Sensitivity Analysis of Price Indices in Models of Demand Systems

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Abstract

The primary motivation of the paper is to point out the sensitivity of price indices calculated by the model of demand systems in different price and quantitative levels. We simulated different prices and quantities, so increasing the values of the variables reduced their variance. We would like to point out the significant role of variability and thus the deviation of input data, which is the basis for identifying consumer behavior.

From the methodology, we used the Linearized Almost Ideal Demand System and focused on the partial output of specific expenditure elasticities calculated by price indices – Stone, Laspeyres and Törnqvist index. Following we are wondering which index can consider as the most trustworthy?

In the analysis, we realized that the price variance would affect the indices' values more significantly, than the more considerable variance of the quantity consumed. It means that elasticities characterize consumer behavior in terms of prices, not in terms of quantity consumed.

Keywords	***JEL code***
Variance, expenditure elasticity, consumer demand model, meat items	*D11, D12, C43*

INTRODUCTION

The review of the literature is focusing on the presentation of experience with price indices calculated in models of demand systems.

The Linearized Almost Ideal Demand System (AIDS) developed by Deaton and Muellbauer (1980) remains one of the most popular systems in the applied demand analysis. This seeming nonlinear system's linearization usually uses the Stone price index to approximate the model's accurate price index, Buse and Chan (2000). Deaton and Muellbauer (1980) found that the Stone price index provided an excellent approximation to the valid price index given high prices' high positive collinearity.

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The choice of price index did not influence the likelihood ratio tests. Anderson and Blundell (1987) confirmed these features. Buse and Chan (2000) stated that Pashardes (1993), and Buse (1994; 1998) were interested in the presumption that the Stone index will always provide a good approximation. Buse (1994) showed that the Seemingly Unrelated Regression (SUR) estimator was inconsistent and that it was impossible to derive a consistent instrumental variable estimator for this model. In a subsequent study of tests of homogeneity in AIDS, Buse (1998) examined the role of price collinearity and concluded that high collinearity was neither necessary nor sufficient to obtain a good approximation of the actual index by the Stone index.

Moschini (1995) offered and described other deficiency of the Stone index. He referred that if the prices were given in natural units, as opposed to indices, the Stone index and derived parameters such as elasticities are not invariant to measurement units. Three alternative indexes proposed to deal with this deficiency. Using a single experiment in Monte Carlo, Moschini confirmed the invariant indices' adequacy, as only the Stone index could not represent a suitable approximation of the right elasticity of expenditure. The question arises as to whether these results are more general or whether Moschini's results are specific to a particular Monte Carlo design. Considering variations in properties, such as price collinearity and sample size change, could well use to calculate the Stone index.

Furthermore, it is necessary to examine the Stone index's sensitivity to measurement units' changes to obtain some meaningful final range of values that could adequately described by the previously published result. Finally, although the Stone index appears to be generally lower, the question remains whether all invariant indices are as good as Moschini's results suggest. The validity of this question turns out when we note that Moschini's advocacy of average price escalation (variants of the Paasche index) has been uncritically accepted in the literature; they can see Mittelhammer, Shi and Wahl (1996), Asche, Bjorndal, Salvanes (1998), and La France (1998).

The main motivation of the contribution is to point out the sensitivity of price indices calculated using the model of demand systems into the context of different levels of price and quantity. We simulated different prices and quantities, so increasing items causing the reduction of their variability. Besides, the variability we consider as a key factor impact on behavior represented by price indexes. The aim of the contribution is not to compare different models of AIDS. We focus on the partial output of specific expenditure elasticities calculated by price indices.

1 METHODOLOGY

It is necessary to describe methodologically price indices, representing partial calculations to the final output of demand system models (in our case, we computed LA/AIDS-Linearized Approximation of the Almost Ideal Demand System) in software R 3.6.3 (Henningsen and Hamann, 2007). Defectively, three basic price indices are generating. There is confusion about which of the price indices is relevant and suitable for interpreting the given situation's decision-maker. For this reason, we decided to describe them in detail and specify their advantages and disadvantages.

Contributions of LA/AIDS models, it is a standard linearization model implementing through the Stone nonparametric index to obtain approximate income deflator parameters. This approximation has some drawbacks. Pashardes (1993) pointed out that this mistake could be understood based on an approximation because of a missed variable. The resulting estimates of query function parameters

Summary 1 Summary about AIDS price indexes

Paasche price index	Calculates the price change between two periods by comparing the cost of purchasing in each period the bundle of goods purchased in the final period
Laspeyres price index	Calculates the price change between two periods by comparing the cost of purchasing in each period the bundle of goods purchased in the initial period
Tornqvist price index	Is a weighted average of the growth rates of the individual prices, with weights equal to the average of the expenditure shares in the two periods used to compute the growth rates

Source: Diewert (1987)

may be biased. Besides, Moschini (1995) pointed out that the Stone index failed to satisfy ownership "adequacy" because the price index's growth rate is not invariant for the unit of measurement of prices. Moschini proposes three alternative indexes: two normalized forms of the Stone index and the Törnqvist index. The indices in Summary 1 will be described in detail in the following chapter.

1.1 Paasche index (the first modified Stone index)

The Stone index typically used in estimating linear almost ideal demand systems is not invariant to changes in measurement units, which may seriously affect the model's approximation properties. A modification to the Stone index or a regular price index is both desirable practices in estimating linear, almost ideal models.

As Moschini (1995) points out, the Stone index can be adjusted to be invariant to unit rates. It is referred to as a modified Paasche index because it uses the current period of expenditure weights. The resulting price index in logarithms is:

$$\log P_t^P = \sum_{k=1}^{n} w_{kt} \log \frac{p_{kt}}{p_k^0} . \qquad (1)$$

Using this price index, the price elasticity is slightly different from the elasticity that formed the Stone index.

Assuming that prices will never be completely collinear, the literature states that a variable called measurement error was introduced when using the Stone index (see Alston, Foster, Green, 1994; Asche and Wessells, 1997; Moschini, 1995). Index Stone does not respect the basic property of index numbers because it changes in measurement units of prices. One solution for correcting units of measurement error is that prices is adjusted by averaging. Moschini (1995) proposes in his paper to use the Laspeyres price index to overcome measurement error because it is a change in units of measurement.

1.2 Laspeyres index (the second modified Stone index)

Historically, the Laspeyres index was created as the first price index. The formula, which formed in 1871 the German economist Etienne Laspeyres:

$$P_L = \frac{\sum(p_t \cdot q_0)}{\sum(p_0 \cdot q_0)} , \qquad (2)$$

calculates the price change between two periods by comparing the cost of purchasing in each period the bundle of goods purchased in the initial period, according to Wynne and Sigalla (1994).

Index measures the change in the price of the basket of goods and services concerning the set weights of the base period. The Laspeyres price index is referred to as a method of weighing quantities in a base year.

Moschini (1995) also proposed a Laspeyres modification, P^L. This modification is analogous to the Laspeyres index in logarithms, with weights calculated based on the expenditure period under review. The index is the following:

$$\log P_t^L = \sum_j^n w_j^0 \log \frac{p_{jt}}{p_j^0} . \qquad (3)$$

Then the expression holds:

$$\frac{\log \delta p_t^L}{\log \delta p_{jt}} = w_j^0 . \qquad (4)$$

Price elasticity of goods i concerning the price of goods j given by expression:

$$\varepsilon_{ij} = -\delta_{ij} + \frac{\gamma_{ij}}{w_{it}} - \frac{\beta_i}{w_{it}} w_j^0. \tag{5}$$

The income elasticity of the goods i is expressed by:

$$\mu_i = 1 + \frac{\beta_i}{w_{it}}, \tag{6}$$

which is the same as the expression for the complete nonlinear model of AIDS. In this case, elasticity patterns are closer to full nonlinear AIDS formula but not identical to the price elasticity.

The index's main disadvantages are that it is upward-biased and tends to overstate price increases (compared to other price indices). It tends to overestimate price levels and inflation.

1.3 Törnqvist price index

The combination (or average) of quantities in both periods is a typical character of the Fisher and Törnqvist price index. This approach seeks to overcome some of the difficulties associated with using a fixed basket at any time. If there are no clear indications that it will be better to use both periods as a basis or benchmark, a combination of both periods appears to be a reasonable compromise.

Paasche, Laspeyres, or any superlative index number can be regarded as discrete approximations to the continuous line integral Divisia index, which has some useful optimality properties from economic theory. These discrete approximations are closer to the Divisia index if the chain principle is used, argued Diewert (1993). The Törnqvist price index is a weighted geometric mean of the price relatives where the weights are the average expenditure shares in the two periods.

Törnqvist indexes are described as symmetrically weighted indexes because they treat the weights from the two periods equally.

The axiomatic approach also referred to as the test approach, consists of formulating 'desirable' properties, which price and quantity indices should satisfy. Prices and quantities of commodities thereby regard as separate variables. A set of functional equations characterizes a price (or quantity) index if it is the unique solution to this set, described by Balk and Diewert (2001).

The economic approach assumes optimization, such as cost minimization or revenue maximization, which implies a relation between prices and quantities. Within the economic approach, Diewert (1976) obtained a characterization of the Törnqvist price index, namely the economical price index corresponding to a linearly homogeneous translog unit cost or revenue function.

This note provides a characterization of the Törnqvist price index from the axiomatic approach. We consider rather broad class of aggregated price relatives. We show that the imposition of two rather natural requirements reduces this class to a single element, the Törnqvist price index. Diewert (1976) has shown that the Törnqvist index is accurate for the function of translogged unit costs:

$$\log c_0(p) = \alpha_0 + \sum_{k=1}^{n} \alpha_k logp_k + \frac{1}{2}\sum_{k=1}^{n} \sum_{j=1}^{n} \gamma_{kj}^{*} logp_k \, logp_j \,, \tag{7}$$

where:

$$\sum_{i=1}^{n} \alpha_i = 1, \sum_{k=1}^{n} \alpha_{kj} = 0, , \gamma_{ij}^{\square} = \gamma_{ji}^{\square}, \text{k} = 1, 2, \dots \text{N}.$$

The Törnqvist index is considered a discrete approximation to the already existing Divisia index, which is defined as:

$$\log P_t^T = \frac{1}{2}\sum_j \left(w_{jt} + w_j^0\right) \log \frac{p_{jt}}{p_j^0}. \tag{8}$$

And therefore:

$$\frac{\log p_t^T}{\log p_{jt}} = \frac{1}{2}\left[w_{jt} + \sum_{k=1}^{n} \log p_{kt} \frac{\delta w_{kt}}{\delta \log p_{jt}}\right] - \frac{1}{2}\sum_{k=1}^{n} \log p_k^0 + \frac{1}{2} w_j^0 = \frac{1}{2}\left[(w_{jt} + w_j^0) + \sum_{k=1}^{n} w_{kt} \log \frac{p_{kt}}{p_k^0} \frac{\delta w_{kt}}{\delta \log p_{jt}}\right] = \left[(w_{jt} + w_j^0) + \sum_{k=1}^{n} w_{kt} \log \frac{p_{kt}}{p_k^0} (\varepsilon_{kj} + \delta_{kj})\right], \tag{9}$$

where ε_{kj} is Marshall cross price elasticity and δ_{kj} is Kronecker delta.

By replacing the last equation of the AIDS model, an uncompensated cross-price elasticity of the demand for the goods *i* arises concerning the price of the goods *j*.

$$\varepsilon_{ij} = \delta_{ij} + \frac{\gamma_{ij}}{w_i} - \left(\frac{1}{2}\right) \frac{\beta_i}{w_i}\left[(w_{jt} + w_j^0) + \sum_{k=1}^{n} w_{kt} \log \frac{p_{kt}}{p_k^0} (\varepsilon_{kj} + \delta_{kj})\right]. \tag{10}$$

The derivation of income elasticity is analogous:

$$\frac{\delta \log_p^T}{\delta \log_m} = \frac{1}{2}\sum_{k=1}^{n} \log p_{kt} \frac{p_{kt}}{p_k^0} \frac{\delta w_k}{\delta \log m} = \frac{1}{2}\sum_{k=1}^{n} w_{kt} \log \frac{p_{kt}}{p_k^0} (\mu_k - 1). \tag{11}$$

We consider the Divisia Index called Törnqvist (1936) to be a theoretical construct of a continuously weighted sum of the growth rates of the individual components. Scales represent the share of a component in the total value. The so-called growth rates are determined for the Törnqvist index as the difference in the natural logarithms of the successive observations of the components (their log-change). The Divisia or Törnqvist indices have advantages over constant base year weighted indices because they include changes in both purchased quantities and relative prices when relative input prices change.

The Törnqvist index is excellent; thus, it can approximate a continuous production or cost function. "Smooth" means small changes in the relative prices of goodwill associated with small changes in its quantity. Törnqvist corresponds exactly to the production function of the translog, which means that with a change in prices and an optimal response in quantities, the level of the index changes exactly as well as a change in production or utility.

1.4 The Linearized Almost Ideal Demand System (AIDS)

All the above indexes are included in the calculations generating cross and own-price elasticity of demand models systems. This paper has used the demand system LA/AIDS, the linear approximate almost ideal demand system. Each equation in the AIDS given as:

$$w_i = \alpha_i + \sum_i \gamma_{ij}\, lnP_i + \beta_i \ln\left(\frac{X}{P}\right) + \mu_i, \tag{12}$$

where w_i is share of the *i*th good (because $wi = PiQi/X$), P_j is the price of the *j*th good, *X* is the total expenditure on all goods in the system, *P* si a price index, μt is the residuals, and assumed to have zero mean and constant variance, α_i, β_i, and γ_{ij} are parameters.

The price index (*P*) is a translog index:

$$lnP = \alpha_0 + \sum_i \alpha_i\, lnP_i + \frac{1}{2}\sum_j \square\, \gamma_{ij}\, lnP_i\, lnP_j. \tag{13}$$

The price index from Formula (12) makes Formula (13) a nonlinear estimation, raising estimation difficulties. To avoid nonlinear estimation, many empirical studies used Stone (1953) price index (P^*) instead of P, as suggested by Deaton and Muellbauer (1980):

$$lnp^* = \sum_i w_i lnP_i \,. \tag{14}$$

The model that uses the Stone geometric price index is called the Linear Approximate AIDS (LA/AIDS). It shows that if prices are highly collinear, then the LA/AIDS model can estimate the AIDS model's parameters because the factor of proportionally of P to P* incorporated in the intercept term (Green and Alston, 1990; Hsiao, 1986).

1.5 Characteristics data needed for LA/AIDS model and index calculations

The analysis will be performing for the period 1993–2017. For the analysis, we used data obtained from the Statistical Office of the Slovak Republic, the excel database:

- Consumption of selected kinds of foodstuffs per capita.
- Prices from the consumer prices of the consumer basket of food.

We have selected only four meat types. For a better view, we provide short descriptive characteristics of input data.

Table 1 Descriptive statistics of original input data

	beef_q	pork_q	poultry_q	fish_q	beef_p	pork_p	poultry_p	fish_p
Mean	43 687	183 726	93 565	24 916	6.44	4.98	2.32	0.73
Standard deviation	23 432.12	17 457.23	16 106.65	2 824.04	1.44	1.55	0.17	0.19
Minimum	19 171.00	151 555.00	63 081.00	20 196.00	4.07	3.04	2.04	0.42
Maximum	115 070.00	235 766.00	119 932.00	31 254.00	8.68	8.31	2.71	1.03
Skewness	1.19	0.79	–0.29	0.46	0.00	0.74	0.85	0.14
Kurtosis	1.13	1.07	–1.12	–0.53	–1.32	–0.75	0.01	–0.99

Note: p is price in EUR and q is the quantity in kg.
Source: Own calculation

We provided descriptive statistics for original and adjusted, so simulated data for deeper insight into the analysed data. From Table 1, we can review that data have different characteristics. The range between max and min values is wider. If we look at the mean and standard deviation, we can see the standard

Table 2 Descriptive statistics of adjusted input data

	beef_q	pork_q	poultry_q	fish_q	beef_p	pork_p	poultry_p	fish_p
Mean	46 696	198 912	101 420	27 136	7.0	5.3	2.5	0.8
Standard deviation	25 852.73	47 848.43	31 491.64	7 561.65	2.41	1.67	0.56	0.29
Minimum	17 253.90	126 409.50	56 142.09	16 958.25	4.07	2.51	1.68	0.43
Maximum	126 577.00	262 149.00	179 898.00	41 232.00	10.91	8.93	3.77	1.24
Skewness	1.15	**–0.21**	1.06	0.32	0.38	0.36	0.45	0.41
Kurtosis	1.26	–1.54	0.57	–1.20	–1.46	–0.47	–0.50	–1.40

Note: p is price in EUR and q is the quantity in kg.
Source: Own calculation

deviations presented by small values. Therefore, the time series of values are closer to the mean value, so they are not significantly fluctuating values. The skew and kurtosis values also indicate a descriptive condition. All the variables examined, except beef_q and pork_q, show a more pointed distribution of values.

Table 2 shows that the values of standard deviations have changed and the values of skewness and kurtosis have changed, concurrently. As an example, we mention pork_q where skewness and kurtosis have changed, and the value of kurtosis has changed in the variable poultry_p.

For clarification our simulation, we present the analyzed data in line graphs. The blue line represents the original price or the original value of consumption. The red line represents simulated changes in the original time series.

The gradual increase in beef and fish prices can follow up in Figure 1. We can evaluate that poultry prices did not increase significantly during the period under review, so they fluctuated significantly over twenty years. On the other hand, pork became cheaper during the period under review. The year-on-year change in prices is not significant. The time series of original prices showed low values of year-on-year changes, which caused similarity in the calculated values of indices in the demand model. This reason,

Figure 1 Prices and quantity of meat items-beef, pork, poultry, and fish, 1991–2018 and simulations of shocks in adjusted data of prices and quantities

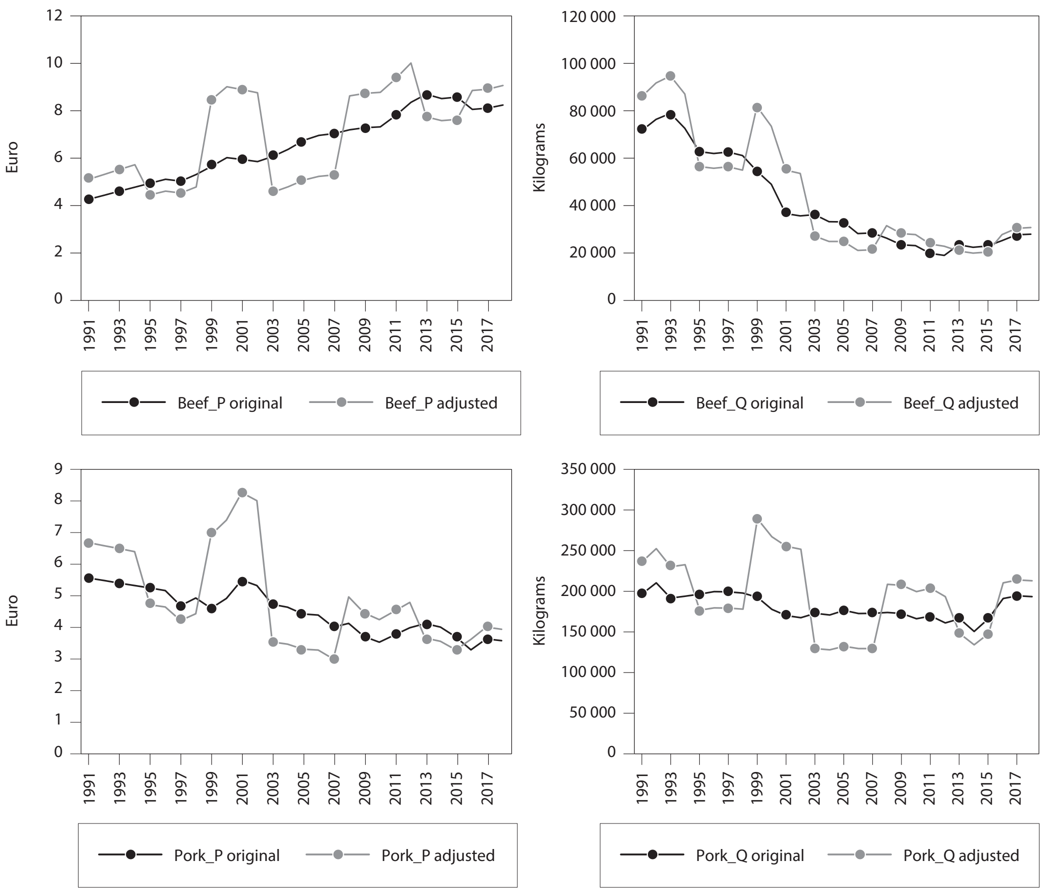
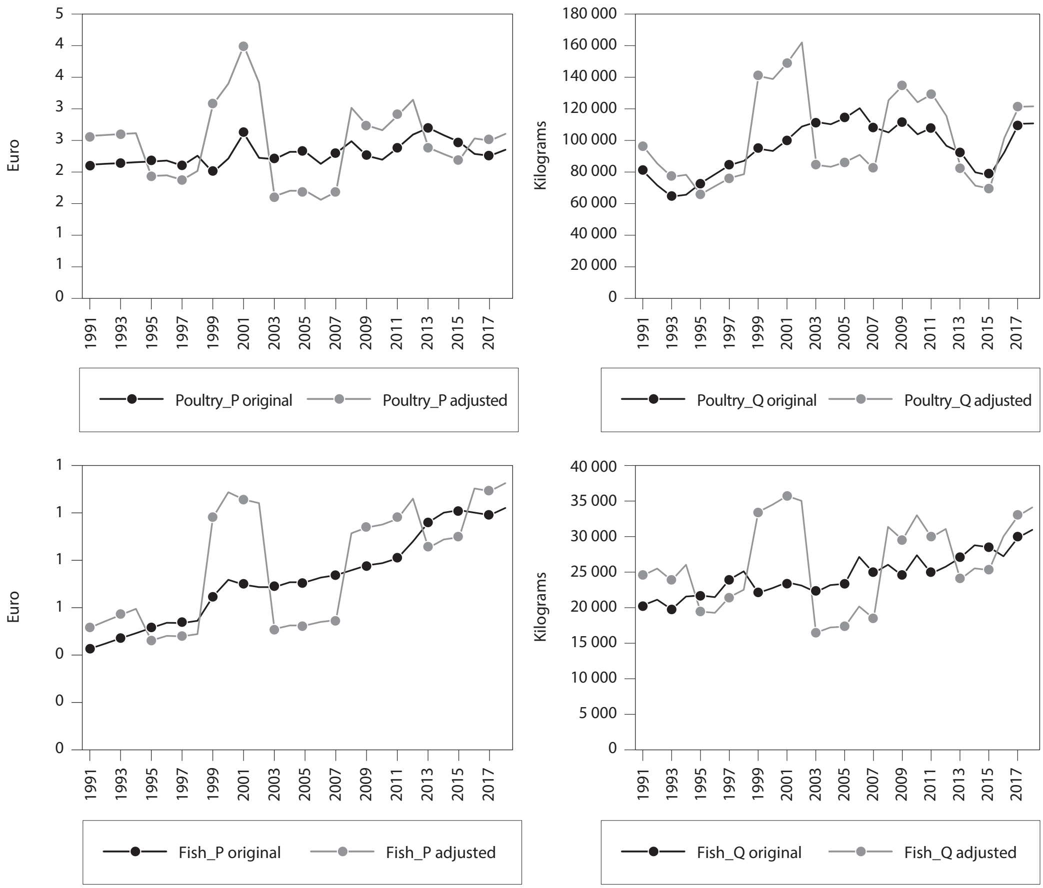


Figure 1 (continuation)



Note: p is price in EUR and q is the quantity in kg.
Source: Data from the Statistical Office of the Slovak Republic (available at: <*www.statistics.sk*>)

we decided to simulate price shocks that would affect the value of the price index. We simulated price random shocks with a 10% decrease and increased compared to the original value.

2 RESULTS AND DISCUSSION

In the results chapter we present the values of price indices of the original data and compare them with simulated data and at the same time we point out the values of price indices of other authors, for comparison and understanding of the presented situation. Following the demand model generates three different price indices and we will examine which of them presents the most relevant picture of the value of income elasticities.

2.1 Comparison of different price approximations through price indices

The analysis aims to point out the issue of variance in the researched data. The already mentioned elasticities calculation is one of the necessary moments in consumer behavior economic analysis. We decided to examine and assess specific price indices' suitability in the output AIDS model based on the studied issues.

The elasticities calculated by the AIDS model themselves indicated a situation that is identical to the real situation. It is worth mentioning that the values of the indices did not differ, although each has been calculated from a different methodological basis. We investigated what does affect this condition. The first table shows the index values of the actual data. We see that no different values were found in the indices.

Income elasticity measures the sensitivity of demand to changes in consumer income and influenced by the time during which elasticity is measured (shorter period shows lower elasticity of demand) and the level of demand for the goods (if there are more essential goods in the model, the model results from income elasticity of demand), Sloman and Norris (2002). Income elasticity can be interpreted as a percentage change in the quantity demanded if other income changes by approximately 1% while factors remain ceteris paribus. Given that the elasticity is independent of measuring units in which demand measured, consumers' most meaningful rate sensitivity demands a change in income or prices described (Benda-Prokeinova, 2016).

Table 3 Income elasticities computed by the AIDS model

	Törnqvist index	Stone index	Laspeyres
Pork	0.908564	0.908564	0.908564
Poultry	0.871678	0.871691	0.871678
Beef	1.583992	1.583982	1.583992
Fish	–0.20235	–0.20234	–0.20235

Note: Data from the Statistical Office of the Slovak Republic (available at: <*www.statistics.sk*>).
Source: Own calculations

Suppose we estimate the average level of beef expenditure for the whole sample, and the income elasticity of demand for beef was 1.58 in Table 3. In that case, a 10% increase in household income could increase beef demand by 15.8%. The lowest value of income elasticity is shown by poultry 0.88. Suppose household income increased by 10%. The demand for poultry would increase by 8.8%. Estimates of intake elasticity for the studied foods were statistically significant, evaluated Benda-Prokeinova (2016).

We can state that pork, beef, and poultry show a positive value of income elasticity of demand, which indicates that these are normal goods. According to the elasticities values, beef shows a luxury commodity, while other types of meat were income-inelastic, indicating its necessity. The model also has inferior food – fish with the value –0.20.

Table 4 Expenditure elasticities by a contribution of Buse and Chan (2000)

Panel A: Meat Data

	Stone index	Laspeyres index	Paasche index	Törnqvist index
$\eta 1$	1.8865 (0.0981)	1.3375 (0.1759)	1.3515 (0.1771)	1.3445 (0.1765)
$\eta 2$	0.8958 (0.1288)	0.4546 (0.1095)	0.4494 (0.1114)	0.4519 (0.1104)
$\eta 3$	0.7677 (0.1551)	1.4236 (0.2584)	1.3819 (0.2692)	1.4038 (0.2639)
$\eta 4$	0.8825 (0.0210)	0.9835 (0.0328)	0.9832 (0.0332)	0.9833 (0.0330)

Source: Buse and Chan (2000)

Buse and Chan (2000) investigated invariance in price indices calculated by the AIDS model. When we compare their results with our results, we find that they have differences between the indices. Our calculations have the same values, in contrast with the output Buse and Chan, where the values of indices in rows are slightly different. That's, in fact, the subject of our interest. How come that the AIDS model computes the indices of the same value? We were considering that a possible answer lies in the invariance of input data. Another reason causing similar values of price indices may lie in the trend of the examined time series. The prices of the examined meat items grew at a similar pace. It can also be one of the reasons for similar values of price indices.

For better understanding, we provided part of the research Buse and Chan paper in Table 4. Subsequently, we focused on increasing the invariance first in beef consumption, while prices remained ceteris paribus. We found that the indices' values did not differ from each other except for poultry and fish in these two commodities. Input`s prices are significantly distorted. For example, we can find the canned fish price only in the consumer basket and not for a real fish piece. A paper provided similar results from Sheng et al. (2008), where the Stone and Laspeyres index acquired different values of elasticity expenditure.

Table 5 Estimated expenditure and own-price elasticities for food items, Malaysia

Food item	LA/AIDS with Stone price index	LA/AIDS with Laspeyres price index
Rice	1.334	0.9091
Bread	0.7536	0.3177
Meat	1.0318	1.4064
Fish	0.9425	1.244
Milk and dairy	0.6284	0.8698
Eggs	1.1429	0.7675
Oils and Fats	1.1158	1.1054
Fruits	1.0325	1.0905
Vegetables	1.1955	1.1729
Sugar	0.9607	0.7458
Other	0.8651	1.4395

Source: Sheng et al. (2008)

The study provides a complete food demand system in Malaysia by analyzing data from the 2004/2005 household expenditure survey using LA/AIDS models to include the Stone price index and the Laspeyres price index. The study results suggest that the Laspeyres price index's application leads to more likely estimates of the elasticity of expenditure and own prices in Malaysia. This discovery is similar to the discovery described by Alston et al. (1994), Asche and Wessells (1997), and Moschini (1995). Therefore, this study's further implications are based on LA / AIDS models that include the Laspeyres price index. Evaluated by the research of Sheng et al. (2008) in Table 5.

For this reason, we focused only on beef, pork, poultry, and fish. Beef has experienced a renaissance in consumption over the last 20 years, and pork is the second national dish of Slovaks after dumplings. Poultry has been considered meat for the poor in Slovakia and we consume woefully low amounts of fish below the recommended doses.

If we follow the values of price indices in both simulations, we should notice one problem. We assumed that the values of the indices would not fluctuate significantly when the quantity changed. We calculated the values of price indices from data in which we simulated an increase and decrease of 10% of the quantity consumed. Thus, a significant change in the variability of meat consumption will not affect the resulting values of the indices. This assumption has been fulfilling. We conclude that

Table 6 Simulations of invariance in price and quantity in the meat items – beef, pork, poultry, and fish

	Original data			Simulation of the Beef quantity			Simulation of the Pork quantity			Simulation of the Poultry quantity			Simulation of the Fish quantity		
	Törnqvist	Stone	Laspeyres	Törnqvist	Stone	Laspeyres	Törnqvist	Stone	Laspeyres	Törnqvist	Stone	Laspeyres	Törnqvist	Stone	Laspeyres
Pork	0.9086	0.9086	0.9086	1.1514	1.1661	1.1598	0.9068	0.9038	0.9059	0.7671	0.7706	0.7681	0.8992	0.8992	0.8992
Poultry	0.8717	0.8717	0.8717	–0.9682	0.0812	–0.9522	1.37	1.12	1.99	–3.1690	1.35	–4.1086	0.7428	0.7428	0.7428
Beef	1.40	1.40	1.40	1.31	1.11	1.09	1.0675	1.0823	1.0720	0.9204	0.6814	0.9306	1.51	1.51	1.51
Fish	–0.2023	–0.2023	–0.2023	0.7266	0.7324	0.7355	0.5044	0.4983	0.5025	0.5618	–1.2179	0.5593	0.8990	0.8990	0.8990
	Original data			Simulation of the Beef price			Simulation of the Pork price			Simulation of the Poultry price			Simulation of the Fish price		
	Törnqvist	Stone	Laspeyres	Törnqvist	Stone	Laspeyres	Törnqvist	Stone	Laspeyres	Törnqvist	Stone	Laspeyres	Törnqvist	Stone	Laspeyres
Pork	0.9086	0.9086	0.9086	1.0999	0.9292	0.7754	1.1228	1.1301	0.9171	0.9878	1.0789	0.9525	0.8960	0.8960	0.8960
Poultry	0.8717	0.8717	0.8717	0.4678	0.7768	19.0109	0.8298	0.2337	2.34	0.1711	1.96	0.4808	0.6969	0.6969	0.6969
Beef	1.40	1.40	1.40	1.54	1.10	1.91	0.9230	1.81	1.26	1.33	1.92	1.14	1.63	1.63	1.64
Fish	–0.2023	–0.2023	–0.2023	0.8153	–3.3637	6.82	0.6952	0.8542	0.2910	0.6990	1.1116	0.4254	0.9278	0.9278	0.9278

Note: Data from the Statistical Office of the Slovak Republic.
Source: Own calculations

the calculated indices copy the original values. This result suggests that the resulting price index has been affected by price and not quantity. For this reason, we will not even interpret price indices calculated from the simulated quantity consumed. We will focus only on price indices calculated from simulated prices. In the second assumption, we assumed that the change in price variability would affect the values of price indices. The simulations of meat product prices has adjusted so that price shocks occurred in time series, which increased and decreased by 10%. This assumption has yielded interesting findings. If we look at the values of the beef index, it is considered a luxury item, but the values vary from 1.31 to 1.69. An interesting situation occurred in pork, where according to the outcome of the original values, pork is normal good. If we changed prices of the beef (increasing and decreasing by 10%, see Figure 1), pork has changed from normal to luxury good. Similar situation is at poultry. In the original analysis, we consider poultry as a normal good. After changing variability in beef prices, poultry is presented as inferior. Increasing the variability of original price caused by the price shocks can significantly affect the calculation of income elasticities in the AIDS model.

Subsequently, we performed the same simulation on pork and poultry. The interpretation of the results is similar. There has been a change in the income elasticities of poultry. According to the original analysis, poultry was one of the standard goods, but after the change in pork prices, we have to reclassify poultry into luxury goods. Again, we do not record changes in the values of individual indices. Subsequently, we performed the same price shocks in poultry prices, while the other variables entering the AIDS model are ceteris paribus. More important for us was the finding that the indices take on different values. If we compare the original data indices with the values of simulation data indices, we can conclude that the calculated price indices are different. From an interpretative point of view, the output of the analysis confirms the fact that if there are small changes in time series of prices represented by low variability, it is clear that the calculated price indices representing income elasticity will show a distortion, which represents very similar values in all calculated indices.

However, one peculiarity occurred again in fish. We simulated price shocks. The values of the price indices are identical. They differ from the original values, but Laspeyres and Törnqvist are the same compared to Stone. It is quite likely that the price changes did not bring about such a massive change that would affect the final value of price indices. To clarify the reasons why income elasticities react in this way, it is appropriate to see the price in Figure 1 for beef, pork, poultry and fish. Figure 1 shows prices (blue line); we see that price changes occurred gradually. No price shocks and jumps took place in Slovakia.

In the analysis, we also came to cases (as an example is the output of a beef simulation), that according to the Törnqvist index, pork meat is considered luxury good and according to the Stone and Laspeyres index it is a normal commodity. Several similar situations have arisen in the simulation of individual prices. The ambiguity of the output led us in the search for an answer to the question which of the indices can we consider the most trustworthy? After studying the detailed information about each of the indexes, we came to the answers.

As a typical index used to estimate linear almost ideal demand systems, the Stone index is not constant against changes in measurement units that can affect the model's approximation properties. In other words, if the data changes show a low variability, it is possible to use interpretations of the Stone index or its modified and improved version of the Paasche index.

The values of the Laspeyres index are skewed upwards and, as a result. It overestimates price increases (compared to other price indices) and thus overestimates price levels and inflation.

The Törnqvist index is almost "consistent", which means that the result is almost the same index values formed by combining many prices and quantities or by combining their subgroups and then combining these indices. It follows from the above that the Törnqvist index expresses the most realistic characteristic identifying the consumer's relationship to the goods under investigation (Diewert, 1976).

Balk and Diewert (2001) developed the theorem on the Törnqvist index. The price index is a unique member of the class of aggregate related prices, which has the property that it is linearly homogeneous in prices of the comparable period. At the same time, it satisfies the time-reversal test.

Buse and Chan (2000) present another approach for identification and using an appropriate index. There are some practical suggestions for applied research in the field of price indices. Suppose, for any reason; it is necessary to estimate the LAI model. In that case, examining the price correlation structure it is necessary to decide whether to use the Laspeyres or Tornqvist index. The Laspeyres index performs exceptionally well with strong positive collinearity, with the latter being better below zero or mixed collinearity. In the mostly time-series studies, price data are strongly positively correlated; Theil (l976) provides an example. We also decided to investigate the structure of price correlation as Buse and Chan (2000).

Table 7 Correlation matrix of the meat prices

Pearson Correlation Coefficients, N = 29
Prob > |r| under H0: Rho = 0

	beef_p	pork_p	poultry_p	fish_p
beef_p	1.00000	–0.89984 <.0001	0.70405 <.0001	0.97258 <.0001
pork_p	–0.89984 <.0001	1.00000	–0.40532 0.0292	–0.86770 <.0001
poultry_p	0.70405 <.0001	–0.40532 0.0292	1.00000	0.65786 0.0001
fish_p	0.97258 <.0001	–0.86770 <.0001	0.65786 0.0001	1.00000

Source: Own calculation, computed in SAS 9.4

From the analysis in Table 7 it is clear that all correlation coefficients are significant. Three coefficients acquired a negative value and three a positive value. We can talk about mixed values. According to Buse and Chan (2000) theory, based on the values of price correlations, the Tornqvist index is more suitable for further analysis. Following we verify the collinearity in the price data using the variance inflation factor (VIF) for the theories described above.

Table 8 Variance inflation factors and testing of variabilities of the meat items

	Original data		Adjusted data		
	R Square	VIF	R Square	VIF	F-test of variability (p-value)
Pork	0.801534	5.038646	0.2788	1.386578	0.0457861
Poultry	0.101108	1.11248	0.6743	3.07031	5.802E-05
Beef	0.033365	1.034517	0.1516	1.178689	2.896E-09
Fish	0.753128	4.050677	0.5682	2.315887	0.0755607

Source: Own calculations

VIF quantifies how much the scatter inflates. The standard errors – and hence the variances – of the estimated coefficients inflated when multi-collinearity exists. A variance inflation factor exists for each of the predictors in a multiple regression model (Course material, Regression Pitfalls, 2018).

According to Han (2018), the general rule of thumb is that VIFs exceeded 4-warrant further investigation, while VIFs exceeding value 10 are signs of serious multi-collinearity and is requiring correction. In our case, investigated variables do not exceed the value of VIF 10, neither in the original nor the modified data.

Based on the assumption of zero multi-collinearity, we can say that it would be most appropriate to interpret only the Tornqvist index. We evaluated this conclusion by studying the literature and assessing the input values and statistical verification's real state.

Due to the invariance of the analyzed data, we were still interested in whether there is a difference in variability between the original and modified data. It was best to test the agreement of the variables in the researched variables. Using the F-Test Two-Sample for Variances, we found a difference in the variance values between the original and the modified data. This condition seemed quite substantial.

CONCLUSION

The primary motivation of the paper was to point out the sensitivity of price indices calculated by the model of demand systems at different price and quantitative levels. We pointed out the important role of variability of input data, which is the basis for identifying consumer behavior. Followed, we assumed that the change in price variability would affect the values of price indices. This assumption was correct.

More important for us was the finding that the indices take on different values. If we compare the original data indices with the values of simulation data indices, we can conclude that the calculated price indices are different. From an interpretative point of view, the output of the analysis confirms the fact that if there are small changes in time series of prices represented by low variability, it is clear that the calculated price indices representing income elasticity will show a bias, which represents very similar values in all calculated indices (our case).

The main reason of the values of the identical price indices are small changes. The time series of original prices showed low values of year-on-year changes. The variability of price values did not change significantly in the observed period (we mean extreme increases and decreases).

Actual data suggest that all types of meat studied are constantly growing without massive price shocks. If the price of one type of meat increases, other types of meat will increase as well. The rise in price is negligible for the consumer, because the changes are in the tenths or hundreds. The observed similarity of expenditure elastics indicates that all studied types of meat have an upward trend in time series, not only in consumption but also in price.

From the simulations we found out even if the price of one product increases, it will not affect the consumer's behavior towards other goods. Extreme conditions would have to exist, such as the complete failure of the production one or two types of meat, and then a significant change in behavior could occur.

Several similar situations arose in the simulation of individual prices. We used the theory of Buse and Chan (2000) to find out which of the indices we can consider the most trustworthy. We examined the price correlation structure which is necessary for decide whether to use the Laspeyres or Tornqvist index.

All correlation coefficients are significant. Three coefficients acquired a negative value and three a positive value. We can talk about mixed values. According to Buse and Chan (2000) theory, based on the values of price correlations, the Tornqvist index is more suitable for further analysis. Similar results we gained by testing multi-collinearity. Based on the assumption of zero multi-collinearity, we can say that it would be most appropriate to interpret only the Tornqvist index.

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