The Real Equilibrium Exchange Rate of the Czech Koruna – the BEER Approach

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Abstract

The aim of this paper is to develop a model of the real equilibrium CZK/EUR exchange rate with all relevant explanatory variables. Emphasis will be placed on examining the longest possible time horizon, with a period of real convergence, but also economic stagnation. The model is based on the theoretical BEER approach using a cointegration or error correction model that distinguishes between short-term and long-term relationships. The results show that the Czech koruna strengthens when the productivity differential, terms of trade differential or gross fixed capital formation increase, while the koruna weakens when the VIX index representing global risk aversion increases. For the real interest rate differential, the hypothesis that the koruna strengthens in the short run during the improvement but weakens in the long run probably due to the country's risk premium, was confirmed. Moreover, it turned out that the start of the European Central Bank's quantitative easing led to strengthening of the Czech koruna, while the start of the CNB's foreign exchange interventions in 2013 led to weakening of the Czech koruna.

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Equilibrium exchange rate, Czech Koruna, exchange rate misalignments, cointegration, error correction model	*https://doi.org/10.54694/stat.2023.50*	*C22, E44, F31, F41*

INTRODUCTION

The area of equilibrium exchange rates plays an important role in macroeconomics as well as economic policy, and the quest to explain medium- and long-term exchange rate developments has been a source of inquiry for over a century. It should be noted that the concept of equilibrium is purely theoretical, and the specific level of the equilibrium exchange rate depends on the model chosen (there is no consensus view). In general, the term equilibrium exchange rate is often used in public discourse, but it is non-trivial to precisely define and specify it using a model. One of the most common theories for assessing the long-term evolution of the exchange rate is purchasing power parity (PPP). For converging economies whose exchange rate shows appreciation symptoms (non-stationary time series), this theory is not complete (at least in the medium term) and the model needs to be supplemented with other determinants.

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Two theoretical approaches are currently distinguished in exchange rate equilibrium analysis. The first group of models introduces the conditions necessary to create an equilibrium into the model relationships and back-calculates the change in the current variables to achieve this goal. This is the so-called normative approach and includes, for example, the Fundamental Equilibrium Exchange Rate (FEER) (Williamson, 1985). The second group of models considers current variables and policies to determine future exchange rate equilibrium. This approach is called the positive concept and includes, for example, the Behavioural Equilibrium Exchange Rate (BEER) (Clark and MacDonald, 1998). This paper creates a real equilibrium exchange rate model based on the positive concept.

The analysis of the equilibrium exchange rate of the Czech koruna was dealt with by many authors (Čihák, 1999; Kreidl, 1997; Lazarová, 1997, etc.), mainly in the 1990s and 2000s during the robust transformation of the Czech economy. Disadvantages of the analyses were generally insufficiently short time series. The aim of this paper is to develop a model of the real equilibrium CZK/EUR exchange rate with all relevant explanatory variables. Emphasis will be placed on examining the longest possible time horizon (i.e. since the creation of the euro in 1999), because only then can the long-run relationships between the variables be observed. Over the past two decades, the Czech economy has undergone a period of significant real convergence but also economic stagnation, which has also been reflected in exchange rate movements.

In the first chapter, a literature review including a summary table of empirical models is developed. The methodology of the thesis, the variables included in the models and the results of the empirical analysis will be described. Chapter two describes the theoretical background of the included explanatory variables used for the long-run relationship and chapter three contains the empirical analysis itself, including a final evaluation of the evolution of the actual exchange rate against the model results.

1 LITERATURE REVIEW

This section summarizes the findings to date and describes the authors' selected models.[2] The emphasis will be on models of the Czech koruna, but also on other currency pairs. Finally, a summary table of existing models is created.

In the 1990s, the authors Frait and Komárek (1999) produced a thorough analysis of the Czech koruna. The model was built by combining the Natural Real Exchange Rate model (NATREX) and BEER and the main explanatory variables used are terms of trade, productivity, savings rate, and world real interest rates. Productivity was approximated by both real GDP per capita and labour productivity in industry.[3] The authors are aware of the limitations of the short time series, possible structural breaks, and insufficient data on fiscal policy. Cointegration analysis was used to estimate the model using both the Johansen approach based on the VAR model and the ARDL approach. In the first model using Johansen's approach, all signs came out as expected, i.e. an improvement in terms of trade, productivity and savings rate led to an appreciation of the domestic currency and an increase in the world interest rate led to a depreciation of the domestic currency. The productivity variable did not turn out to be statistically significant. In the second model using the ARDL approach, the model again came out with the expected signs. The ECM term came out negative and high indicating a rapid return of the model to equilibrium. All explanatory variables came out statistically significant. Finally, the authors compare their model with real data and conclude that the Czech koruna was overvalued just before 1997, which led to the currency crisis.

The author Rubaszek (2004) created an exchange rate model for the analysis of the Polish zloty. The model called the Balance of Payments Equilibrium Exchange Rate Model (BPEER) is based on the BEER

[2] Real equilibrium exchange rate models have also been discussed with a different methodology by the authors: Lazarová (1997), Šmídková (1999), Škop and Vejmělek (2009), Dudzich (2021), etc.

[3] The authors are aware of the limitations that both variables contain.

model but differs in its economic background. The BEER is described using the balance of payments identity as opposed to the BEER which is based on real uncovered interest parity (UIP). The main explanatory variables were domestic demand proxied by Polish real GDP, external demand proxied by euro area real GDP, net foreign assets, and the interest rate differential (three-month interbank rates). Cointegration analysis using Johansen's approach was used. The author was aware of the short time series, so he created an additional Fully Modified OLS model. In both models, all explanatory variables were statistically significant and consistent with economic theory. Changes in domestic and foreign demand (changes in demand on foreign trade) are offset by the exchange rate. Growth in net foreign assets and the interest rate differential led to appreciation of the domestic currency. Finally, the author mentions that according to this model, between 1995–99 the Polish zloty was close to its equilibrium value and in 2000–01 the Polish zloty was overvalued by about 10 to 15%. After a significant depreciation in 2002, the Polish currency returned to its equilibrium.

Komárek and Melecký (2005) used the BEER model to analyse the evolution of the Czech koruna. The model specification included an interest rate differential (real Czech and German lending rates), a productivity differential (real Czech and German GDP to employment), foreign direct investment, terms of trade, openness (a sum of exports and imports to GDP), net foreign assets and government consumption. For robustness, the authors developed three cointegration models namely Dynamic OLS, ARDL approach and Johansen approach. They divided the time period into short and medium periods. In the short period, they compare the exchange rate evolution using their model and the real evolution, and in the medium period, they apply cyclical adjustment to the model implying sustainable values, which they compare with the real values. The results indicated that the growth of productivity differential, interest differential and terms of trade led to the appreciation of the Czech koruna and these variables were statistically significant. Net foreign assets came out opposite in different models (different signs). The openness rate was significant only in one model with a positive relationship. Government consumption came out positive, although the authors expected a negative relationship because a rising government deficit should lead to a depreciation of the domestic currency. Finally, the authors compare the results with real developments and point out that the Czech koruna was undervalued by about 7% between 1994 and 2004.

Dufrenot and Égert (2005) analysed the equilibrium exchange rate using Central European emerging countries as a case study. Specifically, the authors analyzed the Czech Republic, Poland, Hungary, Slovakia and Slovenia. They used a combination of the BEER model and structural vector autoregression (SVAR) for the analysis. They analysed the BEER model using the Johansen cointegration approach. The structural model contains a two-equation approach. The exchange rate is explained by productivity differential, government deficit or surplus to GDP and current account balance of payments to GDP.[4] Further, the relative price level of non-tradable goods is explained by productivity differential and time trend. The authors are constrained not to build models for Slovakia and Slovenia because the cointegration relationship has not been econometrically confirmed. The results suggest that productivity growth leads to domestic currency appreciation for all three countries, suggesting the validity of the Balassa-Samuelson effect. The effect was weakest for the Czech Republic. The growth of the government deficit and the growth of the current account deficit of the balance of payments led to a depreciation of the domestic currency. Moreover, the so-called twin deficits were more strongly reflected in the depreciation of the domestic currency of Poland and Hungary. Finally, the authors conclude that productivity growth affects the exchange rate not only through prices in the services sector, but also through the non-tradable components of tradable goods prices and the growing ability of the tradable goods industry to produce higher quality goods.

[4] The authors deliberately omit the interest rate differential, whose impact on the exchange rate is not clear.

Pošta (2010) developed an equilibrium exchange rate analysis of the Czech koruna based on the BEER model. For the econometric analysis he used cointegration using the Johansen approach. The main explanatory variables he chose were the real interest rate differential, the productivity differential as the difference between the price level in the Czech Republic and the euro area, the real oil price in Czech koruna deflated by the PPI, net foreign assets in relation to GDP expressed as the accumulation of the current account of the balance of payments, government consumption to GDP and the ratio of total government debt to GDP. The author created three models where he included the explanatory variables in different ways. In the modelling, he included the interest differential variable outside the cointegrating vector. The largest positive effects on the appreciation of the Czech koruna were productivity growth, net foreign assets, and the interest rate differential. The government consumption, government debt and oil price variables did not turn out to be statistically significant. Finally, the author mentions that the Czech koruna was overvalued until 2008 according to the model, whereas in the financial crisis it tended to return to its equilibrium.

Pour and Illichmann (2022) attempted to develop an econometric model explaining the exchange rate using a modified BEER model. The authors rely on absolute purchasing power parity (PPP), but this needs to be extended to include other variables as this theory does not hold in the medium term for many countries. For the analysis, they used panel regression on a sample of 34 countries that had a floating currency regime during the time period. In addition to the mentioned exchange rate using PPP, the variables used were GDP per capita, interest rates (annual average of central banks), inflation, investment freedom, urbanization rate or exchange rate. The result shows that absolute PPP is appropriate to include in the exchange rate model (it comes out significantly with a parameter around one). Furthermore, the domestic currency tends to appreciate when GDP per capita, interest rates, investment freedom, urbanization rate and terms of trade rise. Conversely, when domestic inflation rises, the exchange rate tends to depreciate.[5] Finally, the authors conclude that the deviations of the exchange rate according to the constructed model from real developments are smaller than the deviations of the exchange rate according to purchasing power parity from real developments, and half of the deviation from the model tends to return to its equilibrium over a three-year period.

For clarity, Table 1 summarizes all the important parameters of the empirical work models.

Table 1 Summary review of empirical papers

Author	Estimation period	Background	Econometric method	REER deflator	Country	Expl. variables
Frait and Komárek	1992–1998/Q	BEER, NATREX	Cointegration (ARDL, JM)	CPI	CZ	*ToT, PROD, S, IRf*
Rubaszek	1995–2002/Q	BPEER (BEER)	Cointegration (JM), VECM	PPI	PL	*NFA, IRdif, FTD*
Komárek and Melecký	1994–2004/Q	BEER	Cointegration (DOLS, ARDL, JM), VECM	CPI	CZ	*PRODdif, IRdif, ToT,NFA, OR, GC*
Dufrenot and Égert	1993–2002/M	BEER	Cointegration (JM), VECM	CPI	CZ, PL, SK, SI, HU	*PRODdif, DEF, CA*
Pošta	2000–2009/Q	BEER	Cointegration (JM), VECM	CPI	CZ	*PRODdif, NFA, IRdif, GC, OIL*
Pour and Illichmann	2000–2020/A	BEER	Panel	CPI	34 countries	*PRODdif, IRdif, ToT,IF,UR, CPIdif*

Notes: M – monthly, Q – quarterly, A – annually, *ToT* – terms of trade, *PROD* – productivity, *NFA* – net foreign assets, *S* – savings rate, *IRf* – world interest rate, *IRdif* – interest differential, *PRODdif* – productivity differential, *FTD* – foreign trade demand, *OR* – openness rate, *GC* – government consumption, *DEF* – public deficit/surplus, *CA* – current account balance, *IF* – investment freedom, *UR* – urbanization rate, *OIL* – real price of oil, *CPIdif* – inflation differential.
Source: Authorial computation

[5] There is a possibility of endogeneity of inflation, i.e. an inverse relationship.

The table shows that the BEER model was the most frequently used to develop models based on the positive approach. Most of the authors used econometric cointegration method, either using Johansen approach or ARDL approach. Some of the most used explanatory variables were changes in productivity or changes in the interest rate in the home country relative to abroad, which in most models led to an appreciation of the domestic currency. However, in some models, a negative relationship with the real interest rate differential emerged, probably due to an increase in the country risk premium. The models also used the terms of trade variable, i.e., the ratio of export prices to import prices, which led to appreciation of the domestic currency during growth. In most models, growth in net foreign assets led to appreciation of the domestic currency. The authors tried to include the effect of fiscal policy on exchange rate movements, either through government consumption or government budget deficit, but these variables did not show a clear relationship (higher government consumption led to an appreciation of the domestic currency, but the resulting government deficit led to a subsequent depreciation of the domestic currency). Cost variables that the Czech economy must import, such as oil prices, had the expected negative relationship, but were often statistically insignificant.

2 FUNDAMENTAL DETERMINANTS

This paper is based on a positive approach to equilibrium real exchange rate modelling and is also based on the BEER model, which is adapted to the needs of a small export economy such as the Czech Republic. The BEER model in its basic form was defined by Clark and MacDonald (1998) and is based on uncovered interest parity (UIP). The basic form of the model is as follows:

$$SR_t = E_t(SR_{t+k}) \cdot \frac{1 + IR_{F,t}^{t+k}}{1 + IR_{D,t}^{t+k}}, \tag{1}$$

where SR_t is a spot rate, $E_t(SR_{t+k})$ is an expected spot rate, $t+k$ defines the maturity horizon of the bonds, $IR_{D,t}^{t+k}$ is a domestic nominal interest (yield) rate and $IR_{F,t}^{t+k}$ is a foreign nominal interest (yield) rate.
The second equation expresses the expected purchasing power parity in the absolute version:

$$E_t\left(\frac{P_{F,t+k}}{P_{D,t}}\right) = \frac{P_{F,t}}{P_{D,t}} \cdot \frac{1 + E_t(p_{F,t}^{t+k})}{1 + E_t(p_{D,t}^{t+k})}, \tag{2}$$

where $\frac{P_{F,t+k}}{P_{D,t}}$ is the purchasing power parity in the absolute version at the time t, and $\frac{1 + E_t(p_{F,t}^{t+k})}{1 + E_t(p_{D,t}^{t+k})}$ is the relationship for expected changes in foreign and domestic price levels.

Adding Formulas (1) and (2) we get:

$$RSR_t = E_t(RSR_{t+k}) \cdot \frac{1 + RIR_{F,t}^{t+k})}{1 + RIR_{D,t}^{t+k})}, \tag{3}$$

where $RIR_t^{t+k} = IR_t^{t+k} - E_t(p_t^{t+k})$ is the real interest rate (ex-ante, domestic or foreign currency), $RSR_t = SR_t \cdot \frac{p_{F,t}^{t+k}}{p_{D,t}^{t+k}}$ is the real exchange rate.

The real equilibrium exchange rate is therefore explained in Formula (3) by two variables, namely: the real interest differential with maturity $t+k$ and the expectation of the real exchange rate in period $t+k$. Under the market efficiency assumption, this variable can be approximated by a risk premium because the expected return from holding foreign currency is equal to the relative risk premium from holding this currency compared to holding domestic currency.

In this paper, the BEER model is supplemented with other variables that seem to be relevant for the Czech economy. The process of selecting the fundamental determinants entering the BEER model was influenced by known empirical experience and published recommendations. The consistency of the parameter results with the economic theory and their statistical significance were the decisive factors in the assessment of the different variants of the estimation equations. The model specification is based on the current account balance of payments equilibrium and the variables that restore the equilibrium.

2.1 Productivity differential

One of the main explanatory variables that the authors use in their real equilibrium exchange rate models is the labour productivity differential, in other words, the differential evolution of labour productivity in the domestic and foreign economies. There is strong evidence that faster labour productivity growth in the domestic economy relative to the foreign economy ultimately leads to an appreciation of the domestic currency in both nominal and real terms. This is because labour productivity growth tends to be associated with export growth, leading to a foreign trade surplus in the balance of payments, which ultimately leads to nominal and real appreciation of the domestic currency.

An alternative explanation for the appreciation of the real exchange rate (not the appreciation of the nominal exchange rate) through an increase in the domestic price level of internationally non-tradable goods is offered by the Balassa-Samuelson theorem. This argues, in simple terms, that all goods and services consumed in an economy can be divided into tradable (low international arbitrage costs) and non-tradable goods (high international arbitrage costs). The share of labour in value added in the two sectors of a given economy is roughly the same. The nominal exchange rate changes in line with changes in the relationship between domestic and foreign prices of tradable items. Initially, labour productivity is growing faster in the tradable goods sector due to FDI inflows. This allows nominal wages to rise faster. In the long run, however, wages must rise equally in all sectors, otherwise there would be a shift of labour supply to the sector with the higher wage growth rate. The law of supply and demand in the labour market therefore forces the same rate of wage growth in both sectors, regardless of the rate of growth of labour productivity. Maintaining the desired profitability forces firms in the non-tradable goods sector to raise final prices and there is upward pressure on the domestic price level and hence a real appreciation of the domestic currency. Thus, the expected hypothesis in our empirical model is the following: when the labour productivity differential (Czech economy versus euro area economy) increases, the real equilibrium CZK/EUR exchange rate appreciates.

2.2 Real interest rate differential

The inclusion of the real interest differential follows directly from the BEER approach described above. This approach is derived from uncovered interest parity (UIP), which was defined by I. Fisher (1896, 1930) and argues that a currency with a positive interest differential should depreciate to equalize the return in the domestic and foreign currency (speculator's equilibrium):

$$(1 + IR_{D,t}^{t+n}) = \frac{E_t(SR_{t+k})}{SR_t} \cdot (1 + IR_{F,t}^{t+n}), \tag{4}$$

where SR_t is a spot rate, E_t (SR_{t+k}) is an expected spot rate, $IR_{D,t}^{t+n}$ is a domestic interest (yield) rate and $IR_{F,t}^{t+n}$ is a foreign interest (yield) rate. The UIP was originally formulated under the assumption that speculators are risk neutral, i.e. without risk premium. The nominal interest rate can be decomposed using the following formula:

$$IR_t^{t+k} \approx RIR_t^{t+k} + E_t(p_t^{t+k}), \tag{5}$$

where RIR_t^{t+k} is a real interest rate and $E_t(p_t^{t+k})$ is an expected inflation rate. This equation can be supplemented by the effect of risk, which can be captured by the risk premium see Mandel and Vejmělek (2021):

$$RIR_t^{t+k} = rr + rp, \tag{6}$$

where *rr* is a real interest yield and *rp* is a risk premium. The real interest rate differential in relation to the exchange rate should be distinguished in the short and long run. In the short run, a rise in both nominal and real interest rates in the domestic economy attracts speculative capital, which creates demand for the domestic currency in the foreign exchange market, causing the domestic currency to appreciate. However, in the long run, the possibility that higher nominal and real interest rates are associated with a higher risk premium leading to a depreciation of the domestic currency cannot be ruled out. An example of this is the behaviour of speculators when interest rates rise in the face of a short-term government budget deficit, or the opposite behaviour of speculators when the government budget deficit in the long term translates into a risky increase in government debt. A wave of short-term speculative capital inflows and appreciation of the domestic currency is replaced by speculative capital outflows and a depreciation of the domestic currency. The expected hypothesis in our empirical model is the following: when the interest rate differential (Czech economy vs. euro area economy) increases, the real equilibrium CZK/EUR exchange rate appreciates in the short run but depreciates in the long run.

2.3 Terms of trade differential

Terms of trade is the ratio of export prices to import prices in the economy. The basis for realisation prices are invoice prices from major export and import transactions converted into domestic currency. The price indices therefore reflect, in addition to price developments, the effect of changes in foreign exchange rates. Terms of trade differential shows the evolution of this ratio between the domestic and the foreign economy. An improvement in terms of trade in the domestic economy, i.e. an increase in export prices, ultimately leads to a foreign trade surplus in the balance of payments, which leads to an appreciation of the domestic currency in nominal and real terms. Based on the literature review, a positive relationship is expected. i.e., an increase in the terms of trade differential (Czech economy versus euro area economy) leads to an appreciation of the real equilibrium CZK/EUR exchange rate.

2.4 VIX index

The global risk aversion of the financial market plays an important role for the exchange rate. If there is a financial crisis, a global pandemic or other global external shock, the financial market becomes averse to financial risks (risk-off phase) and starts to sell off emerging market currencies. Investors and speculators have observed that emerging economies handle crises worse than safe-haven economies. This situation may arise, among other things, because of the closing of carry trade positions by speculators who cancel deposits in high-yielding currencies (usually emerging markets) and promptly repay debts in low-yielding currencies that they have to buy in the foreign exchange market. (Mandel and Durčáková, 2016). One way to measure this global risk is the VIX index (the Chicago Board Options Exchange's Volatility Index) based on S&P 500 index options (Mandel and Vejmělek, 2021). As the Czech koruna belongs to the group of emerging economies based on historical data, the expected hypothesis is as follows: a rise in the VIX index leads ultimately to a depreciation of the real equilibrium CZK/EUR exchange rate.

2.5 Gross fixed capital formation

Gross fixed capital formation includes the value of the acquisition of tangible and intangible fixed assets (whether purchased, taken over free of charge or produced in-house), less the value of their sale and the value of assets transferred free of charge. In a small open economy, we assume that most of this investment goes into internationally tradable goods. Then, we can expect that the growth in gross fixed capital formation will increase exports of finished goods or reduce imports of finished goods, again leading to a foreign trade surplus in the balance of payments, which will lead to an appreciation of the domestic currency in nominal and real terms. Moreover, if gross fixed capital formation is created with the help of a foreign investor, this will ultimately increase the supply of foreign currency and the demand for domestic currency in the foreign exchange market, which should lead to an appreciation of the domestic currency (nominal and real terms) in the short run. The expected hypothesis in our empirical model is the following: when the gross fixed capital formation increases, the real equilibrium CZK/EUR exchange rate appreciates.

3 EMPIRICAL ANALYSIS

The empirical analysis used primary data available from public sources. These include the database of the Czech Statistical Office, the database of the Czech National Bank, the database of Eurostat, the database of the European Central Bank or the database of S&P Capital IQ. When selecting the data, an attempt was made to use the maximum time range of the data. The euro as the common currency of the euro area was established in 1999, and therefore estimates were made on quarterly data between Q1 1999 and Q2 2022.[6] Since the aim of the empirical analysis is to construct a real equilibrium of the koruna against the euro, bilateral variables of the Czech economy versus the euro area economy are included. All data are converted to a base index (2015 = 100) and are seasonally and calendar adjusted. All calculations will be performed in the statistical program R.

The real exchange rate (*RCZKEUR*) is calculated from the nominal exchange rate, which is deflated by the consumer price index based on the formula:

$$RCZKEUR_t = NCZKEUR_t \cdot \frac{HICPEA_t}{HICPCZ_t}, \quad (7)$$

where $NCZKEUR_t$ is the nominal exchange rate CZK/EUR and $HICP_t$ is the harmonised index of consumer prices according to the Eurostat methodology in the euro area, respectively in the Czechia.[7] The productivity differential (*PRODdif*) is measured as the ratio of real GDP to the number of employed persons (Czech Republic vs. euro area). The terms of trade differential (*ToTdif*) is measured as the ratio of export and import prices (Czechia vs. the euro area). The measurement of global risk was captured by the VIX index (*VIX*), see paragraph above. In calculating the real interest rate differential (*RIRdif*), an attempt was made to create an ex-ante variable that makes more sense from a theoretical point of view. One possibility was to use the inflation expectations published by individual central banks in the creation of the variable. For better statistical significance, the actual consumer price trend was used, see formula:

$$1 + RIRdif_t = \frac{1 + PRIBOR(1Y)_t}{HICPCZ_t} \Big/ \frac{1 + EURIBOR(1Y)_t}{HICPEA_t}, \quad (8)$$

where $PRIBOR(1Y)_t$ is the Prague Inter Bank Offered Rate with maturity of one year and $EURIBOR(1Y)_t$ is the Euro Inter Bank Offered Rate with maturity of one year. The variable real fixed capital formation to GDP (*RGFCF*) in the Czech Republic was also statistically significant and was therefore included in the model. During the model building process, the following variables were tested and eventually excluded due to inferior statistical significance or statistical insignificance. These variables include unit labour cost differential, government debt differential, real price of oil, export to import ratio, export to GDP ratio, foreign direct investment (FDI), current account to GDP, central bank foreign exchange intervention to GDP, etc. The list of time series, shortcuts and data sources used are shown in Table 2. Descriptive time series statistics are presented in Table 3.

6 More recent data are not available due to the way the real interest differential is constructed, see below.
7 A variable with the HICP one year ahead (ex-ante) was also created, but it gave worse statistical results in the model.

Table 2 List of time series used for econometric analysis

Shortcut	Meaning	Data source
RCZKEUR	Real exchange rate CZK/EUR	CNB, Eurostat
PRODdif	Productivity differential (CZ vs EA)	Eurostat
ToTdif	Terms of trade differential (CZ vs EA)	CZSO, ECB
RIRdif	Real interest rate differential (CZ vs EA)	CNB, ECB, Eurostat
VIX	VIX index	S&P Capital IQ
RGFCF	Real gross fixed capital formation to GDP (CZ)	CZSO

Notes: CZSO – Czech Statistical Office, CNB – Czech National Bank, ECB – European Central Bank, Eurostat – Statistical Office of the European Union.
Source: Authorial computation

Table 3 Descriptive time series statistics

Variable	Mean	Median	Maximum	Minimum	Standard deviation
RCZKEUR	104.16	98.75	145.34	79.77	15.48
PRODdif	93.07	99.48	128.03	53.05	19.83
ToTdif	100.28	100.71	103.96	94.94	2.15
RIRdif	100.19	99.69	105.38	90.99	2.58
VIX	116.13	107.73	294.61	57.64	42.39
RGFCF	100.87	100.73	110.97	93.35	3.90

Source: Authorial computation

The econometric model itself will be constructed by cointegration analysis using Johansen's approach based on the VAR model. If a cointegration vector is present, it is possible to build an error correction model (VECM) that puts together the short-run relationships – differentials and the long-run relationships – cointegration (Engle and Granger, 1987; Cipra, 2008; Hindls et al., 2018).

First, it is necessary to check that all time series are integrated in the same order. The stationarity of the time series was tested using the Augmented Dickey-Fuller test and the Phillips-Perron test (both with trend and constant). The results in Table 4 indicate that all time series except the VIX variable are integrated of order 1 and can be tested for cointegration relationship. The VIX index can only be included in the model as an exogenous variable.

Table 4 Unit root tests

Variable	ADF test (p-value)		PP test (p-value)		Order of integration
	Levels	1st diff.	Levels	1st diff.	
RCZKEUR	0.471	<=0.01	0.451	<=0.01	I(1)
PRODdif	0.686	<=0.01	0.638	<=0.01	I(1)
ToTdif	0.562	<=0.01	0.427	<=0.01	I(1)
RIRdif	0.142	<=0.01	0.258	<=0.01	I(1)
VIX	0.010	<=0.01	0.015	<=0.01	I(0)
RGFCF	0.377	<=0.01	0.303	<=0.01	I(1)

Source: Authorial computation

The length order of the leads and lags is determined based on some of the information criteria for the model selection. Based on Akaike's information criterion and Schwarz's criterion, the optimal lag length was chosen to be first order. Based on Johansen's cointegration test in Table 5, exactly one cointegration relationship was found (either with trace or with maximal eigenvalue statistics).

Table 5 Johansen's cointegration test

Trace statistics				Maximal eigenvalue statistics			
H_0	H_1	Test stat.	Crit. values 5%	H_0	H_1	Test stat.	Crit. values 5%
r = 0	r > 0	96.65	53.12	r = 0	r = 1	60.24	28.14
r ≤ 1	r > 1	26.41	34.91	r = 1	r = 2	12.93	22.00

Source: Authorial computation

In the final model, dummy variables were included because it improved the statistical results of the model. The following two dummy variables were created: the exchange rate commitment dummy variable (0,1) (*dummycommit*), where 2013Q4 = 1, and the quantitative easing by the European Central Bank dummy variable (0,1) (*dummyqe*), where 2009Q2 = 1, 2010Q3 = 1 and 2012Q1 = 1. During this period, the European Central Bank gradually began to use the Securities Markets Programme to purchase the total of €250 billion of covered bonds. In the first case, there should be a one-off depreciation

Table 6 Cointegration equation and error correction model for *RCZKEUR*

Cointegration equation		Error correction model:	d RCZKEUR
RCZKEUR(−1)	1	*error correction term*	−0.221
			[−7.752]
RGFCF(−1)	0.669	*d RCZKEUR(−1)*	−0.021
	[2.918]		[−0.231]
RIRdif(−1)	−1.464	*d RGFCF(−1)*	−0.205
	[−3.493]		[−2.026]
ToTdif(−1)	1.148	*d RIRdif(−1)*	−0.587
	[2.102]		[−3.562]
Constant	−172.963	*d ToTdif(−1)*	−0.225
	[−2.777]		[−0.759]
Coefficient of determination (R^2)	0.642	*PRODdif*	−0.113
			[−8.507]
		VIX	0.020
Adjusted R^2	0.598		[5.777]
		dummycommit	2.435
			[1.474]
F-statistic	14.69	*dummyqe*	−3.580
			[−3.504]

Note: The values of t-statistics are in square brackets.
Source: Authorial computation

of the Czech koruna against the euro. The exchange rate commitment between November 2013 and April 2017 was a situation where the Czech National Bank intervened against the appreciation of the Czech koruna below 27 CZK/EUR as part of unconventional monetary policy.[8] In the second case, quantitative easing by the European Central Bank should lead to a weakening of the euro and thus to a strengthening of the Czech koruna. The explanation can be described using the monetary approach to exchange rate, where an increase in the money supply in the domestic economy ultimately leads to a depreciation of the domestic currency (Vejmělek, 2000).

The final model included a constant in the cointegration vector. The variables *VIX* and *PRODdif* were included in the model as exogenous variables, *VIX* because it is cointegrated by a different order of integration and *PRODdif* because the overall quality of the model with respect to the coefficient of determination was enhanced in this combination. The results are summarized in Table 6.

All variables in the model were statistically significant at the 5% significance level except d *ToTdif* and *dummycommit* variables, which were statistically insignificant, which means significant at the 15% significance level. The adjustment parameter (*error correction term*) in the error correction model has a negative sign and is in the desired interval (–1, 0), which indicates a tendency for the exchange rate to return to equilibrium. Productivity growth in the Czech economy relative to the euro area leads to a real appreciation of CZK/EUR in the short run[9] and the Balassa-Samuelson effect can be confirmed. The Czech koruna also appreciated when terms of trade in the Czech economy improved relatively to the euro area. The real interest rate differential should be divided into short and long periods. In the short run, when the real interest rate differential rises, the koruna appreciates due to the attraction of short-term speculative capital; in the long run, on the other hand, the koruna depreciates because the real interest rate differential is mainly influenced by the country's risk premium. A model was also tested where the real interest differential is included as an exogenous variable (the hypothesis that the real interest rate is equal between countries in the long run and therefore has no effect on the exchange rate). However, this model showed much worse statistical results and since the variable is statistically significant even in the long run, it is concluded that this hypothesis does not hold for the Czech koruna. It has been shown that fixed capital formation does indeed lead to an appreciation of the Czech koruna in the short term.[10] The inclusion of the VIX index demonstrates that the Czech koruna is under pressure to depreciate in the face of rising global uncertainty, placing it in the category of riskier currencies. The dummy variable for quantitative easing by the European Central Bank shows a one-off appreciation of the Czech koruna confirming the effect of money supply on the exchange rate. The one-off depreciation of the koruna occurred after the announcement of foreign exchange interventions by the Czech National Bank.

Based on the model, the results expressed in basis index[11] were compared with the actual real CZK/EUR exchange rate deflated by the CPI. The results of the model, see Figure 1, show an appreciation trend of the Czech koruna, especially between 1999 and 2009, when the Czech economy grew faster relative to the euro area and thus real convergence occurred. Between 2010 and 2020, on the other hand, there is some appreciation stagnation of the Czech koruna. Since the onset of the financial crisis and the subsequent European debt crisis, the pace of real convergence has almost stopped for several years. Moreover, developments in this period are strongly influenced by the CNB's foreign exchange interventions between 2013 and 2017. The Czech koruna has been overvalued since 2010, but at the time of the exchange rate commitment the difference to the model has been minimized. Since 2019, the Czech koruna has been slightly undervalued and since 2021 the values have been rebalanced against the model.

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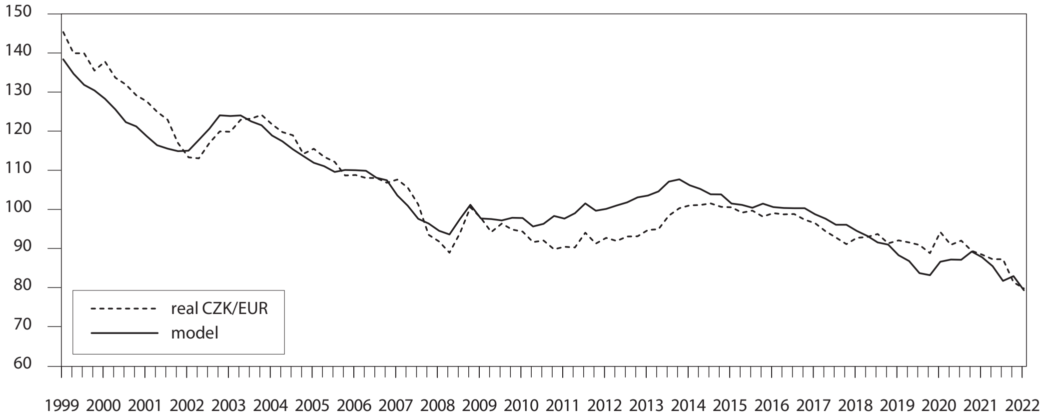
8 The dummy variable with the end of foreign exchange interventions in 2017 was not statistically significant.

9 In the case of a cointegrating vector, the signs of the variables must be perceived in reverse.

10 In the long run, this relationship is no longer significant. It is likely that gross fixed capital formation will feed through into productivity growth.

11 A drop in the index means a real appreciation of the Czech koruna.

Figure 1 Range of the equilibrium real CZK/EUR exchange rate

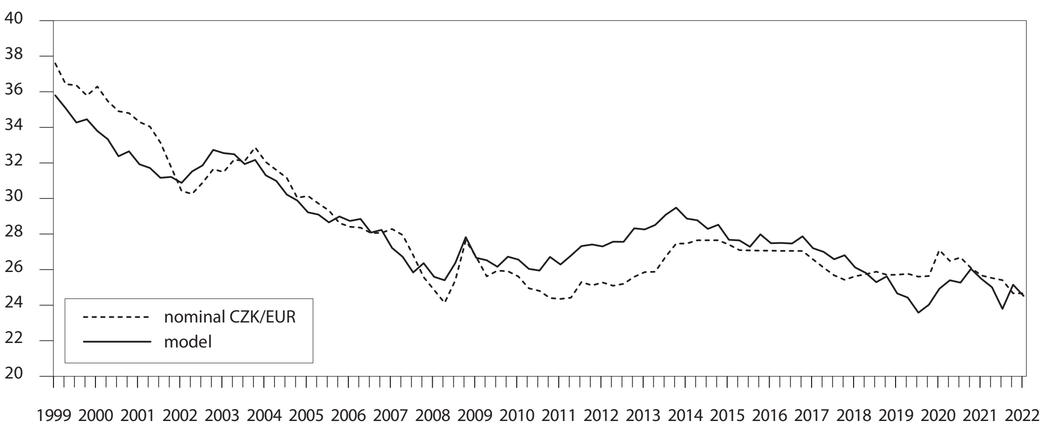


Source: Authorial computation

The resulting model can be converted into an equilibrium exchange rate in nominal terms based on knowledge of inflation in the Czech Republic and the euro area, see Figure 2. Subsequently, the percentage overvaluation or undervaluation of the actual CZK/EUR exchange rate relative to the model results was calculated, see Figure 3.

The results suggest that between 1999 and 2002 the Czech koruna was undervalued relative to the model by around 6%. From 2003 to 2009, the evolution of the real exchange rate was broadly in line with the model. In contrast, between 2010 and 2013, the Czech koruna was overvalued by around 9%. Between 2013 and 2017, the overvaluation of the Czech koruna has fallen to around 2%. Thus, if the CNB had not announced an exchange rate commitment, the Czech koruna would have been stronger. It is evident that the foreign exchange interventions had a long-term impact on the development of the Czech koruna (undervaluation) even after their termination. The deviation from the equilibrium of the model peaked in 2013, when the equilibrium exchange rate should have been around 29 CZK/EUR. After the outbreak

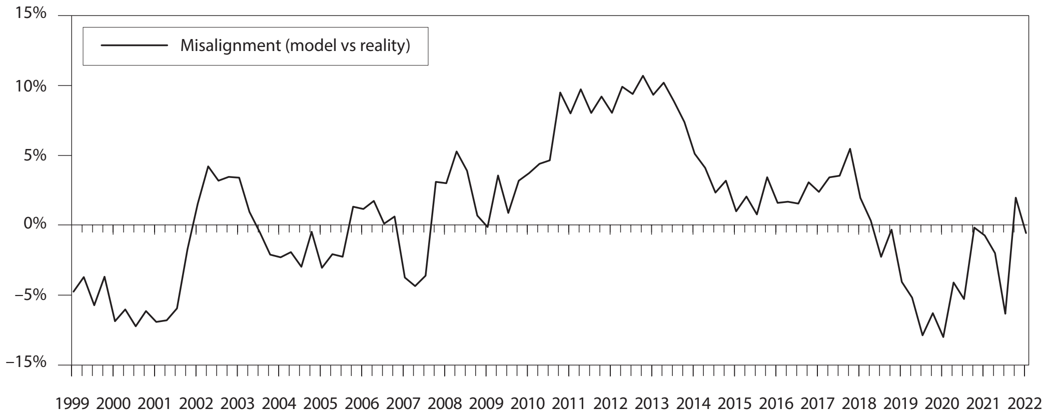
Figure 2 Range of the equilibrium CZK/EUR exchange rate in nominal terms



Source: Authorial computation

of the Covid-19 pandemic in 2020, the deviation decreased, and the exchange rate was in line with the model in 2022. Based on the actual CZK/EUR development after this date, it is known that the Czech koruna continued to appreciate in nominal terms until 2023, when it fell below the 24 CZK/EUR level.

Figure 3 Misalignment of the CZK/EUR exchange rate in nominal terms



Note: For misalignment, (+) indicates overvaluation and (-) undervaluation.
Source: Authorial computation

CONCLUSION

The aim of the academic paper was to analyse the determinants of the real equilibrium exchange rate with an emphasis on the long-run aspects. The empirical analysis was developed for the CZK/EUR currency pair over a long-time horizon between 1999 and 2022. The advantage of the long time series is the inclusion of the development of the Czech economy in the period of real convergence, but also in the period of economic stagnation, which is reflected, among other things, in the development of the Czech koruna. The model is based on the theoretical BEER approach, which is suitable for the study of a small open economy. The econometric method chosen was cointegration, or the error correction model, which has the advantage of distinguishing between short-run and long-run relationships. One of the most important variables for the analysis of the real equilibrium exchange rate appears to be the labour productivity differential between the Czech Republic and the euro area. When improving, the Czech koruna appreciated in both the short and long run, consistent with the Balassa-Samuelson effect. The terms of trade differential also proved to be an important variable, especially in the long run, when the Czech koruna appreciates during growth. The inclusion of the Czech Republic's real gross fixed capital formation also proved to be significant in the short and long run, with a positive effect on the appreciation of the Czech koruna.

For the real interest rate differential, the hypothesis that the Czech koruna appreciates in the short run when it rises because higher rates attract speculative capital was confirmed, while in the long run the Czech koruna depreciates because the country risk premium starts to be written into the real interest rate. Also included in the short-run relationship were the VIX index, which represents the global financial market's risk aversion. The growth of the VIX index leads to a weakening of the Czech koruna. It turned out that the Czech koruna is still one of the risky currencies and in case of an increase in global risk aversion the Czech koruna tends to depreciate. It was also a statistically significant variable when foreign exchange interventions started in 2013 and that led to a one-off depreciation of the Czech koruna. On the contrary, the dummy variable of the European Central Bank's quantitative easing led to a one-off depreciation of the euro or appreciation of the Czech koruna. Comparing the model results with the real data, it was possible to conclude that until 2002, the Czech koruna was significantly undervalued by around 6%, while between 2003 and 2009 it was roughly in line with the model. Since 2010, after the financial crisis, the Czech koruna has been overvalued by around 10%. Since the start of the CNB's foreign exchange interventions, the real exchange rate has moved closer to the model results and since 2019 the Czech koruna has been undervalued by around 8%. In 2022, the real exchange rate has returned to equilibrium in line with the model.

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