Illustration of Single-Regional and Inter-Regional Approach in Regional Input-Output Analysis

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Abstract

Analytical works usually use single-regional approach which does not demand so much data. However, this approach disregards flows of output among regions. This leads to a misrepresentation of results which can be eliminated by using Inter-regional input-output model that requires more data to be employed. This paper illustrates the differences between the two different approaches of regional input-output model construction and their results. We construct inter-regional and single-regional models for all 14 regions of the Czech Republic and with 82 products according to the Classification of Products CZ-CPA. The results are compared on the level of Leontief's matrix and multipliers. We use graphical illustrations to depict the systematicness of differences. The single-regional approach proves a systematic undervaluation of specific products and regions contrary to other regions. The graphical analysis shows the significance of the connection among regions. This illustrates the disadvantage of the single regional approach. Finally, the results confirm the idea of a significant analytical misrepresentation of impacts modelled by this approach in the case of data for the Czech Republic.

Keywords

Regional Input-Output Tables, Input-Output analysis, Leontief`s multipliers, IRIO

JEL code

C67, R13, E21

INTRODUCTION

Regional input-output analysis represents a detailed tool of economic analysis on the sub-national level. Contrary to input-output analysis (IOA), on the national level the regional IOA offers detailed information on the exact structure of impacts. An advantage of the regional IOA lies in an accurate evaluation of effects in individual regions and products. The regional analysis of national policies in context of environment (Miller and Blair, 2009) represents the most common analysis. The detailed output of IOA actually enables a connection to the environmental matrix (Suttinon et al., 2013).

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The regional IOA divides according to two categories of models. One category represents models based on one region with no connection to other regions while the other category comprises inter-regional analysis where researchers simultaneously consider export and import to other regions (Miller and Blair, 2009). If we disregard the connection to other regions, we can use the single-regional input-output model (SRIO). For a comparison, we use the inter-regional input-output model (IRIO). The inconsistency of SRIO and IRIO ties to a so-called problem of aggregation of regional input-output tables. When using SRIO, the aggregation of RIOTs does not lead to national IOT (Crown, 1990).

This paper aims at comparing the SRIO and IRIO approaches using the results of Leontief's matrix and multipliers. The calculations prepared according to the CZ-CPA 2 digit are demostrated at the aggregate level for individual products and regions to give a true picture of the main differences between the approaches. We expect systematic structural differences caused by disregarding relations among regions. These differences are illustrated for the visualization of their systematicness and homogeneity (heterogeneity) across individual regions and products. Moreover, we illustrate the results using figures for the Czech Republic. They clearly show the strength of the connection among geographically close regions.

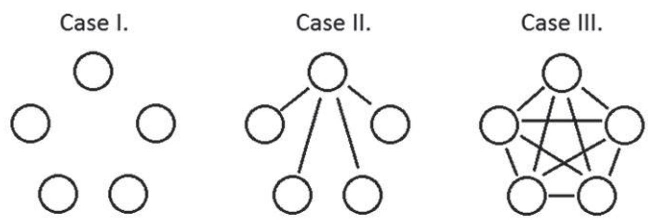
1 LITERATURE REVIEW

The growing number of methods used for regionalization of national input-output tables and the increasing amount of individual analyses (Daniels et al., 2011; Okadera et al., 2014; or Kim et al., 2004) both confirm the rising importance of regional input-output analysis. This phenomenon justifies the existence of multi-regional input-output tables for individual states (Timmer et al., 2015). The main role of regional input-output tables lies in the clarification of the decomposition process of the national impact on regional bases where individual impacts may act unequally even if the weighted sum of effects corresponds to the national analysis.

The following case illustrates this situation: a country which produces a product (*Q*) mostly in one region (*Rq*). It affects the intermediate consumption of the region as well. Thus, in the case of an exogenous impulse in another region and another product the demand for the product *Q* intermediate consumption could increase even if this product does not fall within the intermediate consumption of the region. This is caused by different regional structure where the product *Q* enters intermediate consumption. These differences bring about an inconsistency of estimations at the national level compared to the regional level.

Many regional input-output models exist which analyse multi-regional effects triggered by regional export and import (Miller and Blair, 2009). According to Lenzen et al. (2004) these models divide into three cases (Figure 1).

Figure 1 Three approaches of modelling regional IOA



Source: Authors' own elaboration based on Lenzen et al. (2004)

Case I represents the situation where individual regions create individual units with no export and import among each other (SRIO approach). Any export and import constitutes only exogenous variables. This is an analogy to the national model (Miller and Blair, 2009; EUROSTAT, 2008). Case II illustrates the model where an increase of output in one region causes an increase of output in another region.

However, this other region has no influence on the remaining regions. In this case, we are unable to discuss the so-called 'backward-linked multipliers' (Steinback, 2004). Case III works with relations among all regions. This model allows us to observe backward regional effects. The inconsistence among all cases is called information bias. It can be proved that the consistence of IRIO and national input-output tables creates a neutral bias (Crown, 1990).

Taking no account of inter-regional flows within the IRIO model comprises the root of the inconsistency between the SRIO and IRIO approaches. There is also a certain synthesis of the approaches in question, i.e. Leontief's international model (Leontief, 1953; Leontief and Strout, 1963). This model finds a way between by dividing an economy to an individual region and the rest of the given economy (e.g. Miller and Blair, 1985). This model does not allow researchers to evaluate backward effects or distinguish the target regions to which the production of the examined region multiplies. However, the construction of such model requires a calculation of the flow between the region and the rest of the economy.

Two basic regional input-output models exist for Case III, i.e. Isard's IRIO model (Isard et al., 1960) and Chenery's Multi-regional input-output model, abbreviated as MRIO (Chenery, 1953). The main difference between these models lies in the detail of calculation. While MRIO does not consider a detailed allocation of flows among regions, IRIO requires such data. With respect to the detail of IRIO, we decided to use this approach. IRIO allows us to investigate detailed differences among regions and effects of the flows among regions.

Even though several inputs (e.g. Lahr, 1993) indicate that data sources constructed without survey could produce biased results, we decided to base the flows among regions on minimization of distance (Šafr, 2016). Several input lead us to this choice: firstly, the homogeneity of methods used; secondly, we assume this bias as insignificant in the case of flows among regions (Sargento, Ramos and Hewings, 2012); finally, indirect estimates are accepted disregarding other available data sources.

2 METHODOLOGY[3]

2.1 National Input-Output Methodology (NIO)

The core of IOA consists in the matrix of intermediate consumption $\boldsymbol{X}$. Components of this matrix represent the flow of output from industry i to industry j. If we summarize everything that industry i supplies to other industries and add total final consumption (y) and export (e) in this industry, we get the total output of this industry. The following formula represents the basic equation of IOA (EUROSTAT, 2008):

$$\sum_{j}^{n} x_{ij} + y_i = x_i\,,\;\; j = 1,2,3\ldots\ldots, n. \tag{1}$$

x_{ij} represents the flow of intermediate consumption from industry i to industry j; y_i comprises the final use of product i (final consumption together with export). The proportions of intermediate consumption flows from industry i to industry j on total production of industry j represent technical coefficients:

$$\text{Matrix}: \mathbf{A} = (a_{ij})_{nn}, \text{where: } \; a_{ij} = \frac{x_{ij}}{x_j}\,,\;\; j,i = 1,2,3\ldots\ldots, n. \tag{2}$$

Technical coefficients represent production functions of individual industries which remain stable over a long time period. Moreover, they show how many inputs of intermediate consumption one unit

[3] This part of the paper was published in Šafr and Vltavská (2016): *The evaluation of economic impact using the regional input-output model: the case study of Czech regions in context of national input-output tables* (14th International Scientific Conference 'Economic Policy in the European Union Member Countries'). As we think it necessary for the clarification of the method used, we publish this part in this paper as well.

of output of the industry i requires. We constructed the fundamental input-output model from equation (2) which describes the value of total output necessary for fulfilling final use:

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y}, \tag{3}$$

where $\mathbf{I}$ represents the identity matrix and $(\mathbf{I}–\mathbf{A})^{-1}$ Leontief inversion.

2.2 Single Region Input-Output Analysis (SRIO)

Regional relations are similar to national ones. Miller and Blair (2009) describe the fundamental production function:

$$\sum_{j}^{n} x_{ij}^{R} + y_{ij}^{R} = x_{i}^{R}\,,\ j = 1,2,3\ldots\ldots, n, \tag{4}$$

where x_{ij}^{R} represents the flow of intermediate consumption from industry i to industry j in region R; y_{ij}^{R} represents final use of product i in region R. The difference between the national and regional model lies in export as part of final use. The regional model includes not only export outside the country ($e_i^{N_R}$) but export to other regions within the country (e_i^{R}) as well.

Following description characterises technical coefficients:

$$\text{Matrix: } \mathrm{A}^{R} = (a_{ij}^{R}),\ \text{where: } a_{ij}^{R} = \frac{x_{ij}^{R}}{x_{i}^{R}},\ j, i = 1,2,3\ldots\ldots, n. \tag{5}$$

Regional technical coefficients differ from national technical coefficients. Šafr (2016) described their relation as follows:

$$\left[\sum_{R=1}^{m} \mathbf{A}^{R} diag(\mathbf{x}^{R})\right] diag(\mathbf{x})^{-1} = \mathbf{A}. \tag{6}$$

Finally, formula (3) is adjusted for the regional model:

$$\mathbf{x}^{R} = (\mathbf{I} - \mathbf{A}^{R})^{-1}\mathbf{y}^{R}. \tag{7}$$

2.3 Inter-regional Input-Output Analysis (IRIO) – Isard's approach

IRIO is based on decomposition of matrix A (Miller and Blair, 2009). Our goal is the construction of a matrix of intermediate consumption X^T that simultaneously differentiates individual products and individual industries. This matrix consists of n products and m regions (this matrix has $m \times n$ columns and rows in total). The diagonal of X^T represents the regional matrix of intermediate consumption (X^T). Matrices outside the diagonal represent the allocation of import from region i to region j:

$$\mathrm{X}^{\mathrm{T}} = \begin{bmatrix} \mathrm{X}^{1} & \mathrm{F}^{1,2} & \cdots & & \cdots & \mathrm{F}^{1,m-1} & \mathrm{F}^{1,m} \\ \mathrm{F}^{2,1} & \mathrm{X}^{2} & \cdots & & \cdots & \mathrm{F}^{2,m-1} & \mathrm{F}^{2,m} \\ \vdots & \vdots & \ddots & & & \vdots & \vdots \\ & & & \mathrm{X}^{R} & & & \\ \vdots & \vdots & & & \ddots & \vdots & \vdots \\ \mathrm{F}^{m-1,1} & \mathrm{F}^{m-1,2} & \cdots & & \cdots & \mathrm{X}^{m-1} & \mathrm{F}^{m-1,m} \\ \mathrm{F}^{m,1} & \mathrm{F}^{m,2} & \cdots & & \cdots & \mathrm{F}^{m,m-1} & \mathrm{X}^{m} \end{bmatrix} \tag{8}$$

Beside others, the condition of the transition from X^T to national matrix X applies:

$$\sum_{R=1}^{m}\sum_{\beta=1}^{m*n} x^{T}_{[(R-1)*82+i],\beta} = \sum_{j}^{n} x_{i,j}. \tag{9}$$

Columns of X^T has to follow the same condition:

$$\sum_{R=1}^{m}\sum_{\alpha=1}^{m*n} x^{T}_{\alpha,[(R-1)*82+j]} = \sum_{i}^{n} x_{i,j}. \tag{10}$$

Formulas (9) and (10) ensure comparability of inter-regional impacts and national impacts. Thus, this represents inter-regional decomposition of national matrix with respect to inter-regional particularity.

The final part presents the construction of matrix **F**. An unsolved problem lies in the construction of regional matrices $\mathbf{F}^{R,P}$where rows represent export of output from industries in region R to region P. This matrix has the same number of columns and rows as the matrix of intermediate consumption and has to respect the volume of inter-regional flows (Šafr, 2016). Using the matrix of intermediate consumption of import ($\mathbf{X}^{R,imp}$) we approximate the structure of $\mathbf{F}^{R,P}$:

$$\sum_{R=1}^{m} \mathbf{F}^{R,P} = \mathbf{X}^{P,imp}. \tag{11}$$

Moreover we gain information about $\mathbf{F}^{R,P}$:

$$\begin{array}{ccc} \begin{bmatrix} F^{R,P}_{1,1} & \cdots & F^{R,P}_{1,j} & \cdots & F^{R,P}_{1,n} \\ \vdots & \ddots & & & \vdots \\ F^{R,P}_{i,1} & & F^{R,P}_{i,j} & & F^{R,P}_{i,n} \\ \vdots & & & \ddots & \vdots \\ F^{R,P}_{n,1} & \cdots & F^{R,P}_{n,j} & \cdots & F^{R,P}_{n,n} \end{bmatrix} & \sum = & \begin{matrix} e^{R,P}_{1} \\ \vdots \\ e^{R,P}_{i} \\ \vdots \\ e^{R,P}_{n} \end{matrix} \\ \sum = & & \\ \begin{matrix} i^{R,P}_{1} & \cdots & i^{R,P}_{j} & \cdots & i^{R,P}_{n} \end{matrix} & & \end{array} \tag{11}$$

As we know sums of both rows and columns of $\mathbf{F}^{R,P}$, we can use the RAS method (Sargento et al., 2012) for the approximation of the structure of matrix $\mathbf{F}^{R,P}$ with the condition that the structure of $\mathbf{F}^{R,P}$ and $\mathbf{X}^{R,imp}$ are similar. This concept ensures the consistency between regional and national Leontief's coefficients.

Such approach ensures the consistency between regional (IRIO) and national input-output model where we disregard regions. Thus, multiregional Leontief's coefficients a represent weighted decomposition of national Leontief's multipliers. Using the IRIO approach:

$$\sum (\mathrm{I}^{T} - \mathrm{A}^{T})^{-1} \mathrm{y}^{T} = (\mathrm{I} - \mathrm{A})^{-1}, \tag{12}$$

because in general regional import and export do not exist:

$$\sum (\mathrm{I}^{R} - \mathrm{A}^{R})^{-1} \mathrm{y}^{R} = (\mathrm{I}^{T} - \mathrm{A}^{T})^{-1} \mathrm{y}^{T}. \tag{13}$$

This is the reason why the effects calculated by means of the SRIO approach (the left side of the formula 13) do not correspond with the effect calculated by means of IRIO (the right side of the formula 13). The sum of IRIO equals the national matrix (formula 12). In other words, the sum of effects through individual regions in all products in IRIO has to correspond with the national effects calculated at the national level. Due to the regional export and import this does not apply in SRIO.

3 DATA

For the analysis, we use regional input-output tables (RIOTs) constructed for the reference year 2011 by the Department of Economic Statistics from the University of Economics, Prague (Sixta and Vltavská, 2016; Sixta et al., 2014; Department of Economic Statistics, 2016). RIOTs describe the structure of output in individual regions (NUTS 3 level) corresponding to national input-output tables (IOTs) published by the Czech Statistical Office. Moreover, each region has its own IOT of imported goods. We need both these tables for the analysis and information of regional flows of import and export. However, this data source is not available. RIOTs provide us only with the total amount of import and export for individual industries without any information which region represents the resource side and which region features as the recipient. However, for the construction of IRIO we need more detailed information about the trade, such as which region imports and exports to another region. Therefore, we calculated this information using Karush-Kuhn-Tucker theorem (Šafr, 2016). We proportionally adjusted export into FOB[4] prices for the flows between regions. This ensures the consistency of intermediate consumption matrix.

Table 1 Regional import and export, share on regional output, mil CZK, %

Region	Import	%	Export	%
Jhc	49 914	7.68	43 754	6.73
Jhm	64 432	5.13	49 979	3.98
Kar	44 841	17.01	18 903	7.17
Krh	44 318	7.46	22 010	3.7
Lib	42 311	9.74	15 116	3.48
Mrs	107 683	7.26	63 420	4.28
Olm	47 814	8.48	29 996	5.32
Par	54 351	7.93	37 592	5.48
Pha	222 911	7.37	507 833	16.78
Plz	45 949	7.19	30 567	4.78
Stc	166 479	9.46	89 103	5.06
Ust	79 864	8.17	72 787	7.44
Vys	50 770	9.26	41 331	7.54
Zln	58 437	9.06	57 683	8.94
CZE	1 080 074	7.99	1 080 074	7.99

Note: CZE – the Czech Republic, Pha –Prague, Stc – Central Bohemia Region, Jhc – South Bohemia Region, Plz – the Plzen Region, Kar – the Karlovy Vary Region, Ust – the Usti Region, Lib – the Liberec Region, Krh – the Hradec Kralove Region, Par – the Pardubice Region, Vys – the Vysocina Region, Jhm – the South Moravian Region, Olm – the Olomouc Region, Zln – the Zlin Region, Mrs – the Moravian-Silesian Region.
Source: Authors' calculation

Table 1 shows that the highest absolute value of import and export reaches Prague. However, the results differ if one uses the relative share on the region's output. From such perspective, Prague comprises the most important exporter (16.78%) and an average importer (7.37%). On the contrary, Karlovy Vary Region records the most important relative import with 17.01%.

If we provisionally assume that the regional structure of output and intermediate consumption keep the same level in all regions, we conclude that the Hradec Králové Region is the most undervalued and

[4] FOB prices – Free On Board pricing.

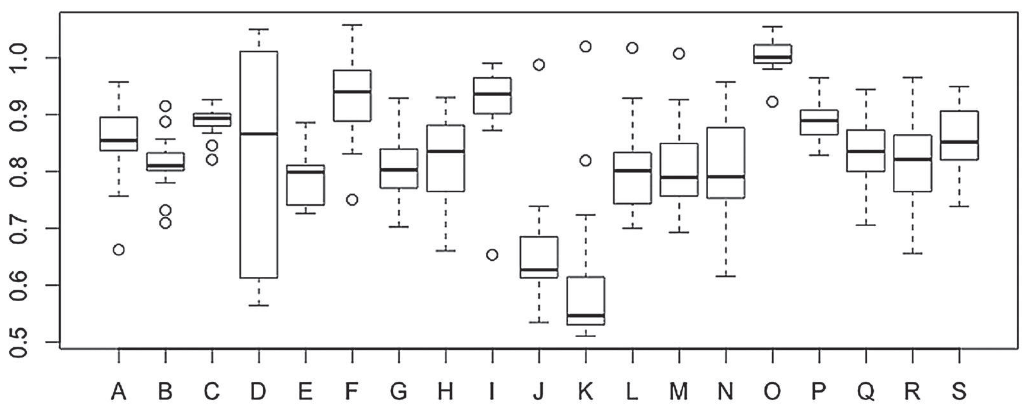
the South Moravian Region the least undervalued area. The highest share of multiplication flows to Prague. However, these results depend on the precise structure of RIOTs which differ from region to region to a certain extent and on the structure of suppliers and consumers of regional output. Moreover, it depends on the right links among individual regions.

4 RESULTS

Leontief's multipliers represent one of the most common tools of IOA. Using IRIO the part of multiplication comprises transfer to import among regions with no influence by other multiplication and it is considered a final quantity. On the other hand, the interregional approach assumes export and import among regions as endogenous variables. This causes an increase of the share of import on output. It further influences export, which constitutes a part of final use. The increase of final use leads to another multiplication. When using IRIO, the export and import among regions ensure the consistency of impacts calculated at regional level with the national level.

For an illustration of bias between SRIO and IRIO, we used the share of SRIO multipliers on IRIO multipliers (Figure 2). Products K (Financial and insurance activities), J (Information and Communication), E (Water supply; sewerage, waste management and remediation activities), A (Agriculture, forestry and fishing) and B (Mining and quarrying) show the most significant differences. On average across all products, SRIO multipliers are undervalued by 14% compared to IRIO. The offer of product K mostly concentrates in Prague, which also causes the underestimation of the product. This induced the fact that product K has notably a role of regional import. Similar situation applies for product J.

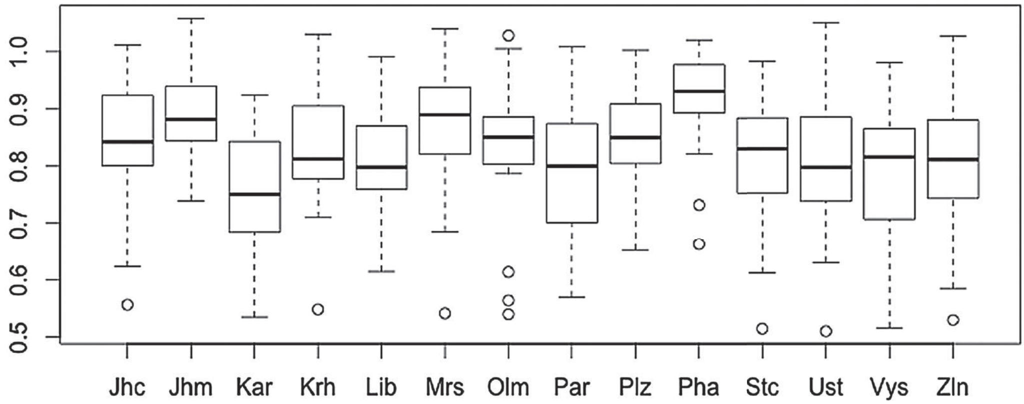
Figure 2 The share of SRIO and IRIO multipliers on products, %



Note: A – Agriculture, forestry and fishing, B – Mining and quarrying, C – Manufacturing, D – Electricity, gas, steam and air conditioning supply, E – Water supply; sewerage, waste management and remediation activities, F – Construction, Services: G – Wholesale and retail trade; repair of motor vehicles and motorcycles, H – Transportation and storage, I – Accommodation and food service activities, J – Information and communication, K – Financial and insurance activities, L – Real estate activities, M – Professional, scientific and technical activities, N – Administrative and support service activities, O – Public administration and defence; compulsory social security, P – Education, Q – Human health and social work activities, R - Arts, entertainment and recreation, S – Other service activities.
Source: Authors' calculation

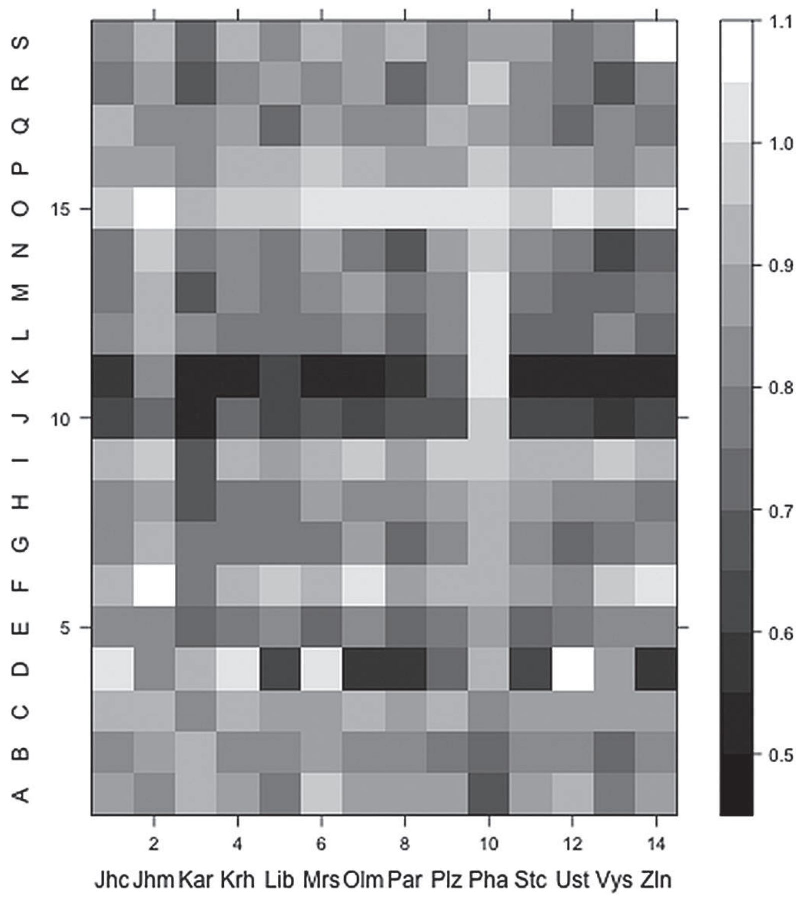
This analysis finds use not only for products but for regions as well. Figure 3 shows that the least undervalued region is Prague, due to a share of export and low share of import on output. This allows us to expect that a lot of output of other regions is multiplied in Prague. This represents an opposite situation than for example in the South Moravian Region. This region demonstrates a low share of import on the output but a low share of export as well.

Figure 3 The share of SRIO and IRIO multipliers in regions, %



Source: Authors' calculation

Figure 4 Level-map of fraction of SRIO and IRIO multipliers at matrix products by regions

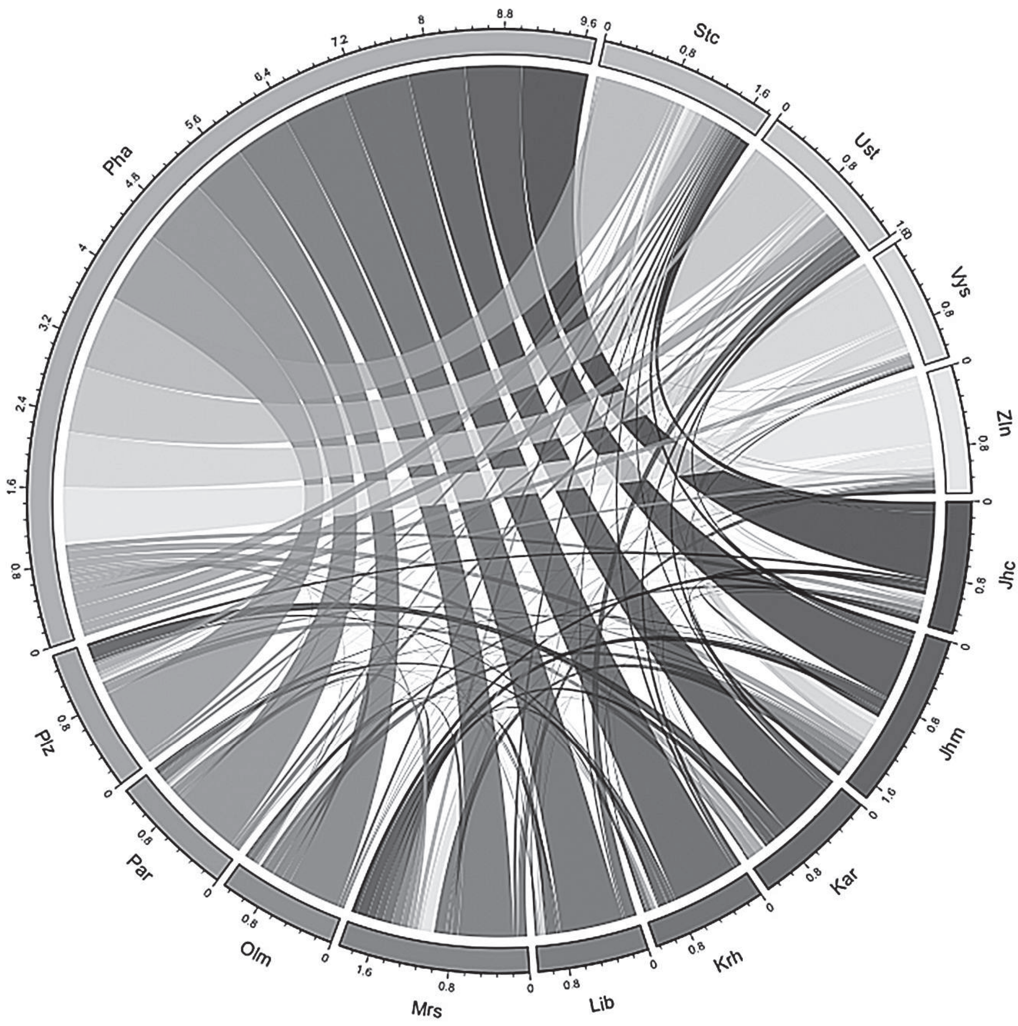


Source: Authors' calculation

We use a level map for an illustration of simultaneous analysis (Figure 4). One can see that mainly products J and K show systematic undervaluing of multipliers. On the other hand, product O (Public administration and defence) has the least undervalued multiplier. Therefore, it distinctly corresponds with the multiregional approach. This figure confirms characteristics of Prague and the South Moravian Region. Their SRIO multipliers correspond more to IRIO than multipliers in other regions.

All these characteristics prove the unique status of Prague among all regions. Figure 5 confirms the idea that if export reaches high and import stays as low as the share on the regional output, the output of other regions multiplies in Prague.

Figure 5 Chord diagram/network circle of multiplication out of regions



Source: Authors' calculation

Figure 5 illustrates multiplication outside regions. The boldness of the link describes the flow from the given region to other regions. If the colour of the flow is different from the one used for the sector, it designates the multiplication into the region. This figure effectively illustrates to which region the output is multiplied due to an increase of uses in the region. The whole diagram corroborates the idea that a significant value of output from other regions multiplies in Prague.

Table 2 describes the strength of IRIO multipliers, i.e. how much of the average multiplier flows outside the region on average.

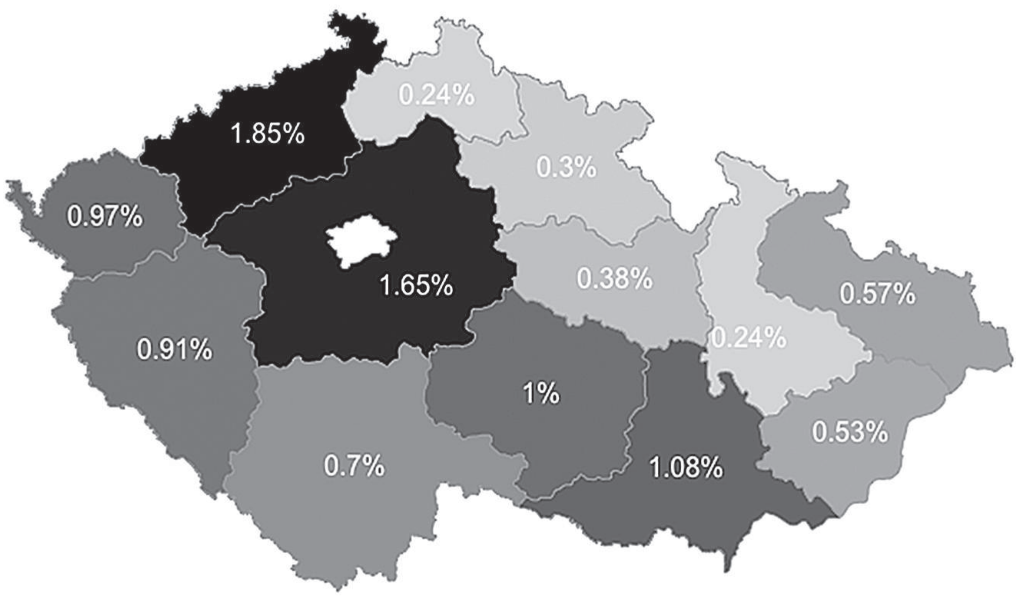
Table 2 Summary statistics of IRIO multipliers, %

Region	Average IRIO multipliers	Part of m. outside region	%	Region	Average IRIO multipliers	Part of m. outside region	%
Jhc	1.61	0.18	0.11	**Par**	1.62	0.20	0.13
Jhm	1.59	0.10	0.06	**Plz**	1.57	0.14	0.09
Kar	1.62	0.25	0.15	**Pha**	1.67	0.10	0.06
Krh	1.60	0.18	0.11	**Stc**	1.62	0.18	0.11
Lib	1.61	0.20	0.12	**Ust**	1.64	0.21	0.13
Mrs	1.57	0.15	0.09	**Vys**	1.66	0.24	0.14
Olm	1.57	0.17	0.11	**Zln**	1.62	0.20	0.12

Source: Authors' calculation

Figures 2 to 5 and Tables 1 and 2 confirm the important status of Prague among the regions. Aiming directly at Prague allows us to find the source regions with the highest average of multiplied output (Figure 6) with the total multiplication in Prague serving as a baseline.

Figure 6 Source regions of the highest multipliers of output in Prague (% part of Prague multiplier)

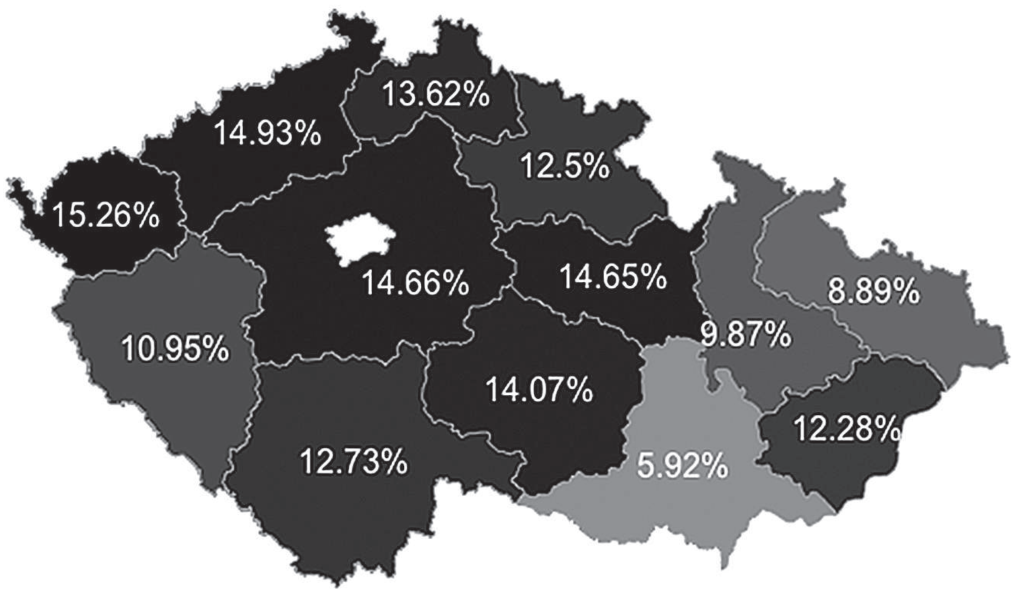


Source: Authors' calculation

Figure 6 proves the strongest connection of Prague and regions geographically close to Prague. The output multiplied in Prague (throughout all categories of products) comes mainly from the Usti Region (1.85%) and the Central Bohemia Region (1.65%).

Figure 7 depicts target regions to which the output of Prague is mostly multiplied. We can even see the decomposition of average multiplication of Prague's output outside the region. A weak connection of the South Moravian Region reflects a generally weak links of this region to the other regions. Table 1 demonstrates it rather clearly.

Figure 7 Regions according to targeting of the multiplied output from Prague (% part of their multipliers)

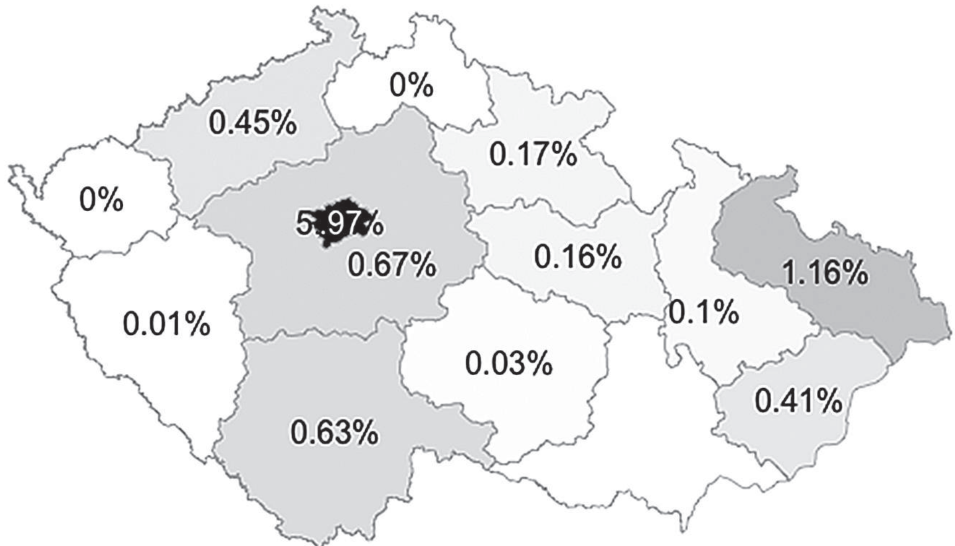


Source: Authors' calculation

The Karlovy Vary Region, the Usti Region and the Central Bohemia Region record the highest multiplication from Prague. Moreover, Figure 7 depicts the influence of the distance of the average connection on the multiplication of the output from Prague. Regions geographically closer to Prague have a stronger link it than a distant region. The costs on import may be the principal cause here. It goes along with economic assumptions about consumers' and producers' behaviours. Minimized costs on import lead to minimizing the import distance which results in strong multiplication mostly to the surrounding regions.

The second example focuses on the region with trends opposite of Prague, i.e. the South Moravian region with the weakest connection to Prague (see Figures 6 and 7) among Czech regions. Figures 8 and 9 summarize the connection of the South Moravian region to all other regions.

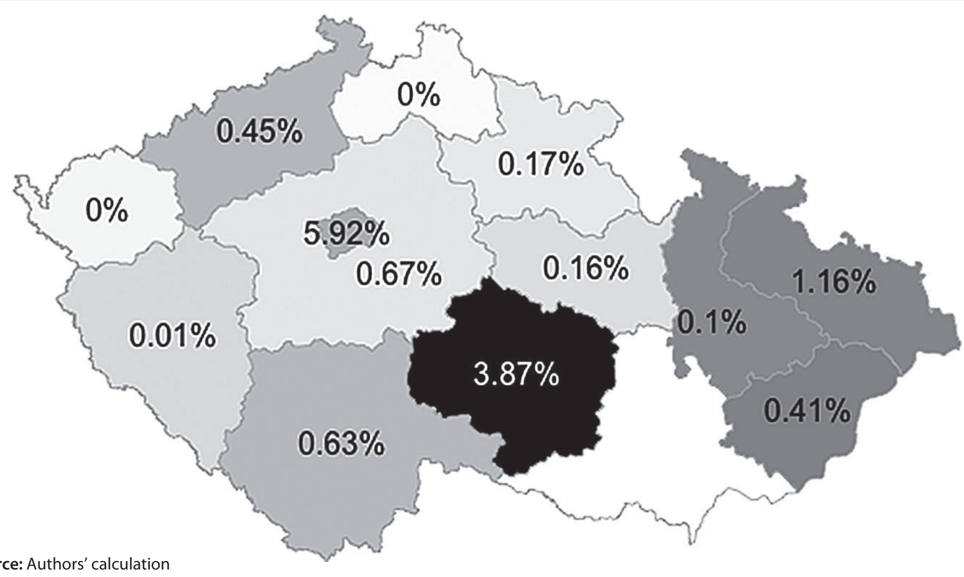
Figure 8 Source regions of the highest multipliers of output to the South Moravian region (% part of the South Moravian region multiplier)



Source: Authors' calculation

Multiplication of the output of the South Moravian region is mainly allocated in Prague, which correspond with the rest of the regions. Prague maintains the strongest position among all regions. Figure 9 shows the ratio of multiplication for the South Moravian region from others regions. This figure proves the minimization of costs on the imported products. Thus, the highest share of the multiplication transfers to the neighbouring regions.

Figure 9 Target regions for the multiplied output of the South Moravian region (% part of their multipliers)



Source: Authors' calculation

DISCUSSION AND CONCLUSION

This paper aimed to illustrate the differences between SRIO and IRIO approaches in IOA. These approaches differ in their construction as well as in the required data. SRIO analyses only one region with no relations to other regions. Thus, this approach proves less data demanding. IRIO approach analysed each region in context of all regions. The different construction and data sources give rise to the following two hypotheses. Firstly, regions analysed by means of SRIO are systematically undervalued. Secondly, Prague has a unique position among the regions.

We employed Leontief matrix for an analytical illustration of the differences between these two approaches. Using the matrix, we illustrate a significant undervalue of single-regional Leontief multipliers' in comparison to IRIO. This undervalue characterizes both average Leontief multipliers for whole regions and partial multipliers for individual products (Figures 2 and 3). Detailed estimations on the level of individual products show wide variability. On the other hand, one can see identical undervalue calculated in individual regions.

Figure 4 proves these expectations for aggregated Leontiefs' products in individual regions. A similar structure of the undervaluation applies to virtually all regions. While the undervaluation which we evaluate at the level of individual regions (in all regions) varies a lot, mainly in the structure of products. These results prove the highest undervaluation of products J and K. Figures 2 to 4 confirm the idea about the undervalue which we expected when using SRIO. The undervaluation is clearly systematic (mainly seen in Figure 4). The undervaluation is traceable in narrow quantiles of individual products using boxplots (Figures 2 and 3).

Figure 5 confirms the hypothesis about a specific position of Prague to which all regions maintain a strong link. Thus, all regions are unequally strongly connected to Prague opposed to the other regions. Figures 6 to 9 illustrate this phenomenon for Prague and South Moravian region. The most significant undervaluation takes place in regions and products which are import demanding on their own output.

The main goal of the paper was achieved mainly by using graphical analyses of Leontiefs matrix. Graphical analysis was necessary due to the impossibility of testing of the data by means of common statistical methods (e.g. t-tests). This analysis was supplemented with an estimation of the undervaluation in individual regions and in the Czech Republic as a whole. Two minor hypotheses (the systematicness of the undervaluation and a unique position of Prague) were illustrated using graphical analysis as well. The results and illustrations confirm both these minor hypotheses. The undervaluation gives ground for questioning of the results by means of SRIO.

The data and the model demonstrate regional heterogeneity of Leontiefs coefficients calculated by means of SRIO and IRIO, as we expected. The fact that various researchers use different data confirms this heterogeneity (e.g. Freeman, Alperovich and Weksler; 1985). The results (undervaluation, systematicness etc.) follow economic and statistical theory behind these models (Lezen et al., 2004).

Recommendation for future work lies in the idea that disregarding the measurement of the connection of a region to other regions produces systematically undervalued results. This bias is noticeable in all regions. Using a more sophisticated regional model, such as MRIO (or used IRIO), although more data demanding, can eliminate such bias. Therefore, a further research lies mainly in more precise and robust methods of the extrapolation of the data sources needed, for example methods for the estimation of the flows of output among regions, or ensuring the consistency of regional estimations with national figures (e.g. 3D RAS etc.).

ACKNOWLEDGEMENT

This paper is prepared under the support of the "Regional estimates of gross domestic product based on the expenditure approach" project of the Czech Science Foundation, project No. 13-15771S and by Institutional Support for Long Period and Conceptual Development of Research and Science at Faculty of Informatics and Statistics, University of Economics, Prague.

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