Comparing Two Non-Compensatory Composite Indices to Measure Changes over Time: a Case Study

Matteo Mazziotta[1] | *Italian National Institute of Statistics, Rome, Italy*
Adriano Pareto[2] | *Italian National Institute of Statistics, Rome, Italy*

Abstract

Composite indices are increasingly recognized as a useful tool to measure socio-economic phenomena such as quality of life, competitiveness, development, and poverty. Considerable attention has been devoted in recent years to the methodological issues associated with composite index construction, particularly non-compensability and comparability of the data over time. In this paper, we compare two non-compensatory composite indices for measuring multidimensional phenomena and monitoring their changes over time: the Adjusted Mazziotta-Pareto Index (AMPI) and the Mean-Min Function (MMF). The AMPI is a non-linear composite index that rewards the units with balanced values of the individual indicators. The MMF is a two-parameter function that allows compensability among dimensions with a cost that increases with unbalance and can be seen as an intermediate case between a compensatory and a full non-compensatory index. An application to a set of individual indicators of development in the Italian regions is also presented.

Keywords	***JEL code***
Composite index, compensability, normalization, aggregation, ranking	*C43, I31*

INTRODUCTION

In the last years, a large number of composite indices to assess countries, according to some socio-economic measure, have been proposed in literature (Bandura, 2008). Composite indices are based on several individual indicators or sub-indices (pillars). These indicators or sub-indices are aggregated by analytical methods to give an overall score for each country or geographical area. The results are used to either create a ranking or to simply summarize the data (Freudenberg, 2003; OECD, 2008).

However, there is no part of the composite index construction that cannot be questioned. For example, additive methods assume a full compensability among the different components of the index (e.g., a high

[1] ISTAT, Viale Oceano Pacifico 171, Rome, Italy.
[2] ISTAT, Viale Oceano Pacifico 171, Rome, Italy. Corresponding author: e-mail: pareto@istat.it.

GDP per capita may offset any educational deficit and vice versa), but a complete compensability among the main dimensions of the phenomenon is often not desirable (Munda and Nardo, 2009). For this reason, more and more often a non-compensatory approach has been adopted. For example, in 2010, the aggregation method of the United Nations' *Human Development Index* (HDI) was changed from the arithmetic mean to the geometric mean in order to penalize unbalanced or skewed development across dimensions (UNDP, 2010). Another important issue is the level of comparability of the data over time (Tarantola, 2008). All the methods allow for space comparisons, whereas time comparisons may be difficult to make or to interpret. For example, standardization with respect to the mean and standard deviation allows the performance of countries to be followed over time only in relative terms, whereas it is not possible to appreciate any absolute change.

In this work, we compare two non-compensatory composite indices which allow for time comparisons in absolute terms: the Adjusted Mazziotta-Pareto Index (AMPI) and the Mean-Min Function (MMF).

The AMPI[3] is a non-linear composite index which, starting from a linear aggregation, introduces a penalty for the units with unbalanced values of the indicators. It is composed of two parts (a measure of the mean level and a measure of the amount of unbalance) and, differently from other methods, may be used for constructing both 'positive' and 'negative' composite indices[4] (Mazziotta and Pareto, 2013c).

The MMF is an intermediate case between arithmetic mean, according to which no unbalance is penalized, and min function, according to which the penalization is maximum, because the other values cannot increase the value of the index. It depends on two parameters that are respectively related to the intensity of penalization of unbalance and intensity of complementarity between indicators (Casadio Tarabusi and Guarini, 2013).

In Section 1, the main steps to implement a composite index are reported and some methodological issues, such as non-compensability and comparability of the data over time, are discussed. In Sections 2 and 3, a brief description of AMPI and MMF is presented. In Section 4, an empirical comparison is made by using a set of regional indicators of development in Italy, in 2004 and 2011. Finally, some comments about the results are given.

1 CONSTRUCTING A COMPOSITE INDEX

Constructing a composite index is a complex task. Its phases involve several alternatives and possibilities that affect the quality and reliability of the results. The main problems, in this approach, concern the choice of theoretical framework, the availability of the data, the selection of the more representative indicators and their treatment in order to compare and aggregate them.

It is possible, shortly, to identify the following steps to do (Salzman, 2003; OECD, 2008; Mazziotta and Pareto, 2013c):

1. *Defining the phenomenon to be measured*. The definition of the concept should give a clear sense of what is being measured by the composite index. It should refer to a theoretical framework, linking various sub-groups and underlying indicators. If causality is from the concept to the indicators we have a *reflective* measurement model; if causality is from the indicators to the concept we have a *formative* model (Diamantopoulos, 2008).

[3] The AMPI has been proposed within the BES Project. The goal of this project – born of a joint initiative of the Italian National Institute of Statistics (Istat) and National Council for Economy and Labour (Cnel) – is to measure equitable and sustainable well-being in Italy.

[4] A composite index is 'positive' if increasing values of the index correspond to positive variations (i.e., an improvement) of the phenomenon (e.g., well-being). On the contrary, a composite index is 'negative' if increasing values of the index correspond to negative variations (i.e., a worsening) of the phenomenon (e.g., poverty).

2. *Selecting a group of individual indicators*. Ideally, indicators should be selected according to their relevance, analytical soundness, timeliness, accessibility and so on. The selection step is the result of a trade-off between possible redundancies caused by overlapping information and the risk of losing information. A statistical approach to the choice of indicators involves calculating the correlation between potential indicators, and including the ones that are less correlated in order to minimize redundancy.
3. *Normalizing the individual indicators*. This step aims to make the indicators comparable as they often have different measurement units. Another motivation for the normalization is the fact that some indicators may be positively correlated with the phenomenon to be measured (positive 'polarity'), whereas others may be negatively correlated with it (negative 'polarity'). We want to normalize the indicators so that an increase in the normalized indicators corresponds to increase in the composite index. There are various methods of normalization, such as *ranking*, *re-scaling* (or Min-Max), *standardization* (or *z*-scores) and 'distance' from a reference (or *indicization*).
4. *Aggregating the normalized indicators*. It is the combination of all the components to form one or more composite indices (mathematical functions). Different aggregation methods are possible. The most used are additive methods that range from summing up unit ranking in each indicator to aggregating weighted transformations of the original indicators. Multivariate techniques as *Principal Component Analysis* (PCA) are also often used (Dunteman, 1989).[5]

Aggregation step has always been an interesting but controversial topic in composite index construction (Saltelli, 2007). A fundamental issue concerning the aggregation is the degree of compensability or substitutability of the individual indicators or pillars. Compensability among indicators is defined as the possibility of compensating any deficit in one dimension with a suitable surplus in another. Thus we can define an aggregation approach as compensatory or non-compensatory depending on whether it permits compensability or not (Casadio Tarabusi and Guarini, 2013). Compensability is closely related with the concept of unbalance, i.e., a disequilibrium among the indicators that are used to build the composite index. In a non-compensatory approach, all the dimensions of the phenomenon must be balanced and an aggregation function that takes unbalance into account, in terms of penalization, is often used (unbalance-adjusted function). A compensatory approach involves the use of linear functions, such as the arithmetic mean that ignores unbalances. A non-compensatory approach generally requires unbalance-adjusted functions, such as the AMPI and the MMF. *Multi-Criteria Analysis* (MCA) can also be used (Munda and Nardo, 2009). However, the MCA provides results in terms of ranks, and not of an index, so the researcher can only follow the unit rankings though time (Booysen, 2002).

Another important issue concerning composite index construction is the level of comparability of the data across countries and over time. Comparisons over time may be absolute or relative (Mazziotta and Pareto, 2013a). We say that a time comparison is 'relative' when the composite index values, at time *t*, depend on one or more endogenous parameters (e.g., mean and variance of the individual indicators at time *t*). Similarly, we say that a time comparison is 'absolute' when the composite index values, at time *t*, depend on one or more exogenous parameters (e.g., minimum and maximum of the individual indicators fixed by the researcher). Comparability of the values of a composite index firstly depends on the normalization method. *Ranking* and *standardization* allow only for relative comparisons since they are exclusively based on values of the individual indicators at the time of reference. Other methods, such as *re-scaling* and *indicization*, require that the minimum and maximum (e.g., the 'goalposts' of the HDI) or

[5] Note that normalization and aggregation are interconnected issues. For example, if the individual indicators are transformed in z-scores, they cannot be aggregated by a geometric mean because it is defined only for sets of positive values. Furthermore, some methods perform both tasks simultaneously (e.g., PCA).

the base of index numbers are independent from the time of reference in order to perform comparisons in absolute terms (Tarantola, 2008).

2 THE ADJUSTED MAZZIOTTA-PARETO INDEX

The AMPI is a non-compensatory composite index based on a *re-scaling* of the individual indicators in the range (70; 130) according to two 'goalposts', i.e., a minimum and a maximum value which represent the possible range of each variable for all time periods and for all units.

Let $\mathbf{X} = \{x_{ijt}\}$ be a three-way array (or three-dimensional matrix) of size n (number of units) x m (number of indicators) x p (numbers of time periods). A normalized array $\mathbf{R} = \{r_{ijt}\}$ is calculated as follow:

$$r_{ijt} = \frac{(x_{ijt} - \mathrm{Min}_{x_j})}{(\mathrm{Max}_{x_j} - \mathrm{Min}_{x_j})} 60 + 70 , \qquad (1)$$

where x_{ijt} is the value of indicator j for unit i, at time t, and Min_{x_j} and Max_{x_j} are the 'goalposts' for the indicator j. If the indicator j has negative 'polarity', the complement of (1) with respect to 200 is computed.

Denoting with $\mathrm{M}_{r_{it}}$ and $\mathrm{S}_{r_{it}}$, respectively, the mean and the standard deviation of the normalized values for unit i, at time t, the generalized form[6] of the AMPI is given by:

$$\mathrm{AMPI}_{it}^{+/-} = \mathrm{M}_{r_{it}} \pm \mathrm{S}_{r_{it}} \mathrm{cv}_{it} ,$$

where $\mathrm{cv}_{it} = \mathrm{S}_{r_{it}} / \mathrm{M}_{r_{it}}$ is the coefficient of variation for unit i, at time t, and the sign ± depends on the kind of phenomenon to be measured. If the composite index is 'positive' then the AMPI^- is used, else the AMPI^+ is used (De Muro et al., 2011).

To facilitate the interpretation of results, it is possible to choose the 'goalposts' so that 100 represents a reference value (e.g., the average in a given year).

A simple procedure for setting the 'goalposts' is the following.

Let Ref_{x_j} be the reference value for indicator j. Denoting with $\mathrm{Inf}_{x_j} = \min_{it}\{x_{ijt}\}$ and $\mathrm{Sup}_{x_j} = \max_{it}\{x_{ijt}\}$, the 'goalposts' are defined as:

$$\begin{cases} \mathrm{Min}_{x_j} = \mathrm{Ref}_{x_j} - \Delta_{x_j} \\ \mathrm{Max}_{x_j} = \mathrm{Ref}_{x_j} + \Delta_{x_j} \end{cases}$$

where $\Delta_{x_j} = (\mathrm{Sup}_{x_j} - \mathrm{Inf}_{x_j})/2$.[7]

The AMPI allows to compare the trends of the various units over time and it may be simultaneously applied to different type of units (e.g., countries, regions, cities) without loss of comparability.

3 THE MEAN-MIN FUNCTION

The MMF is a two-parameter function that incorporates the two extreme cases of penalization of unbalance: the zero penalization represented by the arithmetic mean (complete compensability) and the maximum penalization represented by the minimum function (full non-compensability). All other possible cases are intermediate.

Given a normalized three-way array $\mathbf{Z} = \{z_{ijt}\}$, the MMF is defined as:

$$\mathrm{MMF}_{it} = \mathrm{M}_{z_{it}} - \alpha \left(\sqrt{(\mathrm{M}_{z_{it}} - \min_j\{z_{ijt}\})^2 + \beta^2} - \beta \right) \qquad (0 \le \alpha \le 1;\ \beta \ge 0) \qquad (2)$$

[6] It is a generalized form since it includes 'two indices in one'.
[7] Normalized values will fall approximately in the range (70; 130).

where $M_{z_{it}}$ is the mean of the normalized values for unit i, at time t, and the parameters α and β are respectively related to the intensity of penalization of unbalance and intensity of complementarity between indicators.

The function reduces to the arithmetic mean for $\alpha = 0$ (in this case β is irrelevant) and to the minimum function for $\alpha = 1$ and $\beta = 0$. So, the interval of definition of the values of the MMF is: $\min_j\{z_{ijt}\} \leq \text{MMF}_{it} \leq M_{z_{it}}$.

The MMF has some properties that other important unbalance-adjusted functions lack, such as an unrestricted domain that is independent from the choice of the normalization procedure. By choosing the values of parameters appropriately one should obtain the aggregation function that best suits the specific theoretical approach. However, there is not a general rule for tuning these values (Mazziotta and Pareto, 2013b).

4 AN APPLICATION TO REAL DATA

In order to compare AMPI and MMF, an application to a set of indicators of development in the Italian regions, in 2004 and 2011, is presented. Five basic dimensions are considered: Health, Income, Work, Education and Environment.

The variables used are the following:[8]

I1) 'Life expectancy at birth', expressed in years (positive polarity);

I2) 'Income distribution inequality' – Gini coefficient (negative polarity);

I3) 'Employment rate for people aged 20–64', expressed in percentage (positive polarity);

I4) 'People aged 25–64 with low education level', expressed in percentage (negative polarity);

I5) 'Greenhouse gas emissions', expressed in CO_2 equivalent tons per capita (negative polarity).

In Table 1 is reported the data matrix, of size 22 (number of regions plus national average) x 5 (number of indicators of development) x 2 (numbers of years).

Table 1 Individual indicators of development in the Italian regions – years 2004, 2011

Region	2004					2011				
	I_1	I_2	I_3	I_4	I_5	I_1	I_2	I_3	I_4	I_5
Piemonte	80.6	0.309	66.9	52.0	9.76	81.8	0.303	68.4	42.7	7.13
Valle d'Aosta	80.6	0.296	70.7	54.9	6.81	81.8	0.282	71.2	48.3	4.95
Liguria	80.9	0.314	63.5	44.2	12.31	81.6	0.341	67.4	37.1	9.08
Lombardia	81.0	0.320	69.1	49.3	9.59	82.3	0.291	69.0	41.6	8.39
Bolzano/Bozen	81.2	0.298	73.0	58.1	6.10	83.2	0.256	76.0	46.3	5.50
Trento	81.2	0.271	69.6	43.3	6.10	82.8	0.274	71.0	34.2	5.50
Veneto	81.3	0.281	67.7	53.6	10.24	82.4	0.276	69.2	42.8	7.70
Friuli-V.G.	80.6	0.273	65.8	49.0	11.58	81.7	0.301	68.2	42.1	10.59
Emilia-R.	81.3	0.299	71.7	48.0	12.16	82.4	0.289	72.1	39.4	9.86
Toscana	81.6	0.268	66.8	51.7	7.56	82.6	0.283	67.6	45.0	5.87
Umbria	81.5	0.286	65.2	43.3	14.01	82.6	0.278	66.6	34.1	9.94
Marche	81.9	0.280	67.8	48.5	6.97	82.9	0.284	67.2	42.1	6.41
Lazio	80.2	0.328	62.6	41.6	7.72	81.8	0.328	63.2	33.9	6.45
Abruzzo	81.0	0.293	60.7	47.0	5.80	82.1	0.279	61.1	38.4	4.15
Molise	81.0	0.286	56.4	51.2	8.28	82.1	0.303	54.7	47.5	7.77
Campania	79.4	0.347	49.2	57.7	3.57	80.4	0.353	43.1	52.9	3.74
Puglia	81.2	0.303	48.8	60.4	14.07	82.1	0.314	48.6	54.1	11.87
Basilicata	80.5	0.298	53.6	53.0	4.66	82.0	0.344	51.7	46.1	2.93
Calabria	80.8	0.333	50.5	53.5	3.38	82.1	0.317	46.2	48.4	3.25
Sicilia	80.2	0.348	47.0	59.5	8.44	81.1	0.334	46.2	53.2	7.67
Sardegna	80.8	0.323	55.0	61.4	11.64	81.9	0.277	55.6	53.5	9.47
Italy	**80.8**	**0.328**	**61.3**	**51.9**	**8.91**	**82.0**	**0.319**	**61.2**	**44.3**	**7.43**

Source: <http://noi-italia.istat.it>

[8] Note that the purpose of the application is purely illustrative. The choice of the indicators is arbitrary and based on data availability.

Since we are measuring the development, the AMPI$^-$ is used. A MMF with proportional compensability ($\beta = 0$) is considered, for $\alpha = 0$ (MMF_1), $\alpha = 0.5$ (MMF_2) and $\alpha = 1$ (MMF_3). The normalization procedure for calculating the MMF is given by (1), thus we have $z_{ijt} = r_{ijt}$ in (2).[9] Furthermore, the 'goalposts' were set so that 100 represents the Italy's value in 2004.

Tables 2 and 3 show the final scores (value) and rankings (rank) of the Italian regions for 2004 and 2011, respectively. The mean absolute difference of rank and the Spearman rank correlation coefficient between AMPI$^-$ and MMF are also reported.

As we can see, the AMPI$^-$ is more similar to the MMF_2, i.e., the MMF with medium penalization (the mean absolute difference of rank is 0.4 for 2004 and 0.2 for 2011; the Spearman rank correlation is 0.992 for 2004 and 0.996 for 2011). This is due to the fact that both AMPI$^-$ and MMF_2 are based on a penalty function (calculated in a different way) subtracted to the arithmetic mean.

The results are very different if we compare the AMPI$^-$ and the MMF_3, i.e., the MMF with maximum penalization (the mean absolute difference of rank is 2.6 for 2004 and 1.8 for 2011; the Spearman rank correlation is 0.813 for 2004 and 0.913 for 2011). In this case, we have large differences of rank and almost all the regions have a different position in the two rankings. For example, in 2004, Umbria ranks 10th with the AMPI$^-$ and 20th according to the MMF3, since the minimum function does not allow indicators I_1–I_4 to compensate for the 'bad' value of I_5.

Finally, differences between AMPI$^-$ and MMF_1, i.e., the MMF with zero penalization or arithmetic mean, represent a middle result between the previous ones (the mean absolute difference of rank is 0.5 both for 2004 and 2011; the Spearman rank correlation is 0.988 for 2004 and 0.991 for 2011).

Table 2 Composite indices of development in the Italian regions – year 2004

Region	AMPI$^-$		MMF_1		MMF_2		MMF_3		Difference of rank		
	Value	Rank	Value	Rank	Value	Rank	Value	Rank	AMPI$^-$ –MMF_1	AMPI$^-$ –MMF_2	AMPI$^-$ –MMF_3
Piemonte	102.1	14	102.5	15	102.3	15	95.4	6	–1	–1	8
Valle d'Aosta	106.3	6	107.4	6	106.9	6	93.3	7	0	0	–1
Liguria	100.9	16	102.2	16	101.6	16	81.7	15	0	0	1
Lombardia	104.4	9	104.7	12	104.6	11	96.4	5	–3	–2	4
Bolzano/Bozen	108.0	5	109.5	5	108.8	5	86.4	11	0	0	–6
Trento	117.1	1	117.9	1	117.5	1	105.6	2	0	0	–1
Veneto	105.9	7	107.4	7	106.7	7	92.9	8	0	0	–1
Friuli-V.G.	103.5	12	105.9	10	104.7	10	85.6	13	2	2	–1
Emilia-R.	105.3	8	107.0	8	106.2	8	82.5	14	0	0	-6
Toscana	112.1	3	113.4	3	112.7	3	100.4	3	0	0	0
Umbria	103.7	10	106.8	9	105.3	9	72.5	20	1	1	–10
Marche	114.6	2	115.1	2	114.9	2	107.4	1	0	0	1
Lazio	103.0	13	104.1	13	103.6	13	89.7	10	0	0	3
Abruzzo	109.4	4	110.0	4	109.7	4	98.8	4	0	0	0
Molise	103.6	11	104.8	11	104.2	12	91.0	9	0	–1	2
Campania	87.8	19	91.8	18	89.8	19	76.9	18	1	0	1
Puglia	87.3	20	90.5	20	88.9	20	72.2	21	0	0	–1
Basilicata	101.7	15	103.7	14	102.7	14	85.8	12	1	1	3
Calabria	97.8	17	100.4	17	99.1	17	80.2	16	0	0	1
Sicilia	86.5	21	87.5	21	87.0	21	73.7	19	0	0	2
Sardegna	90.3	18	91.2	19	90.8	18	79.2	17	–1	0	1
Italy	**100.0**		**100.0**		**100.0**		**100.0**				
Mean absolute difference									**0.5**	**0.4**	**2.6**
Rank correlation									**0.988**	**0.992**	**0.813**

Source: Elaboration of the authors

[9] We normalized the individual indicators by a re-scaling in order to perform time comparisons in absolute terms.

Table 3 Composite indices of development in the Italian regions – year 2011

Region	AMPI⁻		MMF_1		MMF_2		MMF_3		Difference of rank		
	Value	Rank	Value	Rank	Value	Rank	Value	Rank	AMPI⁻ $-MMF_1$	AMPI⁻ $-MMF_2$	AMPI⁻ $-MMF_3$
Piemonte	114.5	11	114.6	11	114.6	11	109.6	5	0	0	6
Valle d'Aosta	117.8	8	118.2	8	118.0	8	107.8	6	0	0	2
Liguria	107.6	14	109.3	14	108.4	14	91.8	13	0	0	1
Lombardia	116.5	10	117.1	10	116.8	10	102.8	8	0	0	2
Bolzano/Bozen	126.6	2	127.7	1	127.2	2	112.2	2	1	0	0
Trento	127.1	1	127.6	2	127.3	1	117.6	1	–1	0	0
Veneto	118.9	6	119.5	7	119.2	6	106.5	7	–1	0	–1
Friuli-V.G.	110.1	13	111.0	13	110.6	13	90.9	14	0	0	–1
Emilia-R.	116.7	9	117.9	9	117.3	9	94.9	11	0	0	–2
Toscana	119.2	5	119.5	6	119.3	5	111.5	3	–1	0	2
Umbria	118.0	7	120.1	5	119.0	7	94.5	12	2	0	–5
Marche	120.4	3	120.9	3	120.6	3	110.6	4	0	0	–1
Lazio	112.6	12	114.3	12	113.4	12	99.9	9	0	0	3
Abruzzo	119.7	4	120.8	4	120.3	4	99.6	10	0	0	–6
Molise	106.5	15	107.6	15	107.1	15	87.8	16	0	0	–1
Campania	89.6	21	93.8	21	91.7	21	66.7	21	0	0	0
Puglia	94.3	19	96.8	19	95.5	19	76.7	18	0	0	1
Basilicata	103.9	16	107.1	17	105.5	17	82.4	17	–1	–1	–1
Calabria	103.7	18	107.3	16	105.5	16	72.4	19	2	2	–1
Sicilia	93.6	20	95.1	20	94.3	20	72.3	20	0	0	0
Sardegna	103.9	17	106.1	18	105.0	18	89.6	15	–1	–1	2
Italy	**109.0**		**109.5**		**109.2**		**99.7**				
Mean absolute difference									**0.5**	**0.2**	**1.8**
Rank correlation									**0.991**	**0.996**	**0.913**

Source: Elaboration of the authors

Table 4 Composite indices of development in the Italian regions – variations 2004–2011

Region	AMPI⁻		MMF_1		MMF_2		MMF_3		Difference of rank		
	Value	Rank	Value	Rank	Value	Rank	Value	Rank	AMPI⁻ $-MMF_1$	AMPI⁻ $-MMF_2$	AMPI⁻ $-MMF_3$
Piemonte	12.5	5	12.1	5	12.3	5	14.2	4	0	0	1
Valle d'Aosta	11.5	7	10.7	9	11.1	8	14.5	3	–2	–1	4
Liguria	6.6	15	7.1	13	6.9	13	10.1	11	2	2	4
Lombardia	12.1	6	12.3	4	12.2	6	6.4	12	2	0	–6
Bolzano/Bozen	18.6	1	18.2	1	18.4	1	25.7	1	0	0	0
Trento	9.9	10	9.7	11	9.8	11	12.0	7	–1	–1	3
Veneto	13.0	4	12.1	6	12.5	4	13.7	5	–2	0	–1
Friuli-V.G.	6.5	16	5.1	18	5.8	17	5.3	13	–2	–1	3
Emilia-R.	11.4	8	11.0	7	11.2	7	12.4	6	1	1	2
Toscana	7.1	13	6.1	16	6.6	15	11.0	8	–3	–2	5
Umbria	14.3	2	13.3	3	13.8	3	21.9	2	–1	–1	0
Marche	5.7	18	5.7	17	5.7	18	3.2	15	1	0	3
Lazio	9.6	11	10.1	10	9.8	10	10.2	10	1	1	1
Abruzzo	10.4	9	10.8	8	10.6	9	0.8	16	1	0	–7
Molise	2.9	19	2.8	20	2.9	19	–3.2	18	–1	0	1
Campania	1.7	21	2.1	21	1.9	21	–10.2	21	0	0	0
Puglia	7.0	14	6.3	15	6.6	14	4.4	14	–1	0	0
Basilicata	2.1	20	3.3	19	2.7	20	–3.4	19	1	0	1
Calabria	5.9	17	6.9	14	6.4	16	–7.8	20	3	1	–3
Sicilia	7.1	12	7.6	12	7.3	12	–1.4	17	0	0	–5
Sardegna	13.5	3	14.9	2	14.2	2	10.4	9	1	1	–6
Italy	**9.0**		**9.5**		**9.2**		**–0.3**				
Mean absolute difference									**1.2**	**0.6**	**2.7**
Rank correlation									**0.969**	**0.990**	**0.839**

Source: Elaboration of the authors

The variations over 2004–2011 can be evaluated in Table 4.

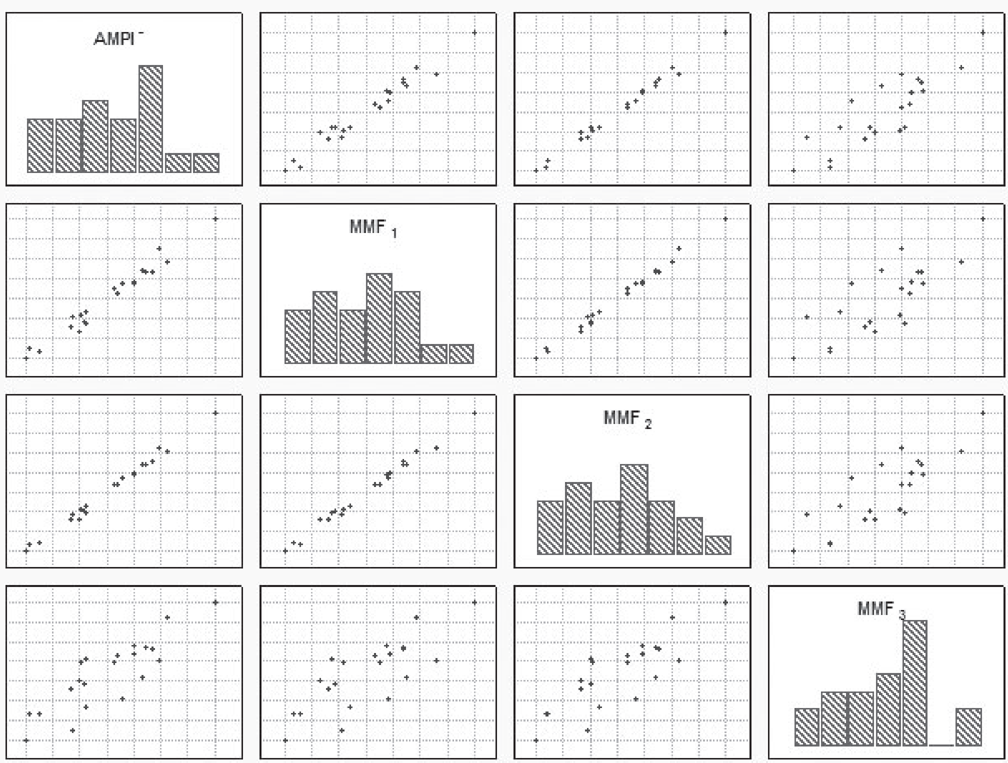
Note that, while the AMPI$^-$, MMF_1 and MMF_2 increase of about 9%, at national level, the MMF_3 decreases of 0.3%. In this case, in fact, the minimum value of the normalized indicators is considered as score and the regions of the South Italy are particularly penalized. In particular, Campania drops from 76.9, in 2004, down to 66.7, in 2011, according to the MMF_3 (variation of –10.2), whereas Calabria shows a reduction from 80.2 to 72.4 (variation of –7.8).

The differences between the two investigated computation methods do not change, by comparing the rankings of the variations over time (the mean absolute difference of rank between AMPI$^-$ and MMF_1 is 1.2, between AMPI$^-$ and MMF_2 is 0.6, between AMPI$^-$ and MMF_3 is 2.7; the Spearman rank correlation between AMPI$^-$ and MMF_1 is 0.969, between AMPI$^-$ and MMF_2 is 0.990, between AMPI$^-$ and MMF_3 is 0.839).

In order to assess the consistency of the results across regions and over time, a matrix-plot is shown in Figure 1, where the variations of the four composite indices are 'crossed' and the crossing of each pair is represented by one *x-y* scatter-plot.

In general, the variations are concordant (most of the points are located around a straight line at 45 deg.) and the nearest results are obtained with AMPI$^-$ and MMF_2, as we have seen already. Note that the use of the minimum function (MMF_3) produces the most irregular distribution of the variations, since no averaging of normalized indicators is made (with or without penalization).

Figure 1 Matrix-plot of the composite indices in the Italian regions – variations 2004–2011



Source: Elaboration of the authors

CONCLUSION

Most of the socio-economic phenomena such as quality of life, competitiveness, development, and poverty have a multidimensional nature and require the definition of a set of individual indicators in order to be properly assessed.

Individual indicators are often summarized and a composite index is created. However, the procedure for constructing a composite index is very far from being aseptic and requires a number of subjective decisions to be taken.

Non-compensability and comparability of the data over time are central issues in the construction of composite indices. Non-compensatory composite indices may be obtained by unbalance-adjusted functions, whereas the question of comparability mainly depends on the normalization method. A *re-scaling* or Min-Max transformation can satisfy this need, when the minimum and maximum values, for each indicator, are found across all the considered time periods or, alternatively, are fixed by the researcher.

In this paper, a comparison between two different non-compensatory approaches for monitoring multidimensional phenomena over time is made. The AMPI is a non-linear composite index that normalizes individual indicators by a *re-scaling* in the range (70; 130), where 100 represents a reference value, and aggregates them with a arithmetic mean adjusted by a penalty function related to the amount of unbalance. The MMF is a two-parameter function that poses no constraint to the choice of the most appropriate normalization procedure, and allows the user to adapt it to different kinds of analysis (with progressive or proportional compensability, with complete or incomplete compensability).

The application to real data shows that the AMPI is very similar to an 'intermediate' MMF. However, it respects both the constraint of time comparisons and the non-compensability by using an easier and more transparent methodology than the MMF.

Aside from the procedure used, composite indices provide an irreplaceable contribution to simplification, but they are based on methods that flatten the information and can lead to a myopic reading of reality, especially if they are not supported by an adequate selection and interpretation of the individual indicators.

Therefore, in order to obtain valid and reliable results, it is absolutely essential to support the choice of the set of individual indicators with an appropriate theoretical framework that defines the social reality in each of its dimensions.

ACKNOWLEDGMENT

The paper is the result of the common work of the authors: in particular M. Mazziotta has written Sections 1, 3 and Conclusion; A. Pareto has written Introduction and Sections 2, 4.

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