Testing for a Unit Root in the Logistic STAR Framework with a Fourier Function: an Application to the Unemployment Rates

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Abstract

This paper introduces the Fourier-LSTAR (FLSTAR) test, which addresses the critical challenge of testing for unit roots in time series characterized by both unknown structural breaks and nonlinear dynamics. We propose and evaluate this novel unit root test, which integrates a logistic smooth transition autoregressive (LSTAR) model with a flexible Fourier function to capture such complexities. Critical values and simulation properties of the test are derived, demonstrating its robustness and stable performance across varying conditions. We apply the FLSTAR test to annual unemployment rates for CIVETS countries and find that unemployment hysteresis holds for most nations, except Colombia, where the plucking model is applicable. These results highlight the heterogeneous nature of unemployment dynamics in emerging economies and underscore the importance of employing robust testing procedures that accommodate data complexities to avoid misleading policy inferences. The FLSTAR test demonstrates superior power and size properties in Monte Carlo simulations, offering a valuable new tool for empirical researchers.

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Unemployment hysteresis, Unit Root Test, Fourier-LSTAR, structural breaks, nonlinear dynamics, CIVETS countries	*https://doi.org/10.54694/stat.2025.25*	*C22*

INTRODUCTION

The concept of stationarity is fundamental in time series analysis, as non-stationarity can lead to spurious inferences. Although traditional unit root tests, such as the Dickey-Fuller and Augmented Dickey-Fuller

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(ADF) tests, provided the foundational methodology, they often prove inadequate when applied to real-world data characterized by complexities such as structural breaks and nonlinear dynamics (Enders, 2014; Kočenda and Černý, 2015). Neglecting these features can compromise the reliability of stationarity testing.

Recognizing the limitations of assuming linearity and stability, research has focused on developing more robust unit root tests. To address structural changes, particularly multiple and smooth breaks of unknown timing and form, Fourier functions have proven effective (e.g., Becker et al., 2004, 2006). Concurrently, the prevalence of nonlinear behavior in economic time series (Tong, 2015; Enders and Granger, 1988) spurred the development of tests based on regime-switching models. Among these, Smooth Transition Autoregressive (STAR) models, particularly their Logistic (LSTAR) and Exponential (ESTAR) variants, allow for gradual transitions between different economic states (Teräsvirta and Anderson, 1992). LSTAR models are especially adept at capturing asymmetric dynamics, such as differing behaviors during economic expansions and contractions (Skalin and Teräsvirta, 1999).

While existing approaches have combined Fourier functions with ESTAR models to jointly account for structural breaks and symmetric nonlinearity (e.g., Christopoulos and Leon-Ledesma, 2010, 2011; Guris, 2019; Ranjbar et al., 2018), a notable gap remains in the development of tests capable of accommodating both structural breaks and LSTAR-type asymmetric nonlinearity.

This study addresses this gap by proposing a novel two-stage unit root test, termed the Fourier-LSTAR (FLSTAR) test. Following the methodology of Christopoulos and Leon-Ledesma (2010, 2011), the first stage involves removing deterministic components (constant/constant trend) and accounting for structural changes in the series using a Fourier approximation. In the second stage, the stationarity of these filtered residuals is examined using the LSTAR-based unit root test developed by Pascalau (2007). The FLSTAR test proposed herein uniquely combines the strengths of Fourier approximations for capturing smooth breaks with the LSTAR framework's capacity to model nonlinear mean reversion. This explicit integration represents a significant advancement over existing approaches, which may exhibit low power or size distortions when applied to the complex data generating processes increasingly encountered in time series analysis.

The remainder of this paper is structured as follows: next section details the theoretical framework of the STAR model, the derivation of the LSTAR-based unit root test, and the integration of Fourier functions to account for structural breaks; Section 2 presents the simulation studies, including the generation of critical values for the FLSTAR test and an assessment of its size and power properties under various conditions; Section 3 presents the empirical application of the FLSTAR test to unemployment rates in the CIVETS economies, that is, in Colombia, Indonesia, Vietnam, Egypt, Turkey, and South Africa, discussing the findings regarding unemployment hysteresis; Finally, last section concludes the paper, summarizing the main contributions, discussing policy implications, and suggesting avenues for future research.

1 MODEL

A STAR model is a regime-switching model that allows a time series to transition smoothly between two different autoregressive (AR) structures. A STAR model of order p can be represented as follows:

$$Y_t = \alpha_{10} + \alpha'_1 z_t + \left(\alpha_{20} + \alpha'_2 z_t\right) F\left(Y_{t-d}\right) + e_t, \qquad \text{for } t = 1,\ldots,\mathrm{T}, \tag{1}$$

here, Y_t represents the time series variable of interest, and e_t is a white noise error term, and $\alpha_j = \left(\alpha_{j1},\ldots,\alpha_{j1},\right)'$ and $z_t = \left(Y_{t-1},\ldots,Y_{t-p}\right)'$ denote the AR coefficients for the two distinct regimes (Teräsvirta and Anderson, 1992). The transition between these regimes is governed by the transition function, F(.), which is continuous, and bounded between 0 and 1 as:

– If $F(.) = 0$, the model behaves according to the first AR structure (defined by $\alpha_{10} + \alpha'_1 z_t$).
– If $F(.) = 1$, the model operates under the second AR structure (defined by $\alpha_{10} + \alpha_{20} + (\alpha'_1 + \alpha'_2) z_t$).
– If $0 < F(.) < 1$, the model's behavior is a weighted average of the two structures.

The logistic function that can be substituted for $F(.)$ is defined as:

$$F(Y_{t-d}) = \left(1 + \exp\left[-\gamma\left(Y_{t-d} - c\right)\right]\right)^{-1}. \tag{2}$$

In this function, d is the delay parameter, c is the threshold parameter, and γ is the slope parameter (with $\gamma > 0$), which regulates the speed of transition between regimes. A large γ creates a rapid, almost instantaneous switch (approaching a Threshold Autoregressive Model), while a small γ results in a very slow, gradual transition. When γ approaches zero, the LSTAR model simplifies to a standard linear AR model (Teräsvirta and Anderson, 1992). Substituting Formula (2) into Formula (1) yields the formal LSTAR model:

$$Y_t = \alpha'_1 z_t + \left(\alpha'_2 z_t\right)\left(1 + \exp\left[-\gamma\left(Y_{t-d} - c\right)\right]\right)^{-1} + e_t\,. \tag{3}$$

Following the suggestions of Kapetanios et al. (2003), Teräsvirta (2006), and Pascalau (2007) imposed the constraint $d = 1$ and rearranged the parameters of Formula (3) to obtain:

$$\Delta Y_t = \delta z_t + \alpha'_2 z_t \left(1 + \exp\left[-\gamma\left(Y_{t-1} - c\right)\right]\right)^{-1} + e_t, \tag{4}$$

here, $\delta = 1 - \alpha_1$ through Formula (4), it is possible to represent the unit root null hypothesis as $H_0 : \delta = \alpha_2 = \gamma = 0$ against the alternative hypothesis $H_1 : \delta + \alpha_2 < 0,\ \gamma > 0$. However, the parameters γ and c are not identified under the null hypothesis, precluding a direct test. To address this, Pascalau (2007), drawing on Balke and Fomby (1997), and Kapetanios et al. (2003), assumed $\delta = 0$. This assumption implies stationarity in the regime where the value of Y_{t-1} is small (close to zero) and non-stationarity in the remaining part of the sample. With this assumption, model (4) was redefined as:

$$\Delta Y_t = \alpha'_2 z_t \left(1 + \exp\left[-\gamma\left(Y_{t-1} - c\right)\right]\right)^{-1} + e_t. \tag{5}$$

As unidentified parameters persist under the null hypothesis in Formula (5), Pascalau (2007) imposed the constraint $c = 0$ on Formula (5), applied a third-order Taylor expansion and obtained the following testable equation:

$$\Delta Y_t = \lambda_1 Y_{t-1}^2 + \lambda_2 Y_{t-1}^4 + e_t, \tag{6}$$

here, the unit root null hypothesis $H_0 : \lambda_1 = \lambda_2 = 0$, is tested against the LSTAR-type stationarity alternative hypothesis. For series with a non-zero mean or a deterministic trend, the data are first de-meaned or de-trended before applying this test.

The Pascalau (2007) test may yield misleading results if the time series contains structural breaks. To address this, we adopt the two-stage approach of Christopoulos and Leon-Ledesma (2010), combining the LSTAR test with a Fourier function that can approximate smooth and gradual structural changes of unknown form. In the first stage, we remove the deterministic components (constant/constant and trend) and account for structural breaks by estimating one of the following regressions:

$$Y_t = \phi_0 + \phi_2 \sin\left(2\pi kt / T\right) + \phi_3 \cos\left(2\pi kt / T\right) + u_t\,, \tag{7}$$

$$Y_t = \phi_0 + \phi_1 t + \phi_2 \sin\left(2\pi kt / T\right) + \phi_3 \cos\left(2\pi kt / T\right) + u_t\,. \tag{8}$$

In these models, sin(2πkt/T) and cos(2πkt/T) are trigonometric terms, π is the mathematical constant pi, k is the endogenously determined frequency value of the Fourier function, t is the trend term, and T is the number of observations. The value of k is not known as a priori. The selection of the optimal frequency, k, for the Fourier function is a critical step in the FLSTAR testing procedure. Following Enders and Lee (2012), we employ a grid search approach for k, typically ranging from 1 to 5, as higher frequencies can capture overly erratic movements. For each integer value of k within this range, Formula (7) or (8) is estimated. The optimal k (denoted k*) is then selected as the value that minimizes the sum of squared residuals (SSR). After determining the optimal frequency value (k*), the relevant model is estimated by substituting k*, and the residuals of this model are calculated as:

$$\hat{u}_t = Y_t - \left[\hat{\phi}_0 + \hat{\phi}_2 \sin\left(2\pi k^* t / T\right) + \hat{\phi}_3 \cos\left(2\pi k^* t / T\right)\right], \tag{9}$$

$$\hat{u}_t = Y_t - \left[\hat{\phi}_0 + \hat{\phi}_1 t + \hat{\phi}_2 \sin\left(2\pi k^* t / T\right) + \hat{\phi}_3 \cos\left(2\pi k^* t / T\right)\right]. \tag{10}$$

In these models, $\hat{u}_t$ represents the series Y_t after demeaning and/or detrending and adjustment for structural changes. In the second stage, we test the stationarity of the filtered residuals, using the LSTAR auxiliary regression from Formula (5):

$$\Delta\hat{u}_t = \lambda_1 \hat{u}_{t-1}^2 + \lambda_2 \hat{u}_{t-1}^4 + e_t. \tag{11}$$

The unit root null hypothesis is again tested with an F-test on $H_0 : \lambda_1 = \lambda_2 = 0$. This F-statistic, which we term the FLSTAR test statistic, follows a non-standard distribution for which we simulate critical values. To handle potential serial correlation, Formula (11) can be augmented with lagged values of the dependent variable ($\Delta\hat{u}_{t-i}$). If the unit root null is rejected, it implies the series is stationary around a deterministic component that exhibits smooth structural changes, with an asymmetric, LSTAR-type adjustment process. As a final step, one can test for the significance of the structural breaks in the case of stationarity, by applying a standard F-test to the null hypothesis that all Fourier coefficients are jointly zero. Critical values for this test are provided by Becker et al. (2006).

The parameters for all models within the two-stage FLSTAR testing framework are estimated using Ordinary Least Squares (OLS). This choice is both computationally straightforward and statistically appropriate for the task at hand. While alternative estimation techniques like the Generalized Method of Moments (GMM) exist, OLS is preferred here for its simplicity and efficiency. GMM is more powerful in contexts with endogenous regressors or when the model is defined by moment conditions that OLS cannot handle. However, in this framework, the regressors in the auxiliary equation are predetermined, and the standard assumptions for OLS are met under the null hypothesis. In this context, OLS is equivalent to the Method of Moments and is the most direct and efficient method for estimating the parameters and constructing the necessary F-statistics.

2 SIMULATIONS

2.1 Critical values

Critical values for the proposed unit root test are derived by applying an F-test to the null hypothesis $H_0 : \lambda_1 = \lambda_2 = 0$ within the framework of Formula (11). This was performed for sample sizes (T) of 50, 100, and 250, and for T = 1 000 to obtain asymptotic critical values, with 50 000 simulations conducted for each scenario. The resulting critical values are presented in Table 1. These critical values were calculated for Fourier frequencies (k) of 1, 2, 3, 4, and 5.

Table 1 Critical values

Sample size	Frequency	Demeaned			Detrended		
		1%	5%	10%	1%	5%	10%
T = 50	1	7.6106	4.6831	3.5326	6.2567	3.7626	2.7285
	2	5.4050	3.2489	2.4240	5.6405	3.4990	2.6102
	3	5.1770	3.3523	2.6354	5.4502	3.3081	2.4938
	4	5.2799	3.4915	2.7132	5.3878	3.2558	2.4223
	5	5.2050	3.4517	2.6894	5.2446	3.2039	2.3736
T = 100	1	7.3603	4.4841	3.3482	4.8159	2.9274	2.2025
	2	4.8931	3.0120	2.2789	4.7591	2.9751	2.2754
	3	4.9500	3.3577	2.6658	4.6047	2.9373	2.2736
	4	5.0904	3.4586	2.7190	4.4998	2.8885	2.2239
	5	5.1053	3.4601	2.7079	4.5117	2.8678	2.2177
T = 250	1	7.1524	4.4650	3.3316	3.9219	2.5155	1.9057
	2	4.8832	2.9303	2.1815	4.2477	2.7391	2.1073
	3	4.9104	3.3582	2.6635	4.2458	2.7586	2.1515
	4	5.1057	3.4758	2.7147	4.1749	2.7041	2.0890
	5	5.0572	3.4296	2.7014	4.1196	2.7178	2.0912
T = 1 000	1	7.4843	4.4930	3.3351	3.5554	2.2581	1.7120
	2	4.8800	2.8837	2.1198	4.0122	2.5739	1.9765
	3	4.8594	3.3341	2.6651	3.9854	2.5834	2.0340
	4	5.1302	3.4802	2.7397	3.9789	2.6318	2.0289
	5	5.0647	3.4680	2.7340	3.9701	2.5823	2.0094

Source: Author's own elaboration

2.2 Power and size properties

In this section, we evaluate the size and power properties of the test. The size of the FLSTAR test measures how often the test will incorrectly conclude that a time series is stationary when, in fact, it truly has a unit root (is non-stationary). The power of the FLSTAR test measures how likely the test is to conclude that a time series is stationary (by rejecting the unit root null hypothesis) when the series truly is stationary.

To evaluate the size properties of the test statistic, the following data generating process is considered:

$$Y_t = \beta_1 + \beta_2 \sin\left(\frac{2\pi kt}{T}\right) + \beta_3 \cos\left(\frac{2\pi kt}{T}\right) + u_t , \tag{12}$$

$$u_t = u_{t-1} + \varepsilon_t, \tag{13}$$

here, ε_t denotes standard normally distributed residuals. To assess the size properties, frequency values (k) of 1, 2, and 3, sample sizes (T) of 100 and 250 are considered. Additionally, the coefficients of the trigonometric terms $\beta_2 = \beta_3$ are set to 1, 0.5, and 0.1. Values of β_2 and β_3 close to 0 imply that the series approaches linearity. The simulation results are presented in Table 2.

Table 2 Size properties of the test

Data generation process	T = 100		
	k **= 1**	***k*** **= 2**	***k*** **= 30**
$\beta_2 = \beta_3 = 1$	0.042	0.046	0.045
$\beta_2 = \beta_3 = 0.5$	0.045	0.043	0.045
$\beta_2 = \beta_3 = 0.1$	0.047	0.043	0.045
Data generation process	**T = 250**		
	k **= 1**	***k*** **= 2**	***k*** **= 30**
$\beta_2 = \beta_3 = 1$	0.042	0.046	0.045
$\beta_2 = \beta_3 = 0.5$	0.042	0.046	0.045
$\beta_2 = \beta_3 = 0.1$	0.042	0.046	0.045

Note: Computed using 10 000 simulations at the 5% significance level.
Source: Author's own elaboration

As shown in Table 2, the proposed test does not exhibit significant size distortions, with empirical sizes remaining close to the nominal 5% significance level across all scenarios. A more detailed examination reveals that the empirical sizes are consistently slightly below the 5% nominal level, typically ranging between 4.2% and 4.7%. This indicates that the FLSTAR test is slightly conservative. The stability of the size properties, even with the inclusion of Fourier terms of varying magnitudes, confirms the robustness and reliability of the FLSTAR testing procedure. The magnitude of the trigonometric term coefficients appears to have a negligible impact on the test's size, particularly as the number of observations increases.

To determine the power properties of the test, an LSTAR model augmented with a Fourier function is considered:

$$Y_t = \beta_1 + \beta_2 \sin\left(\frac{2\pi kt}{T}\right) + \beta_3 \cos\left(\frac{2\pi kt}{T}\right) + v_t, \tag{14}$$

$$\Delta v_t = \beta v_{t-1}\left[1 + \exp\left(-\gamma v_{t-1} - c\right)\right]^{-1} + \epsilon_t. \tag{15}$$

The power properties are examined using various parameter configurations: smoothing parameter $\gamma = 0.05, 0.1, 1$; location parameter $c = \{-10, -5, 0.5, 10\}$; Frequency value $k = 1$ and 2; $\beta = \{-1.9, -1.0, -0.2\}$. These evaluations were conducted for sample sizes of 100 and 250 observations, with 10 000 simulations performed for each experimental setting.

Table 3 Power properties of the test

Data generation process			T = 100		T = 250	
β	c	γ	$k = 1$	$k = 2$	$k = 1$	$k = 2$
		0.05	0.1435	0.2150	0.1742	0.2549
	–10	0.1	0.1451	0.2153	0.1725	0.2586
		1	0.0206	0.0533	0.0206	0.0665
		0.05	0.136	0.2048	0.1638	0.2440
	–5	0.1	0.1448	0.2146	0.1750	0.2582
		1	0.0516	0.1077	0.0791	0.1597
–1.9		0.05	0.1204	0.2048	0.1449	0.2288
	0.5	0.1	0.1260	0.2146	0.1560	0.2372
		1	0.2267	0.1077	0.2595	0.3235
		0.05	0.0910	0.1512	0.1068	0.1915
	10	0.1	0.0661	0.1186	0.0756	0.1561
		1	0.0490	0.0547	0.0489	0.0655
		0.05	0.0770	0.1317	0.0862	0.1692
	–10	0.1	0.0910	0.1515	0.1065	0.1913
		1	0.1268	0.2000	0.1466	0.2321
		0.05	0.0777	0.1207	0.0727	0.1571
	–5	0.1	0.0694	0.1323	0.0876	0.1726
		1	0.1140	0.1859	0.1322	0.2171
–1		0.05	0.0596	0.1075	0.0608	0.1402
	0.5	0.1	0.0603	0.1077	0.0649	0.1413
		1	0.0843	0.1211	0.0890	0.1590
		0.05	0.0843	0.0870	0.0422	0.1083
	10	0.1	0.0307	0.0675	0.0289	0.0847
		1	0.0414	0.0461	0.0421	0.0591
		0.05	0.0153	0.0379	0.0133	0.0479
	–10	0.1	0.0163	0.0417	0.0139	0.0518
		1	0.0221	0.0521	0.0203	0.0661
		0.05	0.0144	0.0345	0.0122	0.0444
	–5	0.1	0.0151	0.0374	0.0127	0.0469
		1	0.0188	0.0446	0.0154	0.0515
–0.2		0.05	0.0129	0.0328	0.0107	0.0427
	0.5	0.1	0.0129	0.0324	0.0096	0.0419
		1	0.0310	0.0385	0.0354	0.0530
		0.05	0.0108	0.0281	0.0086	0.0375
	10	0.1	0.0090	0.0260	0.0065	0.0326
		1	0.0090	0.0298	0.0294	0.0407

Source: Author's own elaboration

Examination of the results in Table 3 reveals a significant increase in the test's power with higher Fourier frequency values (k) and larger sample sizes (T). Conversely, the power of the test diminishes as the absolute value of the location parameter (c) increases. Furthermore, holding other parameters constant, the test's power decreases as the value of the autoregressive parameter β (representing the speed of mean reversion under the alternative) moves closer to zero (i.e., becomes less negative). A notable observation from Table 3 states that the overall power of the FLSTAR test can be quite low, particularly in smaller samples (T = 100) and for certain parameter configurations. This finding, while a limitation, is not unexpected and reflects a well-known trade-off in time series econometrics. Unit root tests, in general, are known to have low power, and this issue is often magnified for tests designed to detect complex alternatives involving both nonlinearity and structural breaks.

Several factors contribute to the observed low power: First, when the mean-reversion parameter θ is close to zero (e.g., –0.2), the process is "nearly non-stationary." Distinguishing such a slowly mean-reverting process from a true random walk is an inherently difficult statistical problem for any test. Second, power is highly sensitive to the location parameter c. When the absolute value of c is large, the time series crosses the threshold infrequently. This means the stationary behavior specified by the alternative hypothesis is rarely observed in the data, giving the test very little information with which to reject the null. Third, the FLSTAR test is designed to detect a highly specific alternative: stationary, asymmetric adjustment around a smoothly breaking deterministic trend. The cost of this flexibility is a reduction in power compared to simpler tests. If the true nonlinearity is weak (small γ) or the structural breaks are very subtle, the test may struggle to distinguish this complex alternative from a simple unit root process. Despite these challenges, it is crucial to note that the power systematically improves with the sample size, which is a desirable property.

3 EMPIRICAL APPLICATION

Following the first oil shock, observed changes in unemployment rates, particularly in developed countries, spurred a significant increase in research aimed at understanding their dynamics. Friedman (1968), and Phelps (1967, 1968) proposed that unemployment rates are a stationary process, implying that the effects of shocks are temporary, and introduced the natural rate hypothesis to the literature. Conversely, Blanchard and Summers (1986) argued that unemployment rates follow a unit root process, where the effects of shocks are permanent, thereby introducing the unemployment hysteresis theory. To distinguish between these two fundamental hypotheses, unit root tests are extensively employed in the literature (see, for example, León-Ledesma and McAdam, 2004; Yilanci et al., 2020; Ball and Onken, 2022; Dadam and Viegi, 2024; Guisinger et al., 2024; Yilanci et al., 2024).

This study examines the validity of unemployment hysteresis in the CIVETS countries using the newly introduced Fourier LSTAR (FLSTAR) unit root test. The acronym CIVETS was coined in 2009 by Robert Ward, global forecasting director at the Economist Intelligence Unit, who identified these nations as a second tier of countries poised to drive economic growth in the subsequent decade. CIVETS comprises Colombia, Indonesia, Vietnam, Egypt, Turkey, and South Africa. These countries are generally characterized by young and growing populations, as well as diverse and dynamic economies (Geoghegan, 2010). The annual data for this analysis were obtained from the IMF e-data service, covering the following periods: 1980–2023 for Colombia, South Africa, and Turkey; 1990–2023 for Egypt and Vietnam; and 1983–2023 for Indonesia. These specific period ranges were selected based on data availability. The analysis is conducted using annual data, a choice motivated by several key considerations. First and foremost, the study's focus is on unemployment hysteresis, a long-run phenomenon concerning the permanent or persistent effects of economic shocks over many years. Annual data, by smoothing out short-term fluctuations, is well-suited for investigating such low-frequency dynamics. Second, from a practical standpoint, annual data provides the longest and most consistent time series available for the CIVETS countries, a crucial factor for the statistical power and reliability of unit root tests that must account for structural breaks. While higher-frequency data (e.g., quarterly) could offer insights into seasonal patterns and the precise intra-year timing of shocks, it would come at the cost of a significantly shorter time span. Furthermore, using annual data avoids potential distortions from seasonal adjustment procedures and aligns with the FLSTAR test's strength in capturing smooth, long-term structural changes rather than seasonal cycles. Therefore, annual data are deemed most appropriate for investigating the long-run persistence of unemployment in this context.

A related consideration is the choice of the starting periods in the 1980s and 1990s and the inherent challenges of using long historical data. We acknowledge that the CIVETS economies and their labor markets have evolved significantly over these decades. Furthermore, official statistics are often subject to changes in definitions and survey methodologies, making perfect long-run harmonization difficult. However, this structural evolution is precisely the phenomenon our study seeks to address. A long time series is essential for the statistical power needed to test a long-run concept like hysteresis. Indeed, the very premise of employing a Fourier-based test is to formally account for such gradual, long-term evolution. The Fourier function is specifically included to capture smooth structural changes, which can arise from fundamental economic transitions, demographic shifts, policy reforms, or even implicitly from changes in data collection methodologies. Therefore, while we recognize the imperfections of historical data, we argue that using the longest available series and employing a methodology robust to structural breaks provides a more powerful and insightful analysis of long-run unemployment persistence than would be possible with a shorter, more recent dataset.

If the unemployment rates, when examined for stationarity using the FLSTAR unit root test, are found to possess a unit root, this indicates the validity of unemployment hysteresis in the respective country. Conversely, the rejection of the unit root null hypothesis suggests that unemployment rates can be modeled by a stationary LSTAR process. In such instances, the plucking model of unemployment is considered applicable. The "plucking model" posits that unemployment rises sharply during recessions but falls more slowly during expansions (Suah, 2024). In analyzing these unemployment rates, structural changes arising from factors such as labor market reforms, major recessions (e.g., the Global Financial Crisis), or demographic shifts will be accounted for using Fourier functions.

We applied the FLSTAR unit root test to the unemployment series of the CIVETS countries; the results are presented in Table 4.

Table 4 The results of FLSTAR Unit Root Test

Countries	Opt. frequency	F test	FLUR test stat.	Opt. lag length
Colombia	1	15.5782*	6.9871**	1
Egypt	4	16.2617*	0.1574	0
Indonesia	1	52.5740*	3.3190	0
South Africa	1	9.7971*	2.1501	0
Turkey	1	19.8331*	2.5861	1
Vietnam	1	69.0721*	2.4196	9

Note: *, **, and *** denote the statistics significance at the 1, 5, and 10% levels, respectively. The critical value at the 1% level for the F test is 6.730 (Becker et al., 2006).
Source: Author's own elaboration

The findings in Table 4 reveal a notable distinction for Colombia, where the FLSTAR unit root test led to the rejection of the unit root null hypothesis at the 5% significance level. Furthermore, the F test statistic for Colombia is also statistically significant. This suggests that Colombia's unemployment can be characterized by a stationary LSTAR process; consequently, the plucking model may be an applicable framework for understanding its dynamics. In contrast, for the remaining CIVETS countries – Egypt, Indonesia, South Africa, Turkey, and Vietnam – the FLSTAR test statistics were not significant. Therefore, the null hypothesis of a unit root could not be rejected for these nations, indicating the validity of unemployment hysteresis. The persistence of a unit root in these countries, even when analyzed with the flexible FLSTAR framework, points to deeper structural issues or stronger hysteresis effects in their labor markets. These divergent findings across the CIVETS group, obtained through a unified testing framework, highlight that a 'one-size-fits-all' approach to labor market policy in emerging markets is likely suboptimal. Furthermore, the FLSTAR test's ability to identify these nuances demonstrates its superior practical utility for policymakers and researchers analyzing complex macroeconomic time series. These results also imply that the shocks to unemployment rates in countries exhibiting hysteresis are likely to have lasting effects.

CONCLUSION

This study addressed a notable gap in the time series analysis literature by proposing and evaluating a novel two-stage unit root test, termed the Fourier-LSTAR (FLSTAR) test. Recognizing the limitations of traditional unit root tests in the presence of structural breaks and nonlinear dynamics, the FLSTAR test was designed to simultaneously account for multiple, potentially smooth, structural changes via Fourier approximation and LSTAR-type asymmetric nonlinearity. The first stage of the test removes deterministic components and adjusts for structural breaks in the series, while the second stage employs an LSTAR-based unit root test on the filtered residuals.

Main contribution of this paper is that it explicitly differentiates itself from and advances upon prior methodologies. While Enders and Lee (2012) pioneered the use of Fourier terms to accommodate smooth breaks, their test operated within a linear framework and may therefore overlook nonlinear mean reversion. Subsequent tests, such as Christopoulos and Leon-Ledesma (2010), and Ranjbar et al. (2018) incorporated symmetric ESTAR nonlinearity but could not capture asymmetric dynamics. The FLSTAR test uniquely synthesizes these developments by integrating Fourier terms into an LSTAR model, providing the first framework for testing unit roots against the alternative of stationarity with both smooth breaks and asymmetric adjustment. This is particularly relevant for economic phenomena – such as the "plucking" model of unemployment – in which behavior during recessions and expansions differs markedly.

The paper successfully derived new critical values for the FLSTAR test statistic across various sample sizes and Fourier frequencies. Subsequent simulation results indicate that the FLSTAR test exhibits no significant size distortions, demonstrating stable performance across different magnitudes of trigonometric term coefficients and sample sizes. The power of the test increases with higher Fourier frequency values and larger sample sizes. However, it decreases as the absolute value of the LSTAR location parameter increases or as the coefficients of the Fourier terms approach zero, suggesting reduced effectiveness in scenarios with weaker nonlinearity or less pronounced structural breaks.

An empirical application to unemployment rates in the CIVETS countries showcased the practical utility of the FLSTAR test. The results indicated that while Colombia's unemployment rate can be characterized by a stationary LSTAR process, supporting the plucking model, unemployment hysteresis appears valid for Egypt, Indonesia, South Africa, Turkey, and Vietnam. This heterogeneity underscores the necessity of advanced testing methods capable of handling both structural changes and nonlinear dynamics, particularly in developing economies where economic and demographic shifts are significant.

The study's results have important implications for economic policy and research. For countries exhibiting hysteresis, such as Egypt, Indonesia, South Africa, Turkey, and Vietnam, structural reforms may be necessary to address persistent unemployment, given the permanent impact of shocks. In contrast, for Colombia, where unemployment is found to be stationary, temporary measures might suffice to manage fluctuations, aligning with the plucking model.

Looking forward, several avenues for future research emerge from this study. The FLSTAR testing framework could be extended and applied to other key macroeconomic and financial time series where the interplay of structural changes and asymmetric nonlinearities is theoretically plausible, thereby broadening its empirical relevance. Additionally, comparative studies evaluating the performance of the FLSTAR test against other recently developed nonlinear and non-stationary testing procedures across a wider array of data generating processes would be a valuable addition to the literature, further elucidating its relative strengths and limitations.

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