Digital Country Rankings for the Visegrád Group Countries with DEA and TOPSIS

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Abstract

Our paper is based on the five principal dimensions of the International Digital Economy and Society Index (I-DESI), but instead of using the original scoring model based on arbitrary pre-determined weights, we apply more objective ranking methods that use the statistical properties of the data series to determine where the Visegrád Group (V4) countries (Czechia, Hungary, Poland and Slovakia) stand in terms of digital development among the countries of the European Union and other developed countries in the data set. The ranking is performed using the DEA-CWA (Data Envelopment Analysis/Common Weights Analysis) method (with six models) and the TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) method. Although the resulting weight vectors differ significantly from the arbitrary weights set by the European Commission, the country rankings remain similar, displaying relatively little sensitivity to the weighting method chosen.

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Data Envelopment Analysis, information and communication technology, International Digital Economy and Society Index, TOPSIS, Visegrád Group	*https://doi.org/10.54694/stat.2024.32*	*C44, L38*

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INTRODUCTION

The International Digital Economy and Society Index (I-DESI) is an indicator system published every two years (from 2016), which is prepared by independent experts (research companies) at the request of the European Commission. Its objective is to provide a comprehensive assessment of the position of the European Union on the "road towards a digital society and economy" compared to non-EU economies.

The defining principle in the methodology is that the indicator system reflects the results of the European Commission's original DESI (Digital Economy and Society Index) indicator, i.e. I-DESI also measures the progress of digital transformation (as DESI does for EU member countries) and expands it to non-EU countries using substitute indicators, which are available for them. DESI and I-DESI are indicator systems combining individual indicators with predefined weights and using similar but not identical scoring models to evaluate and rank individual countries based on their digital performance. Consequently, minor inconsistencies can be observed in the country rankings and scores within the DESI and I-DESI databases, even when non-EU countries are excluded from the analysis. Nevertheless, the authors of I-DESI implemented strategies to ensure a high degree of alignment between the two. To this end, they conducted correlation and covariance tests of indicators, sub-dimensions, and dimensions with the objective of "[minimising] covariance in any new or replacement indicators" (European Commission, 2021: 40).

The 2020 edition of I-DESI measures the achievements in the field of digitisation in 45 countries (of which 27 are EU members, 18 are non-EU countries) in five main policy areas, as shown in Table 1 (European Commission, 2021). In the following sections, we will use the abbreviations for the dimensions, as shown in Table 1.

Table 1 Dimensions of I-DESI 2020 and their interpretations

Dimension	Abbrev.	Interpretation of the given dimension
Connectivity (internet access)	CNC	Availability and quality of broadband network infrastructure.
Human capital	HUC	Level of digital skills necessary to access opportunities offered by the digital society.
Use of internet	UOI	Use of internet services by citizens (consumers).
Integration of digital technology	IDT	Digitisation of businesses and online sales channels.
Digital public services	DPS	Digitisation of public services, focusing on e-government services.

Source: Based on European Commission (2021)

In this study, we used the data of these main dimensions of the year 2018 as a basis for the evaluation of the countries and the calculation of the DEA and TOPSIS efficiencies. It is worth noting that, despite the authors' desire to include more recent data, the most up-to-date edition of I-DESI available at the time of analysis was the 2020 edition. While a call for tenders for the 2022 edition of I-DESI is publicly available (European Commission, 2022), and the publication itself is believed to exist in an eBook form (WorldCat, 2022), it has been inaccessible for an unknown reason.

In the I-DESI 2020 study, the methodology for aggregating indicators into sub-dimensions, sub-dimensions into dimensions, and dimensions into the overall index follows a simple bottom-up approach (scoring model based on subjective weights). Weighted arithmetic averages are applied following the structure of the index to calculate the top-level I-DESI score using Formula (1):

$$IDESI = CNC \cdot 0.25 + HUC \cdot 0.25 + UOI \cdot 0.15 + IDT \cdot 0.2 + DPS \cdot 0.15 \,. \tag{1}$$

It should be noted, however, that since 2021, the Use of Internet (UOI) dimension has been removed from the (EU-only) DESI, and the structure has been simplified, with the remaining four dimensions having equal weights of 25%. Given that the I-DESI was designed to be an international extension of the original index, it is reasonable to expect that these changes would be reflected in the I-DESI 2022 study and all future editions.

The paper is structured as follows. In the next section, we provide a brief overview of the literature related to the DESI and I-DESI indicator systems. In the subsequent sections, we present the methodology of the models used, the Data Envelopment Analysis (DEA), the models based on it, and the TOPSIS method, as well as the results, highlighting the position of the four Visegrád Group (V4) countries (Czechia, Hungary, Poland, Slovakia). Finally, we conclude our study.

1 LITERATURE SURVEY

There are several indicator systems that aim to characterise the degree of digital transformation and digital development at the level of country groups and countries. In this paper, we focused on presenting the main findings in the literature review related to DESI and I-DESI.

As these indices measure the digital economy performance of a specific set of countries, their analyses can have different goals. According to our literature review, these goals are (I) analysing the indices, their dimensions, and methodologies, (II) evaluating the performance of one or some included countries or (III) some other important issues.

Many studies have analysed the indices' different dimensions and their relationships with statistical methods (e.g. correlation and factor analysis). Multivariate statistical methods were used by Bánhidi et al. (2020), and the authors found that the dimensions of DESI are closely related. Tokmergenova et al. (2021) analysed the multicollinearity among the dimensions of I-DESI and found some redundancy among them. These findings are important not only for further analysis of the digital economy's performance but also show that these characteristics can only be developed with a comprehensive strategy.

Other researchers considered these indices as competitors or measures complementary to others. Kotarba (2017) discusses the most important indicator systems of digital development, including DESI, showing the main similarities and differences, and making suggestions for their development. Esses and Szalmáné Csete (2022) examined the digital transformation and sustainability transition of the European capital cities using the Cities In Motion Index (CIMI), with the Sustainable Development Goals (SDG) and the Human Development Index (HDI). They also developed a modified CIMIS index extended with sustainability to rank the cities and assess their strengths and weaknesses.

Focusing on one or some countries, many (I-)DESI-related analyses were done (performing e.g. cluster analyses, data envelopment analyses). Bánhidi et al. (2020) ranked the EU-member countries based on DESI data using many methods. Tarjáni et al. (2023) compared the 27 EU Member countries and 18 non-EU participants based on I-DESI data with discriminant and variance analysis. Bánhidi and Dobos (2021), and Bánhidi et al. (2021) examined Russia's position and performance with a DEA/CI model. Moroz (2017) used the DESI and the Networked Readiness Index (NRI) to evaluate Poland's situation and the dynamics of its development. Laitsou et al. (2020) analysed Greece's digital performance and possible development strategies using the Gompertz model.

Different important parts of the economy and phenomena in our society can be analysed with DESI and I-DESI measures, which lie somewhat outside our analysis's scope but should be mentioned as they highlight the following analysis possibilities. Roukanas (2021) analysed the innovation capabilities of countries based (among others) on DESI, I-DESI data. Skare et al. (2023) meanwhile examined the impact of digital technology on SMEs using DESI data. Finally, Basol and Yalçin (2021) analysed the effects of DESI indicators, especially on the labour market indicators.

2 METHODS

The first DEA model was initiated by Charnes et al. (1978) and was followed by countless other model variants. When using the International Digital Economy and Society Index (I-DESI), the ideal value of the given criterion for each of the five dimensions is the highest possible, so these criteria can be considered outputs in the DEA model. Since the denominator, i.e. the weighted criterion of this DEA model is equal to one, there are no explicit input criteria. In the literature, this latter model is called a DEA model without explicit inputs (DEA/WEI) and/or a DEA-type composite indicators (DEA/CI) method (Bánhidi and Dobos, 2023; Cherchye, 2008; Dobos and Vörösmarty, 2014).

The DEA/WEI model was first developed by Fernandez-Castro and Smith (1994), then applied by Despotis (2005) and Liu and Peng (2008) to practical problems. Due to the model's shape, the earlier mentioned composite indicators were used.

Table 2 summarises the advantages and disadvantages of DEA models. The table is based on the results of Iqbal and Lerme (1997).

Table 2 Advantages and disadvantages of DEA models

Advantages of DEA	Disadvantages of DEA
Versatility: DEA can handle multiple inputs and outputs simultaneously, making it suitable for complex scenarios where various factors influence efficiency.	*Sensitivity to Input and Output Selection*: The results in DEA can be sensitive to the choice of inputs and outputs. Proper analysis of their relative importance is crucial before conducting DEA.
Returns to Scale Consideration: DEA accounts for returns to scale when calculating efficiency. It allows us to assess efficiency changes based on the size and output levels of an organisation.	*Lack of Statistical Inference*: DEA does not provide statistical significance tests, limiting our ability to make formal statistical inferences.
Non-parametric Approach: DEA does not require specific functional forms or assumptions about the production process, making it flexible and applicable across different industries.	*Assumption of Constant Returns to Scale*: While DEA considers returns to scale, it assumes constant returns to scale for all decision-making units (DMUs), which may not always hold in practice.
Competitor Analysis: By evaluating comparative advantage and disadvantage, DEA provides insights into organisations' performance relative to its peers.	*Efficiency Frontier Ambiguity*: The efficiency frontier (the boundary of efficient DMUs) may be ambiguous due to the non-parametric nature of DEA.
Strategic Alliances: DEA helps assess the feasibility and desirability of forming strategic alliances.	

Source: Based on Iqbal and Lerme (1997)

The basic DEA model for ranking cannot be fully applied here because the DEA efficiency of several decision-making units (DMUs, i.e., the countries) can reach an optimum, i.e., a single value; this means that these DMUs cannot be ranked clearly. At the same time, we also have the problem that individual DMUs achieve different efficiencies that can be calculated for every country. This also means that the weights of DMUs cannot be clearly determined in classical DEA models. Therefore, another method must be found that evaluates all possible DMUs with the same weight.

One of the first such applications is the Common Weights Method (CWA) model, first used by Podinovski and Athanassopoulos (1998). The problem is named the MaxiMin DEA model. In this DEA type model, we first look for the country that gives the smallest efficiency for a predefined weight vector, and after that find the weight vector that optimises this minimum. The name of this procedure refers to this hierarchical optimisation procedure, where we first perform minimisation and only then maximise the DEA efficiency.

To solve the problem, it is necessary to find a DEA procedure that evaluates all countries with the same weight. This second procedure is called the method of common weights. The already mentioned MaxiMin model can also be classified in this group. Liu et al. (2011) suggested an easy form of the procedure. The model in this form is a linear programming problem searching for common weights with boundary conditions, including efficiency constraints.

The third type of DEA/CWA model determines common weights by compromise programming. This method was initiated by Kao and Hung (2005). The goal functions here can be e.g. Manhattan, Euclidean, or Chebyshev functions, which are distance functions. However, to apply these methods, a nadir and/or ideal point must be determined in the set of the efficiency of the countries. A possible nadir efficiency can be equal to zero vector. The ideal is the previously defined DEA efficiencies or the maximum efficiency achievable for each DMU, i.e. one. In this study, only the two ideal efficiencies are used. The ideal point requires finding the weights for which the distance between the ideal point and the set of weights is minimal. In the case of non-ideal points, we find the minimal distance between the nadir point and the set of all possible weights. The $\boldsymbol{E}^*$ vector represents the efficiencies that show the optimal efficiency of each country in all possible DEA models.

In the DEA method, a different weighting system is assigned to each DMU. Therefore, the search for common weights often appears in the literature. Roll and Golani (1993) solve the problem of finding a common weight by restricting them. In Kao and Hung (2005), a method of compromise programming (CPM) was proposed to find the common weights with solutions of non-linear programming problems.

The inequality system (2)–(3) determines the set of possible weights. The first of the two systems of inequalities (2) specifies the upper limit of the common weights, while the second (3) specifies the non-negativity condition. The number of DMUs is p, and the vector $\boldsymbol{y}_j$ is the evaluation of the j-th DMU, i.e. country. The vectors $\boldsymbol{y}_j$ can be gathered in the matrix $\mathbf{Y}$. The vector $\mathbf{u}$ contains the DEA weights. The DEA/WEI weights are equal to the $\mathbf{u} \cdot \mathbf{Y}$ vector.

$$\mathbf{u} \cdot \mathbf{y}_j \leq 1; j = 1, 2, ..., p. \tag{2}$$

$$\mathbf{u} \geq 0. \tag{3}$$

The goal functions of the possible Date Envelopment Analysis models are shown in Table 3.

Table 3 Goal functions of the used DEA models

DEA Models	Goal function ($F_i(\mathbf{u})$)	Literature
MaxiMin model (1.)	$F_1(\mathbf{u}) = \min_{1 jp} \mathbf{u} \cdot \boldsymbol{y}_j \to \max$	Podinovski and Athanassopoulos (1998)
CWA model (2.)	$F_2(\mathbf{u}) = \mathbf{u} \cdot \mathbf{Y} \cdot \mathbf{1} \to \max$	Liu and Peng (2008)
CPM with **1** (1; 1; 1;…) as ideal point (3., 4.)	$F_3(\mathbf{u}) = d_2(\mathbf{u} \cdot \mathbf{Y}; \mathbf{1}) \to \min$ $F_4(\mathbf{u}) = d_{+\infty}(\mathbf{u} \cdot \mathbf{Y}; \mathbf{1}) \to \min$	Kao and Hung (2005)
CPM with $\boldsymbol{E}^*$ ideal points (5., 6.)	$F_5(\mathbf{u}) = d_2(\mathbf{u} \cdot \mathbf{Y}; \boldsymbol{E}^*) \to \min$ $F_6(\mathbf{u}) = d_{+\infty}(\mathbf{u} \cdot \mathbf{Y}; \boldsymbol{E}^*) \to \min$	Kao and Hung (2005)

Source: Own elaboration based on Kao and Hung (2005), Liu and Peng (2008), and Podinovski and Athanassopoulos (1998)

Similar results for both the summation **1** and the DEA efficiency $\boldsymbol{E}^*$ vectors are given for the result of the CWA of the Manhattan distances (Toloo, 2014). The distance functions of CPM are (4)–(5):

$$\text{Euclidean distance } (k = 2)\text{: } d_2\left(\mathbf{u}\cdot\mathbf{Y};\boldsymbol{E}\right)=\sqrt{\sum_{\mathrm{j=1}}^{\mathrm{p}}(\mathbf{u}\cdot\mathbf{y}_{\mathrm{j}}-E_{\mathrm{j}})^2}. \tag{4}$$

$$\text{Chebyshev distance } (k = +\infty)\text{: } d_{+\infty}\left(\mathbf{u}\cdot\mathbf{Y};\ \boldsymbol{E}\right)=\max_{1\le \mathrm{j}\le \mathrm{p}}\left|\mathbf{u}\cdot\mathbf{y}_{\mathrm{j}}-E_{\mathrm{j}}\right|. \tag{5}$$

As defined before, vector $\boldsymbol{E}$ is an ideal efficiency vector that can be equal to the efficiency vectors $\boldsymbol{E}^*$ or **1**.

TOPSIS is a decision-theory method based on a simple geometric approach that ranks alternatives according to their similarity to an ideal solution. It has gained popularity due to its relative simplicity, stability, and moderate computational complexity. It has been widely used in various domains (e.g. Omurbek et al., 2021; Shaktawat and Vadhera, 2021), and its effectiveness has been demonstrated in numerous studies. The main limitation of the method is its limited applicability to dynamic problems with critical time dimensions and close interdependence between ranking criteria. However, these limitations are not relevant to our study as we worked with a cross-sectional database.

3 RESULTS AND DISCUSSION

3.1 Ranking with DEA/WEI and CWA methods

Before presenting the results of the calculations, we show that the CWA model in Table 3 and the Manhattan distance CPM analysis do not differ from each other, i.e. they result in the same weighting system. Therefore, of the two models, it is sufficient to solve the CWA model. Since the weighted sum of the predetermined **1** and $\boldsymbol{E}^*$ vectors does not depend explicitly on the weights, we optimise the expression $-\mathbf{u}\cdot\mathbf{Y}\cdot\mathbf{1}$, which means that the negative of a linear function must be optimised. It also means we have got the CWA model back. Hence, minimising the Manhattan distance models also leads to the optimisation problem of the CWA model.

The mathematical model of the six common weighting problems can be characterised as (6)–(8):

$$\mathbf{u}\cdot\mathbf{y}_{\mathrm{j}}\le 1;\ \mathrm{j}=1,2,\ldots,\mathrm{p}. \tag{6}$$

$$\mathbf{u}\ge 0. \tag{7}$$

$$\boldsymbol{F}_i\left(\mathbf{u}\right)\to \min/\max,\ \mathrm{i}=1,2,\ldots,6. \tag{8}$$

Problems (i) 1 and 2 are maximised, while problems 3, 4, 5 and 6 are minimised. The mathematical model of the functions is shown in Table 3. All calculations were performed using Microsoft Excel 2016, except the calculations of the correlation coefficients, which were done in IBM SPSS 28. For the sake of brevity, only the results (in terms of weights and efficiencies) are presented in the paper; the detailed calculations are omitted. The efficiencies of the six DEA models (6)–(8) [i = 1, 2, …, 6] can be found in Table 4. The table shows the efficiency solutions obtained using the six common weighting methods, with the Visegrád Group (V4) countries highlighted in grey.

Table 4 Efficiencies for common weights methods [data of the 2020 I-DESI]

Country	I-DESI	MaxiMin	DEA CWA	Euclidean E*	Euclidean 1	Chebyshev E*	Chebyshev 1	TOPSIS	DEA efficiency
Australia	0.599	0.894	0.899	0.898	0.899	0.866	0.894	0.645	0.934
Austria	0.519	0.810	0.826	0.824	0.826	0.773	0.810	0.503	0.831
Belgium	0.489	0.832	0.836	0.836	0.836	0.808	0.832	0.467	0.853
Brazil	0.365	0.634	0.629	0.628	0.629	0.544	0.634	0.246	0.663
Bulgaria	0.400	0.801	0.805	0.804	0.805	0.698	0.801	0.287	0.805
Canada	0.553	0.824	0.805	0.807	0.805	0.829	0.824	0.582	0.844
Chile	0.353	0.699	0.706	0.705	0.706	0.633	0.699	0.194	0.707
China	0.463	0.766	0.771	0.769	0.771	0.681	0.766	0.406	0.777
Croatia	0.348	0.739	0.752	0.751	0.752	0.655	0.739	0.204	0.760
Cyprus	0.471	0.855	0.849	0.847	0.849	0.746	0.855	0.425	0.855
Czechia	0.476	0.812	0.822	0.821	0.822	0.767	0.812	0.425	0.822
Denmark	0.695	1.000	1.000	1.000	1.000	1.000	1.000	0.837	1.000
Estonia	0.572	0.869	0.861	0.861	0.861	0.844	0.869	0.602	0.911
Finland	0.683	0.953	0.966	0.966	0.966	0.995	0.953	0.809	1.000
France	0.575	0.928	0.913	0.913	0.913	0.882	0.928	0.583	1.000
Germany	0.579	0.844	0.863	0.863	0.863	0.866	0.844	0.623	0.886
Greece	0.403	0.799	0.790	0.788	0.790	0.679	0.799	0.307	0.799
Hungary	0.411	0.726	0.734	0.733	0.734	0.685	0.726	0.309	0.739
Iceland	0.619	0.939	0.977	0.975	0.977	0.948	0.939	0.672	1.000
Ireland	0.597	0.835	0.849	0.848	0.849	0.848	0.835	0.650	0.869
Israel	0.584	0.744	0.759	0.759	0.759	0.818	0.744	0.633	0.946
Italy	0.382	0.792	0.777	0.777	0.777	0.686	0.792	0.265	0.792
Japan	0.577	1.000	1.000	1.000	1.000	0.961	1.000	0.607	1.000
Latvia	0.412	0.750	0.752	0.752	0.752	0.704	0.750	0.327	0.770
Lithuania	0.437	0.827	0.849	0.846	0.849	0.721	0.827	0.366	0.849
Luxembourg	0.620	0.887	0.911	0.910	0.911	0.893	0.887	0.707	0.922
Malta	0.478	0.934	0.933	0.932	0.933	0.831	0.934	0.428	0.934
Mexico	0.370	0.624	0.614	0.613	0.614	0.561	0.624	0.245	0.679
Netherlands	0.682	0.881	0.886	0.887	0.886	0.952	0.881	0.813	1.000
New Zealand	0.542	0.846	0.844	0.844	0.844	0.821	0.846	0.550	0.846
Norway	0.638	0.919	0.908	0.909	0.908	0.930	0.919	0.737	0.985
Poland	0.364	0.730	0.720	0.718	0.720	0.617	0.730	0.235	0.730
Portugal	0.409	0.774	0.760	0.761	0.760	0.727	0.774	0.313	0.774
Romania	0.417	0.738	0.749	0.747	0.749	0.644	0.738	0.324	0.749
Russia	0.427	0.640	0.631	0.630	0.631	0.604	0.640	0.351	0.728
Serbia	0.378	0.673	0.685	0.683	0.685	0.590	0.673	0.240	0.686
Slovakia	0.389	0.718	0.718	0.717	0.718	0.650	0.718	0.269	0.727
Slovenia	0.469	0.793	0.801	0.800	0.801	0.744	0.793	0.404	0.801
South Korea	0.544	0.952	0.917	0.918	0.917	0.872	0.952	0.545	1.000
Spain	0.466	0.825	0.808	0.808	0.808	0.736	0.825	0.412	0.845
Sweden	0.650	0.922	0.953	0.952	0.953	0.946	0.922	0.752	0.977
Switzerland	0.656	0.914	0.947	0.947	0.947	0.973	0.914	0.736	1.000
Turkey	0.336	0.585	0.572	0.572	0.572	0.541	0.585	0.187	0.585
United Kingdom	0.593	0.905	0.902	0.903	0.902	0.913	0.905	0.656	0.927
United States	0.710	0.960	0.975	0.975	0.975	0.987	0.960	0.870	1.000

Source: Own elaboration based on the database of European Commission (2021)

If we take into consideration the relationship between the efficiencies calculated by CWA procedure and the efficiency of DEA, there are two types of connections.

Table 5 presents the correlation coefficients. Since they are continuous variables, i.e. the statistical scale is minimum interval scale; the Pearson correlation can be used. They show high correlations between the I-DESI and the weights of the DEA efficiencies with values above 0.8. We investigated if it is worth to determine efficiencies without determining the DEA efficiencies. Then if we consider the DEA efficiency as an ideal (virtual) DMU, these efficiencies must first be calculated, which implies solving p linear programming (LP) problems. Calculating the CWA efficiencies is the solution of an auxiliary LP, i.e. $p + 1$ LP models have to be solved. On the other hand, if we only determine the maximally achievable efficiency, i.e. the individual efficiency, the mileage from the possible efficiency, then only one (non)linear programming problem must be solved.

Table 5 Pearson correlation between the I-DESI and the six DEA/CWA efficiencies

Pearson	Manhattan	Euclidean	Chebyshev
E*	0.823*	0.845	0.952
1	0.842	0.842	0.823

Note: * the solution of the MaxiMin model is given and not the Manhattan distance.
Source: Own elaboration

Results in Table 5 show that the Chebyshev distance is minimising the distance from the ideal efficiencies vector $\boldsymbol{E}^*$. A correlation of 0.952 is achieved, which is considered a very strong relationship. The efficiency with the biggest correlation from vector **1** gives a value of 0.842, which is also considered relatively high. The two computational efficiencies do not show significant difference from the I-DESI efficiencies. This may lead to the results that it is unnecessary to calculate all p DEA efficiencies, which may save time and costs not solving p LP problems.

Kendall's *tau*-b shows the linear relationship between the rankings (Table 6). This correlation indicates a strong stochastic (linear) relationship when greater than 0.7 and it is close to this value for the I-DESI data. The Chebyshev distance leads to the highest correlation among the six common weightings, with a value of 0.826. This also shows the greatest linear relationship with DEA efficiency.

Table 6 Kendall's *tau*-b correlation between I-DESI and the DEA/CWA models

Kendall	Manhattan	Euclidean	Chebyshev
E*	0.669*	0.693	0.826
1	0.689	0.687	0.669

Note: * the solution of the MaxiMin model is given and not the Manhattan distance.
Source: Own elaboration

Finally, we compare the weight vectors of the six DEA models. As shown in Table 7, the MaxiMin and the Chebyshev model with the unit vector distance produced identical results, as did the Euclidean distance model and the DEA/CWA model. This means that the six models gave only four different solutions for the weight vectors. Table 7 also contains the weight vectors defined by the European Commission. While the Commission's weight vectors are balanced, in all our models the weight vector of the Connectivity dimension is the largest, with at least 60% (but in most models it receives a weight of around 90%). At the same time, the UOI dimension determines efficiency in only one model (and with a very small contribution).

Table 7 I-DESI and DEA common weights of each dimension and their normalised ratio

Weights	CNC	HUC	UOI	IDT	DPS
I-DESI	*0.25*	*0.25*	*0.15*	*0.20*	*0.15*
MaxiMin	1.247	0.000	0.000	0.000	0.108
	0.920	*0.000*	*0.000*	*0.000*	*0.080*
Chebyshev 1	1.247	0.000	0.000	0.000	0.108
	0.920	*0.000*	*0.000*	*0.000*	*0.080*
Euclidean 1	1.246	0.156	0.000	0.000	0.000
	0.889	*0.111*	*0.000*	*0.000*	*0.000*
DEA CWA	1.246	0.156	0.000	0.000	0.000
	0.889	*0.111*	*0.000*	*0.000*	*0.000*
Euclidean E*	1.243	0.145	0.000	0.006	0.005
	0.888	*0.104*	*0.000*	*0.004*	*0.004*
Chebyshev E*	0.944	0.000	0.029	0.279	0.127
	0.685	*0.000*	*0.021*	*0.202*	*0.092*

Source: Own elaboration

The weights of the model with the smallest Chebyshev distance measured by DEA efficiency are the closest to the common weights given by the European Commission, although they are much less balanced than those. Connectivity is by far the most weighted in this model as well. Nevertheless, the deviation of the weight vectors from the weights determined by the Commission does not usually have a significant effect on the ranking position of individual countries. Czechia is ranked 24th in four out of the six DEA models (in line with the Commission's ranking), and 25th in the remaining two models. Hungary is ranked 37th or 38th in five of the six models above, and 33rd in the sixth (and the Commission's ranking). Poland is ranked 38th in three models, 37th in two models, but only 40th in the sixth model and 42nd in the original ranking. Finally, Slovakia is ranked 39th in five out of the six DEA models and 37th in the remaining one and the Commission's scoring model. The larger discrepancy between the rankings of Hungary and Poland in the DEA models and in the scoring model may be attributable to the considerable, perhaps excessive, emphasis placed on the Connectivity dimension in the DEA models and their relative strengths in this domain of digital development.

3.2 Ranking with the TOPSIS method

Before the TOPSIS method is briefly presented, Table 8 summarises the advantages and disadvantages of the procedure. The description was summarised based on Shih (2022). The TOPSIS method determines the order of countries in three steps described below (readers can find a more detailed description of the method in Bánhidi and Dobos, 2021).

Table 8 Advantages and disadvantages of TOPSIS models

Advantages of TOPSIS	Disadvantages of TOPSIS
Comprehensive Comparison: TOPSIS allows decision-makers to evaluate and compare multiple alternatives when faced with various criteria. It provides a holistic view by considering all relevant factors simultaneously.	*Assumption of Monotonicity*: TOPSIS assumes that the criteria are monotonically increasing or decreasing. In practice, this may not always hold true, especially when dealing with complex and dynamic systems.
Compensatory Aggregation: Unlike non-compensatory methods that use hard cut-offs, TOPSIS allows trade-offs between criteria. A poor result in one criterion can be offset by a good result in another. This realistic modelling accounts for the complexities of real-world decision-making.	*Normalisation Challenges*: Normalisation is necessary because criteria often have incongruous dimensions. However, selecting an appropriate normalisation method can be challenging. Linear normalisation and vector normalisation are commonly used approaches.
Geometric Distance Calculation: TOPSIS calculates the geometric distance between each alternative and the ideal solution (both positive and negative). This approach ensures that the chosen alternative is close to the positive ideal solution and far from the negative ideal solution.	*Subjectivity in Weight Assignment*: Assigning weights to criteria involves subjectivity. Different decision-makers may assign different weights, leading to variations in the final rankings.
Weighted Criteria: The method accommodates weighted criteria. Decision-makers can assign weights to each criterion based on their relative importance. These weights influence the overall ranking of alternatives.	*Sensitive to Outliers*: TOPSIS is sensitive to outliers. Extreme values in the data can significantly impact the results, affecting the overall ranking.

Source: Based on Shih (2022)

Step 1: Data normalisation

Let us assume that the data of criterion i is contained in the vector $\boldsymbol{x}_i$ according to each country (based on European Commission, 2021). Then data transformation is then performed as follows:

$$y_{ji} = \frac{x_{ji} - x_j^{min}}{x_j^{max} - x_j^{min}}, \left(j = 1,2,\ldots,n;\ i = 1,2,\ldots,m \right), \tag{9}$$

where the minimum and maximum values of criterion i are x_j^{min} and x_j^{max}, n is the number of DMUs, and m denotes the number of criteria. With this determination, the calculated values of the individual criteria were converted to the interval [0,1] for every country. The values of the new vectors are y_{ji}.

Step 2: Determining the objective weights

We determine the weight of the variables using the entropy-based method (Zou et al., 2006). The conversion formula is as follows:

$$H_i = -\frac{1}{\ln(n)} \cdot \sum_{j=1}^{n} \frac{y_{ji}}{\sum_{j=1}^{n} y_{ji}} \cdot \ln\left(\frac{y_{ji}}{\sum_{j=1}^{n} y_{ji}} \right), \left(i = 1,2,\ldots,m \right). \tag{10}$$

The new weights are:

$$w_i = \frac{1 - H_i}{n - \sum_{i=1}^{m} H_i}, \left(i = 1,2,\ldots,m \right). \tag{11}$$

The weighted normalised values are z_{ji} which are equal to: $z_{ji} = w_i \cdot y_{ji}$. Then the ideal and lowest points are determined using the z_{ji} values.

Step 3: Calculating the TOPSIS efficiencies
Thirdly, based on the newly calculated information, the efficiency index is determined using the highest, the ideal (I_i) and the lowest, the nadir (N_i) points, which are determined as follows:

$$I_i = \max_{j=1,2,\ldots,n} z_{ji},\ N_i = \min_{j=1,2,\ldots,n} z_{ji},\ (i = 1,2,\ldots,m). \tag{12}$$

The distance of the j-th country from the highest ideal and the lowest nadir point is calculated as follows:

$$d_j^I = \sqrt{\sum_{i=1}^{n}\left(z_{ji} - I_i\right)^2},\ d_j^N = \sqrt{\sum_{i=1}^{n}\left(z_{ji} - N_i\right)^2},\ (j=1,2,\ldots,n). \tag{13}$$

The last calculation is to determine the E_j TOPSIS efficiencies, which represent the proportion of the distance from two specified points:

$$E_j = \frac{d_j^N}{d_j^I + d_j^N},\ (j = 1,2,\ldots,n). \tag{14}$$

We present the results of the calculations performed on the dataset using Excel 2016. For the sake of brevity, we have omitted the detailed calculations. The calculated objective weights are listed in Table 9, while the TOPSIS efficiencies themselves are listed in Table 4.

Table 9 Calculated TOPSIS weights of digital dimensions

	CNC	HUC	UOI	IDT	DPS
Weights	0.172	0.196	0.200	0.249	0.184

Source: Own elaboration

Among the weights of the dimensions, IDT's is the largest (with a weight of almost 25%), followed by UOI and HUC (with a weight of around 20%). This indicates that the most advanced countries in terms of education are ranked highest. The DPS and CNC dimensions are given slightly less weight than these. Compared with the weight vector determined by the European Commission, the most striking difference is that the Commission gave more than 20% of the weight to the dimension of basic infrastructure and competencies (CNC, HUC), while TOPSIS gave the highest weight to enterprise applications (IDT). However, it may be worth recalling that in the post-2021 editions of the DESI, the structure of the overall index has been changed, and all four (remaining) principal dimensions, including IDT, have the same weight (25%, which in this case is almost equal to the entropy-based weight). Among the V4 countries, Czechia has the highest TOPSIS ranking (24th), followed by Hungary (34th) Slovakia (37th) and Poland (42nd). These are the same as the original positions, except for Hungary, which ranks 33rd according to the I-DESI scoring model.

First, the ranking obtained with I-DESI weights is compared with the TOPSIS ranking with Kendall's *tau*-b correlation. The value of the correlation is high, with a value of 0.966. This correlation indicator is also significant. Interestingly, despite the difference in weights, the rankings are very similar.

Table 10 shows the correlation between the efficiencies calculated by CWA and the TOPSIS efficiency of using Kendall's *tau*-b linear relationship. The highest correlation (0.812) is obtained again in the case of the maximal solution of the Chebyshev distance by minimising the distance from the $\boldsymbol{E^*}$ vector, which shows a strong stochastic relationship.

Table 10 Kendall's *tau*-b correlation between the TOPSIS and DEA/CWA efficiencies

Kendall	Manhattan	Euclidean	Chebyshev
*E**	0.669*	0.691	0.812
1	0.686	0.685	0.669

Note: * the solution of the MaxiMin model is given and not the Manhattan distance.
Source: Own elaboration

In Table 11, the 45 countries of the database (I-DESI report) were divided into 9 groups of five countries based on the ranking of digital development evaluated based on TOPSIS efficiencies, of which the most developed (with a TOPSIS efficiency higher than 0.75) are the United States, Denmark, the Netherlands, Finland and Sweden. The least developed group includes Poland, Serbia, Croatia, Chile and Turkey. Slovakia is in the 8th group with Bulgaria, Italy, Brazil and Mexico. Hungary was placed in the 7th group, which also includes Latvia, Romania, Portugal and Greece meaning that the country and even more the companies in the country have started on the path to digital maturity. Among the V4 countries, Czechia, together with Austria, Belgium, Malta and Cyprus, is in the highest group (5th), which roughly corresponds to an average level of development among the countries in the dataset, in contrast to the other V4 countries, which are in the bottom three groups. However, it is important to emphasise that the I-DESI only includes countries that are either in the EU itself or are considered to be its competitors, and in a global context even these countries would be considered relatively advanced in terms of digital maturity.

Table 11 TOPSIS efficiencies of I-DESI countries by efficiency category

Tier	Best eff.	Worst eff.	Countries in each tier (grouped by TOPSIS efficiencies)
1	0.870	0.752	United States, Denmark, Netherlands, Finland, Sweden
2	0.737	0.656	Norway, Switzerland, Luxembourg, Iceland, United Kingdom
3	0.650	0.607	Ireland, Australia, Israel, Germany, Japan
4	0.602	0.545	Estonia, France, Canada, New Zealand, South Korea
5	0.503	0.425	Austria, Belgium, Malta, Cyprus, Czechia
6	0.412	0.351	Spain, China, Slovenia, Lithuania, Russia
7	0.327	0.307	Latvia, Romania, Portugal, Hungary, Greece
8	0.287	0.245	Bulgaria, Slovakia, Italy, Brazil, Mexico
9	0.240	0.187	Serbia, Poland, Croatia, Chile, Turkey

Source: Own elaboration

CONCLUSION

The results of our DEA models do not differ significantly from the results of the original I-DESI (scoring) model (Hungary's ranking, for example, is never worse than 38th). However, the weights are very different from those defined by the European Commission. Based on the Pearson correlations, the two different calculation efficiencies used in the models do not show significantly different solutions. This may indicate that it is unnecessary to determine all p DEA efficiencies, which may lead to time and cost savings. The highest correlation coefficient was obtained for Chebyshev distance by minimising the distance from the efficiency vector $\boldsymbol{E}^*$, both in the case of the Pearson correlation and the Kendall *tau*-b linear relationship.

Our weights calculated using the TOPSIS method are relatively balanced, like the original weight vector determined by the European Commission (in contrast to the DEA models), but they place more emphasis on the use of digital technologies for business and private purposes than the Commission which gives the greatest weight to the dimension of basic infrastructure and competences, and smaller weights to the dimensions of various applications (this has changed in the post-2021 editions of DESI). The resulting ranking closely resembles the Commission's original ranking, as determined by the I-DESI scoring model (all V4 countries except for Hungary have the same ranking). However, in the DEA rankings, Hungary is typically ranked lower than in the original model, whereas Poland is ranked higher. We posit that this is the joint consequence of the digital development profiles of these two countries being slightly disparate from their peers (despite a similar overall level) and the DEA models placing considerable emphasis on the Connectivity dimension at the expense of the other dimensions.

Although the rankings obtained were relatively similar to those generated by the Commission's original scoring model, we consider the results derived from methods with objective weights to be valuable for assessing their findings' reliability and providing a useful reference point. Compared to the other Visegrád Group (V4) countries, Czechia is relatively advanced, as it could be characterised as having an average level of development. In contrast, the other V4 countries could be considered as laggards. In a global context, however, even these countries would be considered relatively advanced in terms of digital maturity.

Based on our findings, the TOPSIS methodology appears to be more suitable than DEA-based methods for determining digital country rankings due to its more even distribution of weights. However, methods based on the DEA approach require less methodological consideration and are easier to calculate than those based on TOPSIS. Regarding future research avenues, it may be beneficial to explore hybrid models combining expert judgment with data-driven approaches for determining weights, with the aim of ensuring objectivity and relevance.

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