Application of Hierarchical Cluster Analysis in Educational Research: Distinguishing between Transmissive and Constructivist Oriented Mathematics Teachers

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Abstract

Special questionnaire containing two sets of items related to teachers' beliefs and current pedagogies was completed by 30 mathematics in-service teachers from 26 different lower secondary schools in Slovakia. Their responses were analysed by means of hierarchical cluster analysis. The clustering method was chosen by the correlation coefficient between Euclidean and dendrogram-predicted distances. Cluster analysis grouped the items of this questionnaire into four clusters, describing the following aspects: (1) discipline and classroom culture; (2) pedagogies and problem solving; (3) applications of mathematics and students' activity during the lesson; and (4) teachers' attitudes towards students' individuality and mathematics. Teachers were grouped into two clusters. Based on the differences between responses we consider one cluster of teachers as transmissive-oriented and the second one as constructivist-oriented.

Keywords	***JEL code***
Hierarchical cluster analysis, mathematics education, teachers' beliefs, instructional approaches	*C38, I29*

INTRODUCTION

One of the main aims of mathematics education is to motivate students through the properly posed questions and problems. Students gain understanding, their imageries are developed and concepts become

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crystallised (Hejný et Kuřina, 2001). However, the everyday reality of schools is often very different. Very often, in advance prepared facts are routinely presented to students, and which may lead to rote learning and acquisition of merely formal knowledge of mathematical concepts (Lloyd, 2018). Traditionalist or transmissive teachers believe that they should use "didactic instructional practices, directly offering students standardized, prescriptive methods for completing mathematical task. These methods are practiced on homework tasks, memorized and applied on similar … test tasks for which correct responses indicate mathematical understanding" (Lloyd, 2018). By contrast, Beerenwinkel et von Arx (2017) have described the constructivist-oriented teacher as a person who activates students' pre-knowledge, provides them with suitable problems often related to everyday context. During the problem-solving activity of students, the teacher creates the required and necessary space for independent learning, encourages rethinking and seeks to demonstrate the scientific approach to knowledge generation which fosters critical thinking (Grofčíková et al., 2018).

Teachers have to face a lot of issues and challenges when implementing constructivist approaches. Especially, those teachers who lack any experience in the constructivist classroom work often struggle with setting such a learning environment that is required for the constructivist perspective (Windschitl, 2002).

Even though earlier research has shown that teachers' perception of mathematics does not have significant impact on teachers' work, their perceptions of mathematics teaching form teachers' practice (Andrews et Hatch, 1998).

The objective of this study is to propose a mean that could help classify teachers as preferring either transmissive or constructivist pedagogies. Previously, teachers' beliefs had been investigated via questionnaires (e.g., Andrews et Hatch, 1998) and through the observation of their practice (e.g., Beerenwinkel et von Arx, 2017). Moreover, Lloyd (2018) performed a typological analysis of pre-service mathematics teachers based on analysis of both qualitative data, including video responses and lesson plans, and quantitative data from the Likert-scale questionnaires.

In order to divide teachers into several groups according to their beliefs about teaching and self-reporting their teaching practices the hierarchical cluster analysis (Tan et al., 2019) was performed. Hierarchical cluster analysis had previously been used for grouping teachers according their practice based on analysis of video-recordings of their lessons (Beerenwinkel et von Arx, 2017), for grouping students according their responses to the Likert-scale items (da Recha Seixas et al., 2016), or by their learning styles (Liu et al., 2017). It was also used for grouping items of questionnaires according to similar responses (Hörstermann et Krolak-Schwerdt, 2012).

1 METODOLOGY OF RESEARCH

In order to distinguish between the constructivist and transmissive teachers a questionnaire was designed, taking into account their beliefs and practice into account was designed. This questionnaire comprises of specific items focused on the professional practice of teachers, their professional development, the description of current classroom practice, and their beliefs. The questionnaire had been applied within a larger study, investigating the relation between the teachers' pedagogies and mathematical knowledge for teaching combinatorics.

1.1 Teachers' beliefs and current pedagogies

The items focused on the teachers' beliefs and description of current classroom practices were taken from the questionnaire described in more details in (Engeln et al., 2013; Engeln, 2013), designed within the project PRIMAS (PRIMAS, 2018) where the first author of the here-presented study was involved as well.

The first eight questions were aimed at identification of the teacher alone. The battery 9 contained such items that related to the students' role, including social interaction (9a, 9f and 9o) and use of problems (9e, 9g and 9s). The battery 10 comprised of the items related to teacher's role, with the special focus on applications of mathematics (10a, 10e and 10h) and usual pedagogies (10b. 10f, 10i, 10l and 10n). Item 11 was an open question, asking about the problem the teacher had to deal in mathematics instruction. Remaining three batteries were aimed at the use of inquiry-based learning in participants' teaching.

The aim of the PRIMAS questionnaire was to measure teachers' tendency to use inquiry-based pedagogies in their classroom. As the PRIMAS questionnaire was intended for both mathematics and science teachers, the items related to science education were omitted. When applicable, the word *subject* (meaning math, or science in the original version) was for our purposes substituted by the word *mathematics*. The first set of the adjusted questionnaire (battery 9 of the PRIMAS questionnaire) consisted of 18 statements about students' activity during mathematics lessons, and the second one (battery 10) comprised of 14 statements about the teacher. The teachers were asked to assign the frequency at which the described situations occurred in their classroom to the values on Likert scale from 1 (never or hardly ever) to 4 (almost in each lesson). Henceforth we will address the items by their numbers in the PRIMAS questionnaire.

1.2 Participants

The designed questionnaire was distributed to lower secondary mathematics teachers via online portal for teachers. Altogether 30 teachers from 27 schools in Slovakia completed the questionnaire. The sample contains teachers from all regions in Slovakia. As for gender, there were only two men in the sample. Twenty-one teachers were from mixed-ability schools. Among the rest of the teachers there were teachers from schools for gifted children (2), schools with special programme in mathematics (1), sports (2), music (1) or languages (2), and a special school for students with hearing impairment (1). The sample cannot be considered as representative, so we cannot generalize the conclusions from the concluded analysis.

1.3 Statistical methods

Hierarchical cluster analysis is usually employed in order to divide data into meaningful and/or useful groups (or clusters). It helps to provide understanding of the structure and further classification of involved elements (e.g., participants, samples, items) (Tan et al., 2019).

The inputs to the analysis are usually n objects with d descriptors for each further called x_i, $i \in \{1,2, \dots, d\}$. Altogether they form an $n \times d$ matrix with entries m_{ij}. The descriptors can be continuous, discrete or binary. The data should be standardized with the aim to diminish influences due to measure units of different descriptors (Kráľ et al., 2009). In our case, an integer value between one and four was assigned to each questionnaire item by respondents, thus the standardisation was not needed.

As the first step of the analysis, a distance matrix $\mathbf{D}_{ij} = D(x_i, x_j)$ is constructed. Euclidean distance defined as $D(x_i, x_j) = \sqrt{\sum_{k=1}^{n} (m_{ik} - m_{jk})^2}$ is often used. The minimal value $D(C_i, C_j) = \min_{1\le k,\, l\le n,\, k\ne l} D(C_k, C_l)$ is determined and the clusters C_i and C_j are merged into a new cluster C_{ij}. The row j is omitted and values of distance matrix $\mathbf{D}$ are recalculated by the formula:

$$D(C_k, C_{ij}) = \alpha_i\, D(C_k, C_i) + \alpha_j\, D(C_k, C_j) + \beta D(C_i, C_j) + \gamma \left| D(C_k, C_i) - D(C_k, C_i) \right|, \tag{1}$$

where the way to calculate coefficients α, β, γ determines the clustering method (Tan et al., 2019). The most frequently used methods are summarized in Table 1. The clusters with minimal distance are merged until there is only one cluster left. Then, the dendrogram T is constructed and analysed.

The relation between the Euclidean distance and dendrogram-predicted distance can be expressed by the cophenetic correlation coefficient:

$$c = \frac{\sum_{i<j} (D(x_i, x_j) - \bar{x})(t(x_i, x_j) - \bar{t})}{\sqrt{[\sum_{i<j} (D(x_i, x_j) - \bar{x})^2][\sum_{i<j} (t(x_i, x_j) - \bar{t})^2]}}, \tag{2}$$

Table 1 Table of coefficients for common hierarchical clustering approaches

Clustering method	α_i	α_j	β	γ
Nearest neighbour (single linkage)	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$
Unweighted pair group method using arithmetic mean	$\frac{n_i}{n_i + n_j}$	$\frac{n_j}{n_i + n_j}$	0	0
Weighted pair group method using arithmetic mean	$\frac{1}{2}$	$\frac{1}{2}$	0	0
Centroid	$\frac{n_i}{n_i + n_j}$	$\frac{n_j}{n_i + n_j}$	$-\frac{n_i n_j}{(n_i + n_j)^2}$	0
Ward's	$\frac{n_i + n_k}{n_i + n_j + n_k}$	$\frac{n_j + n_k}{n_i + n_j + n_k}$	$-\frac{n_k}{n_i + n_j + n_k}$	0

Note: n_i is the size of cluster C_i.
Source: Adapted from Tan et al. (2019)

where $t(x_i, x_j)$ is the distance between the objects x_i and x_j in dendrogram. The higher the correlation is, the better the model fits the data.

Cluster analysis of the responses to items in two sets from the PRIMAS questionnaire (Engeln, 2013) was performed, using the Euclidean distance and the five clustering methods in R environment (RCoreTeam, 2018). In order to evaluate the most appropriate clustering method for each direction of analysis, the correlation coefficients between Euclidean and dendrogram-predicted distances were calculated. The final clustering was visualized by a heatmap and dendrograms (Figure 1) using the gplots (Warnes et al., 2016) library. The medians of responses for the items were compared by the Moods' median test using the RVAideMemoire (Hervé, 2018).

2 RESULTS AND DISCUSSION

The five clustering methods listed in Table 1 lead to five dendrograms describing the similarity of responses to the questionnaire items. Next, the responding teachers were taken as objects and items as descriptors, and another five dendrograms were constructed. Correlation coefficients between the predicted and Euclidean distances were calculated for the dendrograms (Table 2). The most appropriate method for each direction of analysis is highlighted. UPGMA was found to be the most appropriate clustering method for items and Ward's method for clustering the responding teachers.

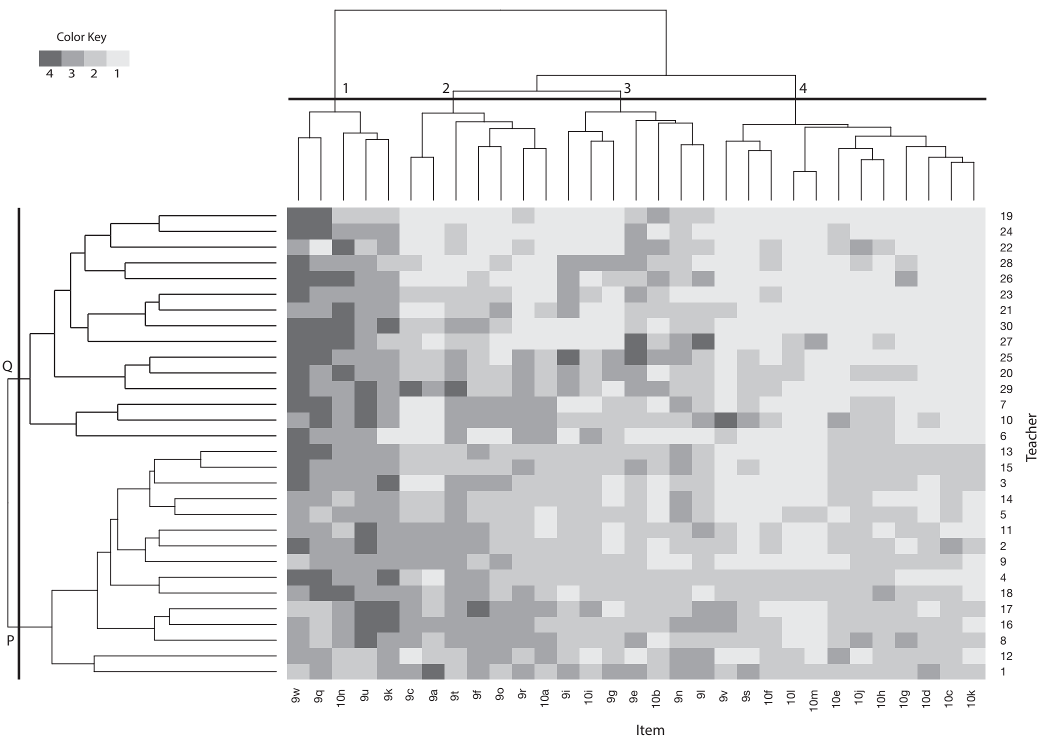
Table 2 Correlation coefficients for dendrograms constructed by different methods

Clustering method	Items	Teachers
Nearest neighbour (single linkage)	0.7085525	0.5842636
Unweighted Pair Group Method using Arithmetic mean (UPGMA)	**0.8613783***	0.6834309
Weighted Pair Group Method using Arithmetic mean (WPGMA)	0.8591654	0.5376688
Centroid	0.8258559	0.6662426
Ward's	0.8347634	**0.7624185***

Note: The highest correlation coefficient is asterisked.
Source: Authors' construction

Hierarchical cluster analysis (Figure 1) grouped all respondents into two clusters, henceforth labelled as cluster P and cluster Q. The questionnaire items were grouped into four clusters, henceforth labelled from cluster 1 to cluster 4. Further, we describe the clusters in more details. The medians and quartite spans for responses to item according the clustering are in the Annex (Table A1).

Figure 1 Heatmap of the teachers' responses to items concerning their beliefs and current classroom practices



Source: Authors' construction

Five items were grouped in cluster *1 Discipline and classroom culture*. All of them are related to discipline (9q, 9w) and classroom culture (9k, 9u, 10n). The cluster P teachers agreed significantly more with statement 9w (The students take long to settle down after the lesson begins, $p = 0.026$). Based on their responses we consider the cluster P teachers as more strict and requiring higher discipline in classroom. Čeretková et Janečková (2015) list the classroom culture supporting students' discussion as one of the five main characteristics of inquiry-based class-room, which is one of the approaches based on constructivism.

Cluster *2 Pedagogies and problem solving* consisted of the items related to teachers' activity during the lessons. Teachers in cluster P agreed more with items 9e (The students repeatedly practice the same method on many questions) and 9i (The students listen to what I say) what is typical for transmissive teachers (Wood, 1995). The teachers in cluster Q responded more affirmatively to item 10i (I summarise content and results, $p = 0.013$). This difference may stem from the different approaches to problem handling. Students taught by the cluster P teachers work out more on routine problems. The higher agreement with statement 10i might indicate more problem-solving activity (Schoenfeld, 1992) in practice of the cluster Q teachers.

There were nine items grouped in cluster *3 Applications of mathematics and students' activity during the lesson*. The items were related either to students' activity during the lessons (SA) in terms of teacher- or student-centred beliefs about teaching according to Murphy et al. (2004), or to applications of mathematics (Ap). Teachers in the cluster Q showed higher agreement with the items 9o (The students are involved in class debate or discussion, $p = 0.009$) and 9t (The students have an influence on what is done in the lesson, $p = 0.024$), what indicates that these teachers tend to offer more room for students' discussion. Windschitl (2002) put a special emphasis on providing students with opportunities for discussion in a constructivist classroom. Higher agreement with statements 9r (The students work on problems that are related to their real life experience, $p < 0.001$) and 10a (I use this subject to help the students understand the world outside school, $p = 0.004$) indicates the teachers' willingness to involve real-life problems in the classroom. Teachers in this cluster manifested their student-centred beliefs about mathematics teaching. Independent learning of students is considered as typical for constructivist teachers (Beerenwinkel et von Arx, 2017).

The last cluster, cluster *4 Teachers' attitudes towards students' individuality and mathematics*, put together two different issues. The items grouped here are reflecting the extent of teachers' individual approach to students (IS) and the teachers' attitude towards mathematics and its teaching (M). Teachers in the cluster Q demonstrated higher agreement with the item 10j (I help students with their learning, $p < 0.001$). Significant differences were confirmed in the items 10e (I show how mathematics is relevant to society, $p = 0.001$) and 10h (I explain the relevance of mathematics to students' daily lives, $p = 0.001$) with that the teachers in the cluster Q agreed more what confirmed their belief that mathematics should be related to everyday life of the students.

Based on the description of clusters and differences between teachers in the clusters P and Q we can see that the cluster P teachers lecture and explain, and, based on teachers' instruction, students are working on routine problems. We can conclude that the teachers in the cluster P tend to do transmissive instructions. On the other hand, according to teachers' responses, students in the cluster Q classrooms discuss and influence the lesson by their activity. Teachers reported themselves as persons summarising results rather than lecturing. The belief that students are able not only to use mathematical concepts, but also to discover and inquire into them, and to relate obtained information with previous knowledge, is usual for constructivist teachers (Krpec, 2015). They tend to involve their students by providing them with opportunities to discuss specific topics related to mathematics and, hence, influencing the lesson. These teachers also incline to pose problems from every-day life of their students. Based on our questionnaire we cannot declare whether they use real problems or pseudo-real tasks. The items related to every-day use of mathematics were present in the two clusters. Cluster 3 contained items related to the choice of problems. Some teachers have never encountered context-based mathematics tasks in their own education (Plothová et al., 2017), so they may differ in attitudes towards mathematics and do not see it useful for society or students' daily lives, as grouped in cluster 4. The teachers in cluster Q see themselves as being the facilitators of the educational process. Their teaching is in good fit with the inquiry-based pedagogies, as described by several scholars (Čeretková et Janečková, 2015; Engeln et al., 2013; Samková et al., 2015), therefore, we consider such teachers as constructivist-oriented.

CONCLUSIONS

This paper tried to shed more light on differences in pedagogies and beliefs of lower-secondary mathematics teachers. As early as 1984, Gonzales Thompson showed that teachers' beliefs play significant role in their instructional practices. Her research was qualitative, comprising several case studies. Subsequently, in the following decades the idea was further elaborated (e.g. Lloyd, 2018). We drew on work of Engeln et al. (2013) within the PRIMAS project who constructed a questionnaire for measuring the teachers' tendency to use inquiry-based learning and therefore the constructivist teaching in their practice.

Having performed the hierarchical cluster analysis while processing the results of questionnaire survey conducted among 30 lower-secondary mathematics teachers, two groups of teachers were identified. Four sets of statements were constructed when the same analysis was performed on transposed data. These four clusters allowed us to study the differences between the two clusters of teachers. We found that the main difference is in students' activity in the classroom. In classrooms taught by the cluster Q teachers there is a space for individual activity of students and strong focus on applications of mathematics.

The students in classrooms of the cluster P teachers more often repeatedly practice the same method on different questions, whereas students in the Q classrooms solve preferably real-life problems and discuss different topics related to mathematical concepts. Based on the literature in the field of mathematics education, we can refer to the cluster P teachers as transmissive-oriented, and the cluster Q teacher as constructivist-oriented.

We have shown that the hierarchical cluster analysis may be used as a reasonable tool for grouping teachers with certain characteristics obtained as an agreement with thoroughly chosen statements. The findings of the study cannot be generalized as the sample was not fully representative and comprised only 30 teachers. If more participants filled the questionnaire, the number of clusters could increase. The new cluster could obtain the more extreme (more transmissive or more constructivist) teachers, or some kind of balanced approach using both transmissive and constructivist pedagogies.

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ANNEX

Table A1 Medians of responses to items according to clustering

Cluster	Item	Statement	Cluster *P* (*n*=18)			Cluster *Q* (*n*=12)		
			***Me*(*P*)**	**Quartile span**		***Me*(*Q*)**	**Quartile span**	
1 Discipline and classroom culture	9k	The students have the possibility to decide how things are done during the lesson.	2	2	2	2	2	2.25
	9q	The students behave noisily and course disorder.	2	1.25	2	1.5	1	2
	9u	The students choose which questions to do or which ideas to discuss.	2	1	2	2	2	2
	9w*	The students take long to settle down after the lesson begins.	2	1	2	1	1	1
	10n	I give a lecture.	2	2	2	1.5	1	2
2 Pedagogies and problem solving	9e	The students repeatedly practice the same method on many questions.	3	3	3	2	2	3
	9g	The students learn through doing exercises.	3	3	3.75	4	2.75	4
	9i	The students listen to what I say.	3	3	3	2	2	4
	10b	I give my students precise instructions.	3	3	3	3	2	3.25
	10i*	I summarise content and results.	3	3	3	4	3	4

Table A1 (continuation)

Cluster		Item	Statement	Cluster *P* (*n*=18)			Cluster *Q* (*n*=12)		
				Me(*P*)	Quartile span		*Me*(*Q*)	Quartile span	
3 Applications of mathematics and students' activity during the lesson	SA	9a	The students are given opportunities to explain their ideas.	3	2.25	3.75	4	3	4
		9cP	The students have the possibility to try out their own ideas.	3	2	3.75	3	3	4
		9fP	The students have discussions about the topics.	2	2	2.75	3	3	4
		9l	The students have no problems to follow the lesson.	3	2	3	3	3	3.25
		9n	The students know enough to understand the lessons.	3	2	3	3	3	3
		9o*	The students are involved in class debate or discussion.	3	2	3	3.5	3	4
		9t*	The students have an influence on what is done in the lesson.	2	2	2	3	2	3.25
	Ap	9r*	The students work on problems that are related to their real life experience.	2.5	2	3	4	2.75	4
		10a*P	I use this subject to help the students understand the world outside school.	3	2	3	4	3	4
4 Teachers' attitudes towards students' individuality and mathematics	IS	9s	The students start with easy questions and move on to harder questions.	3.5	3	4	4	3.75	4
		9v	The students are informed about the aim of the lesson.	3	3	4	4	4	4
		10d	I show interest in every student's learning.	3	3	4	4	4	4
		10f	I give students extra help, if they need it.	3.5	3	4	4	3	4
		10g	I continue teaching until the students understand.	3	3	4	4	3.75	4
		10j*	I help students with their learning.	3	3	3	4	3.75	4
		10l	I outline the most important points of a lesson.	4	3.25	4	4	3.75	4
	M	10cS	I enjoy teaching mathematics.	3	3	3.75	4	4	4
		10e*S	I show how mathematics is relevant to society.	3	3	3	4	4	4
		10h*S	I explain the relevance of mathematics to students' daily lives.	3	3	3	4	3.75	4
		10kS	I really like mathematics.	4	3	4	4	4	4
		10mS	I treat mathematics as important.	4	4	4	4	4	4

Note: SA – students' activity, Ap – applications of mathematics, IS – individual approach to students; M – teachers' attitudes towards mathematics and its teaching. In items marked with the letter S the original wording *the/this subject* was substituted by the word *mathematics*. Items marked with the letter P were used in PISA 2006 (OECD, 2009).

Source: Authors' construction