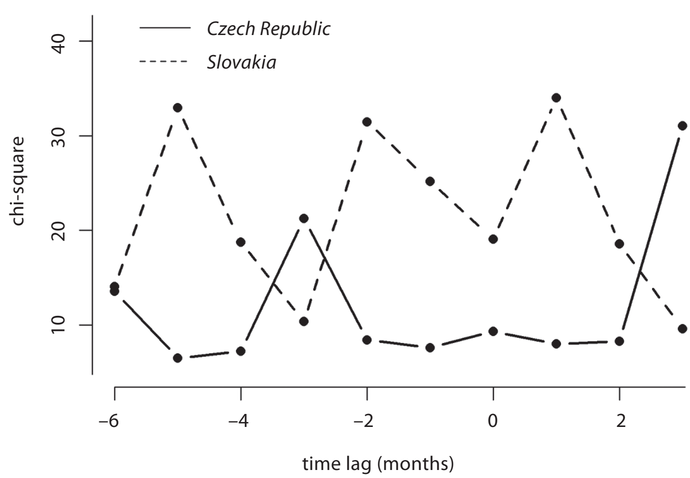
$$\text{cov}(\text{DLC, RWS}) = \text{c1},\ \text{cov}(\text{RWS, LAP}) = \text{c2},\ \text{cov}(\text{LAP, TRO}) = \text{c3},\ \text{cov}(\text{NRR, NTS}) = \text{c4},$$

Slovakia

$$\text{cov}(\text{DLC, LAP}) = \text{c1},\ \text{cov}(\text{DLC, COC}) = \text{c2},\ \text{cov}(\text{RWS, TRO}) = \text{c3},\ \text{cov}(\text{NRR, NTS}) = \text{c4}.$$

Covering Figure 2, the process of time lag for economic parameters are stated in some sense contrary to the spectral time-series data processing, as negative lag demonstrates delayed endogenous tourism variables. Although the maximum likelihood fit function is used for problem solving and has indisputable benefits, the generalized least squares approach is accommodated in the case the numerical algorithm fails. Assuming a true null hypothesis, multivariate normality of data for a maximum likelihood estimation and a sufficient sample size, both fit functions yield an approximate chi-square by multiplying them by (N – 1) and the discrepancy function (Loehlin, 2004, p. 54). No evident trend in the data is observed covering the Czech Republic and Slovakia. But generally, for both types of differencing, the relational pattern remains relatively constant to some degree of time shift. This fact confirms the suitability of selecting a zero lag for interpretational purposes.

Figure 2 Processes for first differences



Source: Authors

For the Czech Republic, the chi-square statistic value 10.24 and corresponding p-value 0.249 supports the model for goodness-of-fit. AIC criterion gains the approximate value 50.24, where about 35.62% variability of NRR, as well as 27.87% variability of NTS, are explained by the model. These are relatively small proportions. In the case of Slovakia, the chi-square statistic value 14.46 and corresponding p-value 0.071 supports the model for goodness-of-fit. AIC criterion gains the approximate value 54.46, where about 70.57% variability of NRR, as well as 76.38% variability of NTS, are explained by the model. Compared to the results of the Czech Republic, these values are significantly higher.

Below, the coefficients are interpreted for p-value < 0.1 covering Table 1. In the Czech Republic for first differencing of data, both LAP and TRO have a significant positive influence on NRR. On the other hand, NTS is most influenced by TRO negatively, then positively by COC, and again negatively by RWS. So, a higher degree of workload and open trade decrease the number of residents. The negative relation of TRO and RWS to NTS can be partially explained by the high input negative correlation of the first differenced NRR to NTS. COC positively influences NTS, which is expected. In Slovakia, the situation is reversed due to the strong positive correlation of NRR to NTS. Here, RWS has an especially positive influence on NRR. TRO has a negative influence on non-residents, while both LAP and COC have a positive influence. RWS, then LAP and COC, positively relate to NTS, here caused by non-residents. TRO negatively relates NTS. This is because at increasing residents, NTS falls with regards to the input data. An RWS, preferring source countries, provides non-residents to this territory. In the case of open trade, Slovaks prefer their own country.

In both countries, the proper model for seasonally differenced data after adopting the necessary covariances, is expressed in equational form as follows:

$$\mathrm{NRR} = \mathrm{a1\ BAP} + \mathrm{a2\ LAP} + \mathrm{a3\ TRO} + \mathrm{a4\ HAU} + \mathrm{a5\ COC}\,,$$

$$\mathrm{NTS} = \mathrm{a6\ BAP} + \mathrm{a7\ TRO} + \mathrm{a8\ HAU} + \mathrm{a9\ COC}\,,$$

Czech Republic
$$\mathrm{cov}(\mathrm{BAP,\ LAP}) = \mathrm{c1},\ \mathrm{cov}(\mathrm{LAP,\ HAU}) = \mathrm{c2},\ \mathrm{cov}(\mathrm{TRO,\ COC}) = \mathrm{c3},$$

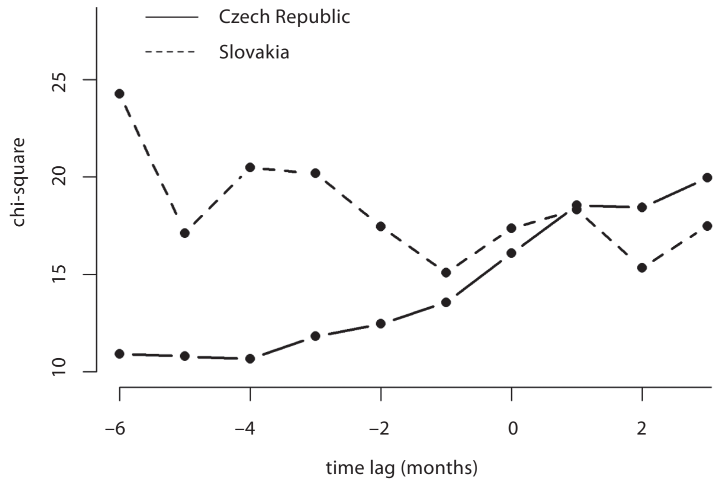
Slovakia
$$\mathrm{cov}(\mathrm{LAP,\ TRO}) = \mathrm{c1},\ \mathrm{cov}(\mathrm{LAP,\ COC}) = \mathrm{c2},\ \mathrm{cov}(\mathrm{BAP,\ LAP}) = \mathrm{c3}.$$

Table 1 Coefficients for first differenced data

Coeff	Czech Republic				Slovakia			
	Estimate	StdError	*t*-statistic	*p*-value	Estimate	StdError	*t*-statistic	*p*-value
a1	−0 .0438	0.0895	−0.489	6.25e-01	−0.0019	0.0573	−0.032	9.74e-01
a2	0.2378	0.1893	1.256	2.09e-01	0.6910	0.0755	9.153	5.54e-20
a3	0.5694	0.1344	4.235	2.29e-05	0.1672	0.0587	2.848	4.40e-03
a4	0.6241	0.1885	3.310	9.33e-04	−0.3786	0.0817	−4.632	3.63e-06
a5	−0.3186	0.2600	−1.226	2.20e-01	0.2256	0.1102	2.048	4.06e-02
a6	−0.3238	0.1915	−1.691	9.08e-02	0.8224	0.0916	8.975	2.83e-19
a7	−0.1752	0.1412	−1.241	2.15e-01	0.4321	0.0705	6.131	8.73e-10
a8	−0.7788	0.1996	−3.902	9.56e-05	−0.5877	0.0992	−5.923	3.16e-09
a9	0.5294	0.2756	1.921	5.47e-02	0.4331	0.1318	3.287	1.01e-03
c1	0.0207	0.0044	4.661	3.15e-06	0.0101	0.0051	2.001	4.54e-02
c2	0.0241	0.0060	3.989	6.65e-05	0.0062	0.0028	2.257	2.40e-02
c3	−0.0163	0.0059	−2.781	5.42e-03	−0.0119	0.0052	−2.305	2.12e-02
c4	−0.0697	0.0126	−5.553	2.82e-08	0.0146	0.0028	5.126	2.96e-07

Source: Authors

Figure 3 Processes for seasonal differences



Source: Authors

The process of time lag for economic parameters is stated in Figure 3. Compared to first differenced data, the range of statistic values is relatively lower, and a growing linear trend is partially revealed in the Czech Republic, excepting tourism lags.

Covering the Czech Republic, the chi-square statistic value 15.27 and corresponding *p*-value 0.084 supports the model for goodness-of-fit. AIC criterion gains the approximate value 53.27, where a 39.06% variability of NRR, as well as a 15.26% variability of NTS, is explained by the model. The value considering NTS is relatively low. In the case of Slovakia, the chi-square statistic value 14.58 and corresponding *p*-value 0.103 supports the model for goodness-of-fit. AIC criterion gains the approximate value 52.58, where a 41.49% variability of NRR, as well as a 34.16% variability of NTS, is explained by the model. Compared to the results of the Czech Republic, these values are significantly higher.

Table 2 Coefficients for seasonally differenced data

Coeff	Czech Republic				Slovakia			
	Estimate	StdError	*t*-statistic	*p*-value	Estimate	StdError	*t*-statistic	*p*-value
a1	0.2486	0.1370	1.815	6.96e-02	–0.0347	0.1032	–0.337	7.36e-01
a2	0.3806	0.1127	3.377	7.33e-04	0.1006	0.1462	0.688	4.92e-01
a3	–0.3085	0.1212	–2.546	1.09e-02	0.3336	0.1107	3.015	2.57e-03
a4	0.2467	0.1201	2.053	4.00e-02	–0.2720	0.0914	–2.976	2.92e-03
a5	0.3270	0.1491	2.194	2.83e-02	0.3908	0.1281	3.051	2.28e-03
a6	0.1134	0.1470	0.771	4.41e-01	0.1143	0.1075	1.063	2.88e-01
a7	–0.0544	0.1442	–0.377	7.06e-01	0.2167	0.0967	2.241	2.50e-02
a8	–0.3317	0.1329	–2.497	1.25e-02	–0.5088	0.1011	–5.031	4.87e-07
a9	–0.3692	0.1774	–2.081	3.75e-02	–0.1019	0.1219	–0.836	4.03e-01
c1	0.0180	0.0058	3.131	1.74e-03	0.0212	0.0051	4.118	3.82e-05
c2	–0.0165	0.0060	–2.735	6.23e-03	0.0129	0.0036	3.584	3.39e-04
c3	–0.0065	0.0039	–1.658	9.72e-02	0.0088	0.0035	2.522	1.17e-02

Source: Authors

In the Czech Republic, all exogenous variables are significant for NRR, where the lowest p-value gains LAP. All BAP, LAP, HAU and COC are positively related to non-residents covering one exception of TRO. Both HAU and COC have a significant negative influence on NTS. The relations are different to first differenced data due to long-lasting processes and the later achievement of economic equilibrium in the data. HAU has a negative influence, especially for residents. The extra behaviour of the TRO and COC parameters is probably due to their relation, although not so significant. Covering Slovakia, TRO and COC are both significantly and positively related to NRR. But the HAU influence is negative. HAU especially relates negatively, and TRO positively, to NTS. Such a behavioural pattern is expected.

CONCLUSION

Tourism is a fundamental part of the global economy depending heavily on human capital. Applying the SEM approach of simultaneous equations to Central European countries for first and seasonally differenced time series data has demonstrated its appropriateness. Despite a fact, the number of scientists argues the economic theory cannot handle differences due to long-run relationships, and the equilibrium stages theory gravitates toward, we interpret the results as shifts in input data. The chi-square statistics for all the explained models are not significant at the 5%, even accompanied by other statistics. It is important to note, the identical models for the Czech Republic and Slovakia are selected, due to their relatively close history and specific economies. Parameters persisting the first and seasonal differencing are perceived especially important. Although a deeper examination is incorporated herein, we summarize the most significant outputs below. The results can capture members of destination management at the national and international levels.

Covering a short-term measure of changes in the sense of first differences, labour productivity and trade openness are the most important parameters in the Czech Republic, while relative wages and salaries parameter is the most significant in Slovakia. An assumption of the importance of relative wages and salaries in both countries is not confirmed. The workload of residents negatively influences total nights spent caused by non-residents in the Czech Republic stay shorter. Because trade openness enhances the number of non-residents relative to residents apparently, this also decreases the number of nights spent. In the Czech Republic, the workload and probably higher level of the wages for selected groups avoid travelling to a great extent, or departures of Czechs go abroad. As tourism is a part of the global economy, the increased import and export have on non-residents a positive influence. In Slovakia, opposite relations in many cases, are observed. Trade openness negatively influences non-residents as Slovaks prefer their own territory and reveal shorter stays compared to non-residents. From this reason, in a short horizon of changes, the import and export decrease nights spent. The higher degree of workload also decreases residents. But in Slovakia, the main factor is relative wages and salaries increasing non-residents which consequently enhances nights spent. Contrarywise, in the Czech Republic, the relative wages and salaries has significant influence only for nights spent. The negative relation is an expression of the reality that increased wage level in source countries should raise the activity of non-residents, contrary spending less time. Both countries separate as the Czech Republic is more concentrated on the global economy and its consequences. Slovaks are more directed to their own country in the short horizon. On the other hand, consumer confidence positively, and trade openness negatively, relate nights spent in both countries. So, confidence and corresponding safety are perceived positively, connected to stays, while the global economy has the effect reversed.

For the Czech Republic and long seasonal horizon, again one of the most significant factors is labour productivity. Between others, harmonized unemployment negatively influences residents, while decreases nights spent. In the case of a first differencing, the trade openness has a positive influence on non-residents, but in a long horizon of mutual changes, the role adopts residents. The consumer confidence relation to nights spent is strange and can be caused by their primary mutual relations. Covering Slovakia, all pattern is expected. Despite for a range of the short time changes, trade openness negatively influences non-residents, moreover here, the relationship is reversed in connection to increased nights spent. For a long horizon of changes, global economy expressed by import and export is more pronounced. While labour productivity persists in the Czech Republic as a significant factor increasing non-residents, trade openness alters its role as supports residents. The influence of import and export in Slovakia is proven opposite completely, as the global economy rather positively relates non-residents on a long horizon. Also, note the significant role of harmonized unemployment differing its influence between both countries for non-residents and residents, as negatively influences nights spent generally. Specific visitors to countries studied are negatively perceived by such a parameter, eliminating seasonal fluctuations. Consumer confidence positively relates non-residents for both territories. The relations included in both short, as well as a long horizon of changes, are especially labour productivity and corresponding wages influence to non-residents in the Czech Republic. Mutual comparison of both countries reveals predominantly different trade openness behaviour as a factor of the global economy in first and seasonally differenced data. Such results can precise the planning capacity of accommodation establishments, the number of bed places, hospitality, and other industries.

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ANNEX

The simultaneous equations system has the general expression:

$$y = \mathbf{B}\, y + \mathbf{\Gamma}\, x + \zeta, \qquad \text{(a)}$$

where both $\mathbf{B}$ $(p \times p)$, $\mathbf{\Gamma}$ $(p \times q)$ are matrices of coefficients with y $(p \times 1)$, x $(q \times 1)$ and ζ $(p \times 1)$ random vectors. Vector y marks endogenous variables, x is the vector of exogenous variables and ζ corresponds to the vector of random errors (Timm, 2002). Here, $E(x) = E(\zeta) = 0$, and thus $E(y) = 0$, variables are centred, and x with ζ are moreover uncorrelated. $\mathbf{\Phi}$ is the covariance matrix of x and $\mathbf{\Psi}$ is the covariance matrix of ζ. Furthermore, we consider $x \sim N_q(0, \mathbf{\Phi})$, $\zeta \sim N_p(0, \mathbf{\Psi})$. If we expect the matrix $(\mathbf{I} - \mathbf{B})$ non-singular, then the model can be written in the form $y = (\mathbf{I} - \mathbf{B})^{-1}\mathbf{\Gamma}x + (\mathbf{I} - \mathbf{B})^{-1}\zeta$.

Covering $\mathbf{B} = \mathbf{0}$, model (a) can be perceived as one of the specific forms of multivariate regression. If $\mathbf{B}$ is lower triangular matrix and the errors are not correlated, then the model is recursive.

The population covariance matrix of vector $(y, x)'$ can be written $\mathbf{\Sigma}$ and its expression in structural parameters $\boldsymbol{\theta} = (\mathbf{B}, \mathbf{\Gamma}, \mathbf{\Phi}, \mathbf{\Psi})$ is as follows:

$$\mathbf{\Sigma}(\boldsymbol{\theta}) = \begin{bmatrix} (\mathbf{I}-\mathbf{B})^{-1}(\mathbf{\Gamma\Phi\Gamma}'+\mathbf{\Psi})(\mathbf{I}-\mathbf{B})^{-1'} & (\mathbf{I}-\mathbf{B})^{-1}\mathbf{\Gamma\Phi} \\ \mathbf{\Phi\Gamma}'(\mathbf{I}-\mathbf{B})^{-1'} & \mathbf{\Phi} \end{bmatrix}. \quad \text{(b)}$$

The term for model identification is a key in structural equations, although the *Error message* is often incorporated in various program environments. According to Bollen (1989), the parameter is identified, if it can be expressed using elements of matrix $\mathbf{\Sigma}$. The model is identified if all elements of $\boldsymbol{\theta}$ can be expressed in this way. The problem of identification is solved in special cases (Skrondal and Rabe-Hesketh, 2004). The matrix $\mathbf{\Sigma}$ often is not available as approximated by sample covariance matrix $\mathbf{S}$.

The symmetric matrix in (b) has the order $(p + q)$. The number of its independent elements is thus $v = \frac{p+q}{2}(p+q+1)$. We concentrate our attention only on the fundamental equality solution $\mathbf{\Sigma} = \mathbf{\Sigma}(\boldsymbol{\theta})$ for the unknown $\boldsymbol{\theta}$ as the number of elements of $\boldsymbol{\theta}$ must be $\leq v$.

Two cases for parameter identification embracing special conditions are:

1) For $\mathbf{B} = \mathbf{0}$ holds $\boldsymbol{\theta} = (\mathbf{\Gamma}, \mathbf{\Phi}, \mathbf{\Psi})$. If $\mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma}_{yy} & \mathbf{\Sigma}_{yx} \\ \mathbf{\Sigma}_{xy} & \mathbf{\Sigma}_{xx} \end{bmatrix}$, then $\mathbf{\Gamma}' = \mathbf{\Sigma}_{xx}^{-1}\mathbf{\Sigma}_{xy}; \mathbf{\Gamma\Phi\Gamma}' + \mathbf{\Psi} = \mathbf{\Sigma}_{yy}$, from where $\mathbf{\Psi} = \mathbf{\Sigma}_{yy} - \mathbf{\Sigma}_{yx}\mathbf{\Sigma}_{xx}^{-1}\mathbf{\Sigma}_{xy}$.
2) If the model is recursive, then identification is related to an algebraic operation, i.e. elimination.

In the following, the model is expected to be identified. We have a joint probability density $f(z, \boldsymbol{\theta})$ of the random vector $z = (z_1, z_2, ..., z_n)$ dependent on the parameters $\boldsymbol{\theta}$. The scalar fit functions are constructed as $F(\mathbf{S}, \mathbf{\Sigma}(\boldsymbol{\theta})) \geq \mathbf{0}$, smooth enough, for which $F(\mathbf{S}, \mathbf{\Sigma}(\boldsymbol{\theta})) = 0 \Leftrightarrow \mathbf{\Sigma}(\boldsymbol{\theta}) = \mathbf{S}$. The sample covariance matrix is $\mathbf{S} = \frac{1}{N-1}\sum_{i=1}^{N}(z_i - \bar{z})(z_i - \bar{z})'$ with the number of observations N. The point $\boldsymbol{\theta}_0$, where the minimum of function F in variable $\boldsymbol{\theta}$ occurs, corresponds to the searched estimate of the structural parameters. We demonstrate the two estimates most often used. The first one is derived from maximum likelihood theory, the second is based on weighted least squares:

1) $F_{ML} = \ln|\mathbf{\Sigma}(\boldsymbol{\theta})| + tr(\mathbf{S\Sigma}^{-1}(\boldsymbol{\theta})) - \ln|\mathbf{S}| - (p + q)$
2) $F_{WLS} = \frac{1}{2}tr[\mathbf{W}^{-1}(\mathbf{S} - \mathbf{\Sigma}(\boldsymbol{\theta})]^2 = \frac{1}{2}\|\mathbf{W}^{-1}(\mathbf{S} - \mathbf{\Sigma}(\boldsymbol{\theta}))\|^2$,

where $\|\mathbf{M}\| = (tr(\mathbf{M}'\mathbf{M}))^{\frac{1}{2}}$ denotes the norm of matrix $\mathbf{M}$, and $\mathbf{W}$ is a weight matrix. Specifically, in case of $\mathbf{W}^{-1} = \mathbf{S}^{-1}$, the estimation method corresponds to generalized least squares, and in the case of $\mathbf{W}^{-1} = \mathbf{I}$, it is the standard method of least squares.

We indicate the solution of first estimation method F_{ML} under the assumption that vector $z = (y, x)'$ can be described by a multivariate normal distribution $\mathrm{N}(0, \mathbf{\Sigma})$. Stationary equations for searching the extrema of fit functions are (excepting special cases) nonlinear, and finding a solution is relatively difficult. There are numerical methods available in current program environments covering various optimization routines. Different gradient approaches are also worth mentioning. The realization of numerical methods in most cases requires knowing the expression for 1st and 2nd derivatives according to their structural parameters. We indicate the computations of the 1st derivative.

To find the function F_{ML} minimum, the following formula is used:

$$F_{ML} = \ln|\mathbf{\Sigma}(\boldsymbol{\theta})| + tr(\mathbf{S}\mathbf{\Sigma}^{-1}(\boldsymbol{\theta})). \quad \text{(c)}$$

Formula (a) is linear, but the induced covariance matrix $\mathbf{\Sigma}(\boldsymbol{\theta})$ given by (b) is not a linear function of the parameters. Expression (c) is a scalar function of matrix variables. As introduced below, the scalar property can be realized by matrix trace. For the computation derivatives of the 1st and 2nd order on F_{ML}, using the results of Jöreskog (1973) or Magnus and Neudecker (2019) enables us to work with its differentials. The extrema type is decided by the Hesse matrix, consisting of partial derivatives of the 2nd order for F_{ML}.

In this study, we for F_{ML} execute the 1st partial derivative which is fundamental for finding the minimum (c). The differential is used instead of the *vec* operator and Hadamard product as an alternative. We use:

$$dF_{ML}\left(\mathbf{\Sigma}(\boldsymbol{\theta})\right) = tr\left(\mathbf{\Sigma}^{-1}(\boldsymbol{\theta})d\mathbf{\Sigma}(\boldsymbol{\theta})\right) - tr(\mathbf{\Sigma}^{-1}(\boldsymbol{\theta})\left(d\mathbf{\Sigma}(\boldsymbol{\theta})\right)\mathbf{\Sigma}^{-1}(\boldsymbol{\theta})\mathbf{S}) = tr\left[\mathbf{\Sigma}^{-1}(\mathbf{\Sigma}-\mathbf{S})\mathbf{\Sigma}^{-1}d\mathbf{\Sigma}\right].$$

The matrix $\mathbf{\Sigma}^{-1}(\mathbf{\Sigma} - \mathbf{S})\mathbf{\Sigma}^{-1} = \mathbf{\Omega}(\boldsymbol{\theta})$ is implemented and divided into blocks $\mathbf{\Omega} = \begin{bmatrix} \mathbf{\Omega}_{yy} & \mathbf{\Omega}_{yx} \\ \mathbf{\Omega}_{xy} & \mathbf{\Omega}_{xx} \end{bmatrix}$ of the same type as matrix $\mathbf{\Sigma}$.

The computation blocks of matrix $\mathbf{\Omega}$ using $\mathbf{\Sigma}$ is executed using the Schur complement. We get:

$$dF_{ML}\left(\mathbf{\Sigma}(\boldsymbol{\theta})\right) = tr\left(\mathbf{\Omega}_{yy}d\mathbf{\Sigma}_{yy}\right) + tr\left(\mathbf{\Omega}_{yx}d\mathbf{\Sigma}_{xy}\right) + tr\left(\mathbf{\Omega}_{xy}d\mathbf{\Sigma}_{yx}\right) + tr\left(\mathbf{\Omega}_{xx}d\mathbf{\Sigma}_{xx}\right).$$

The following demonstrates computation of individual elements. If we introduce a symmetric matrix $\mathbf{A} = \mathbf{\Gamma\Phi\Gamma}' + \mathbf{\Psi}$, then:

$$\begin{aligned}
tr\left(\mathbf{\Omega}_{yy}d\mathbf{\Sigma}_{yy}\right) &= tr\left\{\mathbf{\Omega}_{yy}d\left[(\mathbf{I}-\mathbf{B})^{-1}\mathbf{A}(\mathbf{I}-\mathbf{B})^{-1'}\right]\right\} = 2tr\left\{\mathbf{\Omega}_{yy}d(\mathbf{I}-\mathbf{B})^{-1}\mathbf{A}(\mathbf{I}-\mathbf{B})^{-1'}\right\} \\
&+2tr\left\{\mathbf{\Omega}_{yy}(\mathbf{I}-\mathbf{B})^{-1}d\mathbf{\Gamma\Phi\Gamma}'(\mathbf{I}-\mathbf{B})^{-1'}\right\} + tr\left\{\mathbf{\Omega}_{yy}(\mathbf{I}-\mathbf{B})^{-1}\mathbf{\Gamma}d\mathbf{\Phi\Gamma}'(\mathbf{I}-\mathbf{B})^{-1'}\right\} \\
&+tr\left\{\mathbf{\Omega}_{yy}(\mathbf{I}-\mathbf{B})^{-1}\mathrm{d}\mathbf{\Psi}(\mathbf{I}-\mathbf{B})^{-1'}\right\} = 2tr\left\{(\mathbf{I}-\mathbf{B})^{-1}\mathbf{A}(\mathbf{I}-\mathbf{B})^{-1'}\mathbf{\Omega}_{yy}(\mathbf{I}-\mathbf{B})^{-1}d\mathbf{B}\right\} \\
&+2tr\left\{\mathbf{\Phi\Gamma}'(\mathbf{I}-\mathbf{B})^{-1'}\mathbf{\Omega}_{yy}(\mathbf{I}-\mathbf{B})^{-1}d\mathbf{\Gamma}\right\} + tr\left\{\mathbf{\Gamma}'(\mathbf{I}-\mathbf{B})^{-1'}\mathbf{\Omega}_{yy}(\mathbf{I}-\mathbf{B})^{-1}\mathbf{\Gamma}d\mathbf{\Phi}\right\} \\
&+tr\left\{(\mathbf{I}-\mathbf{B})^{-1'}\mathbf{\Omega}_{yy}(\mathbf{I}-\mathbf{B})^{-1}d\mathbf{\Psi}\right\}.
\end{aligned}$$

$$\begin{aligned}
tr\left(\mathbf{\Omega}_{xy}d\mathbf{\Sigma}_{yx}\right) &= tr\left\{\mathbf{\Omega}_{xy}\left[(\mathbf{I}-\mathbf{B})^{-1}d\mathbf{B}(\mathbf{I}-\mathbf{B})^{-1}\mathbf{\Gamma\Phi} + (\mathbf{I}-\mathbf{B})^{-1}d\mathbf{\Gamma\Phi} + (\mathbf{I}-\mathbf{B})^{-1}\mathbf{\Gamma}d\mathbf{\Phi}\right]\right\} \\
&= tr\left\{(\mathbf{I}-\mathbf{B})^{-1}\mathbf{\Gamma\Phi\Omega}_{xy}(\mathbf{I}-\mathbf{B})^{-1}d\mathbf{B}\right\} + tr\left\{\mathbf{\Phi\Omega}_{xy}(\mathbf{I}-\mathbf{B})^{-1}d\mathbf{\Gamma}\right\} \\
&+tr\left\{\mathbf{\Omega}_{xy}(\mathbf{I}-\mathbf{B})^{-1}\mathbf{\Gamma}d\mathbf{\Phi}\right\}.
\end{aligned}$$

From the property of trace, we know $tr\left\{\mathbf{\Omega}_{yx}d\mathbf{\Sigma}_{xy}\right\} = tr\left\{\mathbf{\Omega}_{xy}d\mathbf{\Sigma}_{yx}\right\}$ and $tr\left(\mathbf{\Omega}_{xx}d\mathbf{\Sigma}_{xx}\right) = tr\left(\mathbf{\Omega}_{xx}d\mathbf{\Phi}\right)$. Thus $dF_{ML}\left(\Sigma(\boldsymbol{\theta})\right)$ can be expressed in partial differentials of the structural parameters:

$$dF_{ML}\left(\boldsymbol{\Sigma}(\boldsymbol{\theta})\right) = tr\left\{2\left(\mathbf{I}-\mathbf{B}\right)^{-1}\left[\mathbf{A}\left(\mathbf{I}-\mathbf{B}\right)^{-1\prime}\boldsymbol{\Omega}_{yy}+\boldsymbol{\Gamma\Phi\Omega}_{xy}\right]\left(\mathbf{I}-\mathbf{B}\right)^{-1}d\mathbf{B}\right\}$$
$$+tr\left\{2\boldsymbol{\Phi}\left[\boldsymbol{\Gamma}'\left(\mathbf{I}-\mathbf{B}\right)^{-1\prime}\boldsymbol{\Omega}_{yy}+\boldsymbol{\Omega}_{xy}\right]\left(\mathbf{I}-\mathbf{B}\right)^{-1}d\boldsymbol{\Gamma}\right\}+tr\left\{\left(\mathbf{I}-\mathbf{B}\right)^{-1\prime}\boldsymbol{\Omega}_{yy}\left(\mathbf{I}-\mathbf{B}\right)^{-1}d\boldsymbol{\Psi}\right\}$$
$$+tr\left\{\left[\left(\boldsymbol{\Gamma}'\left(\mathbf{I}-\mathbf{B}\right)^{-1\prime}\boldsymbol{\Omega}_{yy}\left(\mathbf{I}-\mathbf{B}\right)^{-1}+2\boldsymbol{\Omega}_{xy}\left(\mathbf{I}-\mathbf{B}\right)^{-1}\right)\boldsymbol{\Gamma}+\boldsymbol{\Omega}_{xx}\right]d\boldsymbol{\Phi}\right\}.$$

We do not take into consideration that matrices $\boldsymbol{\Sigma}$**,** $\boldsymbol{\Phi}$ are symmetric and $\boldsymbol{\Psi}$ is moreover diagonal in some cases, which can simplify the computations. The right side of the resulting formulas has the shape $tr(\mathbf{G}_i d\boldsymbol{\theta}_i)$. Then, by the work of Magnus and Neudecker (2019), the matrices $(vec\mathbf{G}_i')'$ are partial derivatives of F_{ML} according to the structural parameters.