Hedonic Price Methods and Real Estate Price Index: an Explanatory Study for Apartments Market in Belo Horizonte, Brazil, from 2004 to 2015

Luiz Andrés Ribeiro Paixão[1] | *Brazilian Institute of Geography and Statistics, Rio de Janeiro, Brazil*

Received 30.9.2022, **Accepted** (reviewed) 16.11.2022, **Published** 16.6.2023

Abstract

Brazil does not have an official real estate price index yet. A 2011 presidential decree stipulated for the Brazilian Institute of Geography and Statistics (IBGE) the assignment to create and spread a real estate price index for Brazil. In this paper, we test some different hedonic model methods to estimate quarterly price indices for apartments in Belo Horizonte, Brazil, from January 2005 to December 2015. Our goals are: i) to measure and compare the different hedonic methods; ii) to present some results that will contribute to the discussion about the development of an official real estate price index in Brazil. The empirical results corroborate the idea of intense apartment prices valuation in Belo Horizonte, mainly between 2007 and 2011, when the annual price index growth remained above 20%. These results cast light on the potential use of both hedonic methods and administrative data base to construct an official real estate price index for Brazil.

Keywords	***DOI***	***JEL code***
Price indices, hedonic price model, housing market	*https://doi.org/10.54694/stat.2022.46*	*C43, E31, R31*

INTRODUCTION

Subprime crises turned attention around the world to the real estate price dynamics question. In Brazil the recent large valuation of real estate price adds more attention to the subject. Academics, news, real estate agents, Brazilians government agencies and statistical institutes started to discuss the importance

[1] Brazilian Institute of Geography and Statistics (IBGE), Avenida República do Chile 500, 5o andar Rio de Janeiro, Brazil. E-mail: luiz.paixao@ibge.gov.br, phone: (+55)2121420194.

of having an adequate measure of real estate price over time. The Federal Government Decree number 7 565, dated 21 September, 2015, established IBGE (Brazilian Institute of Geography and Statistics) as the responsible entity to create and disseminate an official real estate price index for Brazil. Since then, IBGE and others governmental agencies have implemented studies regarding the database and methodology to construct a future Brazilian official real estate price index (Nadalin and Furtado, 2011; Santos and Salazar, 2011).

The Brazilian academy has been studying real estate price index and its application in Brazilian context. In recent years, Rozenbaum (2009), Paixão (2015), and Simões (2017) are examples of doctoral thesis related to this subject. Rozenbaum (2009) used administrative data to construct a hedonic quality adjusted price index for the city of Rio de Janeiro. Simões (2017) also measured the hedonic quality adjusted price index for Rio de Janeiro, using real estate agencies data, while Paixão (2015), also used administrative data to construct a hedonic quality adjusted price index for Belo Horizonte's city.

Some researches in this field published in Brazilian academic journals. Gonzalez (1997) estimated a simple time-dummy hedonic model to construct price index for apartments rents in Porto Alegre. The same approach was applied by Rozenbaum and Macedo-Soares (2007), to estimated real estate valuation in Rio de Janeiro's district of Barra da Tijuca, and by Paixão (2015a), to estimate real estate price indices for Belo Horizonte. Albuquerque et al. (2018) used repeated sales method to construct an index for the city of Brasilia, the capital of Brazil.

Some University's agencies like the Institute of Economic Research Foundation (FIPE), from University of São Paulo (USP), and the Institute of Economic, Administrative and Accounting Research (IPEAD), from Federal University of Minas Gerais (UFMG), released real estate price indices using stratified median methods. The widespread FIPE-ZAP real estate index is calculated by FIPE from the real estate advertised data collected in ZAP's web site platform. The Brazilian Central Bank also estimated and published a monthly stratified median real estate price index, constructed from the real estate loans data, called Financed Residential Properties index (IVG-R). Despite the importance and relevance of those indices for the society, government, academics and real estate agencies some gaps remain. None of those indices used the hedonic quality adjustment methodology, recognized as the best to deal with the nature of real estate's market data (Diewert, 2009; Hill et al., 2018). Besides that, only IPEAD uses administrative data which cover the whole transacted market.

In this paper we try to construct quarterly hedonics quality adjusted real estate prices indices for Belo Horizonte city using administrative data. For this task we will use the hedonic methods proposed by Hill's (2013), and Hill's et al. (2018). We will use the same methods used by Hill et al. (2018) in their analysis of Sidney and Tokyo markets, to produce comparable results.

The last part of the paper is structured as follows. The next section explains the different hedonic price methods used to measure the quality adjusted housing price indices. The second section is focused on the database and introduces the Brazilian city of Belo Horizonte. The objective of the third section is to test the several hedonic methods in Belo Horizonte's real estate market database. Finally, our main results are summarized in the conclusion.

1 HEDONIC QUALITY ADJUSTED REAL ESTATE PRICE METHODS

1.1 The hedonic price model

Estimating housing price indices is a complex task. Housing is a type of complex good (or service), that is, a good where each unit or model differs from the others in qualitative terms. A complex good can be described as a bundle of many characteristics (or attributes), so each unit or model is a peculiar bundle of attributes. Computing index prices for complex good necessarily means controlling the change in the good price by the change in composition of its characteristics.

The hedonic price model establishes a functional relationship between the price of the good and its characteristics. In a hedonic perspective a good is a basket of characteristics *Z*, as represented below:

$$Z = Z\left(z_{1,}z_2,\ldots,z_n\right). \tag{1}$$

The price of a good follows a hedonic function as describes in Formula (2):

$$P = P\left(z_{1,}z_2,\ldots,z_n\right). \tag{2}$$

Although only the price of the good can be observed in the market, the hedonic function establishes that the price of a good is determined by the composition of basket of characteristics. Therefore, each attribute (*i*) has an unobserved price (implicit price) that is represented by the first derivative of the hedonic function with respect to *i*.

$$p_i = \frac{\partial P}{\partial x_i}. \tag{3}$$

The seminal paper of Rosen (1974) validated the hedonic price model in theoretical terms. Empirically, Waugh (1928) was pioneer in apply hedonic regression in vegetables market study. Court (1939) used hedonic price regression to construct automobile price indices. Griliches (1958, 1961) constructed hedonic quality adjusted price indices for fertilizers and automobile markets respectively. From Griliches contributions, the application of hedonic model widespread in the academic world, covering many types of different goods and services like computers, refrigerators, fruits, musical instruments, paints etc. However, it was in the real estate market that the hedonic approach achieved its largest projection.

1.2 The hedonic quality adjusted price indexes: the real estate case

Griliches (1971) argues that a complex good's price change can be divided in two dimensions. The first is the observed price change of the good in the market. The second is the unobserved price change of the basket of characteristics. To estimate the unobserved price change, it is necessary to use the hedonic model regression as a quality adjusted factor. Discounting the price change of the attributes bundle from the observed price of the good results in a "pure" estimate of a complex good price change.

There are several ways how to construct a quality adjusted price index from the hedonic methodology. Court (1939) and Griliches (1961) already advanced some questions, like the possibility of using both cross-section regressions or time-dummy approaches. Tripllet (2004) created a taxonomy of the several hedonic methods used to compute quality adjusted price indices for technological goods. Hill (2013) applied this Tripllet's taxonomy to the housing market case.

Hill et al. (2018) using a Hill (2013) approach compiled the hedonic methods used by the European national statistics institutes. The first category embraces all indices which requires cross-section regressions models and, as a result, involves data imputation. The second category is based on time-dummy regressions.

1.2.1 Imputation approach

1.2.1.1 Repricing model

The first imputation method described by Hill et al. (2018) is the repricing method. Like Hill et al. (2018) in this study we adopted quarterly price indices as default and hedonic quality adjusted price indices could be constructed for any period. Defining the base period is the first task in the repricing model. Then, a hedonic regression is estimated for this period. The price implicit estimated in hedonic regression is used to impute prices for each subsequent quarterly. The base period can be fixed or be updated at regular time intervals.

Hill et al. (2018) recommends estimating one regression for the whole base year:

$$\ln(p)_{(1,q),h} = \sum_{c=1}^{C} \beta_{1,c} z_{(1,q),h,c} + \varepsilon_{(1,q),h}\,, \tag{4}$$

where ln(p) is the natural logarithmic of housing price in the base year (1), q is the quarterly of the sale, h is the dwelling sold and c is each characteristic of the dwelling.

Then, the implicit prices estimated ($\hat{\beta}$) are used to estimated prices for each subsequent quarterly. The repricing method is, therefore, a sort of Laspeyres index. The quality adjustment factor $QAF_{(t,q-1),(t,q)}$ is defined as the ratio of the imputed prices for adjacent quarters, q and $q-1$ for example.

$$QAF_{(t,q-1),(t,q)} = \frac{\exp\left(\sum_{c=1}^{C} \hat{\beta}_{1,c} \overline{z}_{(t,q),c}\right)}{\exp\left(\sum_{c=1}^{C} \hat{\beta}_{1,c} \overline{z}_{(t,q-1),c}\right)}. \tag{5}$$

To construct the repricing method price index (RP) a quality unadjusted price index ($QUPI_{(t,q),(tq-1)}$) defined as a ratio between geometric mean prices ($\tilde{p}$) of adjacent quarters is calculated as follows:

$$QUPI_{(t,q),(tq-1)} = \frac{\tilde{p}_{(t,q)}}{\tilde{p}_{(t,q-1)}}\,. \tag{6}$$

Finally, RP is the ratio between quality unadjusted price factor and quality adjusted price factor.

$$\frac{P_{(t,q)}}{P_{(t,q-1)}} = \frac{QUPI_{(t,q),(t,q-1)}}{QAF_{(t,q),(t,q-1)}}\,. \tag{7}$$

The main attractive feature of RP relies on the fact that it is not regression intensive. In the end, it requires only one regression (Hill et al., 2018). However, to achieve good results, Hill et al. (2018: 224) suggested that "the base year under the repricing method should be updated at regular time intervals". Italy and Luxembourg national statistics institutes are benchmark examples since both updated the base every year (Hill et al., 2018).

1.2.1.2 Average characteristic method

The average characteristic method (AC) requires, as any method, a definition of a base period. After that, the average characteristic of the dwellings sold in the base period are computed. The next step consists in estimating hedonic regressions for each subsequent period, quarterly in our case. Then the imputed prices are calculated, applying the estimated quarterly implicit prices on the average characteristics of the base period.

Following Hill et al. (2018), the European national statistical institutes calculated the basket of average characteristics for a whole year (base year), $\overline{z}_{t-1}$. In this line, the European national statistical institutes adopted a Laspeyres version of AC[1]. The base is updated every year. Then, a hedonic regression is estimated for each quarter (q) of the following year (t):

$$\ln(p)_{(t,q),h} = \sum_{c=1}^{C} \beta_{t,c} z_{(t,q),h,c} + \varepsilon_{(t,q),h}. \tag{8}$$

The quality adjusted price index estimated by AC is given as follows:

$$\frac{P_{(t,q)}}{P_{(t,q-1)}} = \frac{\exp(\sum_{c=1}^{C} \hat{\beta}_{(t,q),c} \overline{z}_{t-1,c})}{\exp(\sum_{c=1}^{C} \hat{\beta}_{(t,q-1),c} \overline{z}_{t-1,c})}\,. \tag{9}$$

1.2.1.3 Hedonic imputation method

Real estate is a threshold situation of complex goods. Each unity of real estate differs from the other. Added to this, the set of dwellings sale in one period differs from the set of dwellings sale in other periods. Therefore, it is not possible to construct a basket of dwellings to follow over time. The hedonic imputation method is a way to estimate the price of each dwelling sold in t would have in another period, $t + 1$ for example. According to Hill et al. (2018: 225–6): once a hedonic model has been estimated, it allows one to ask counterfactual questions such as what a particular dwelling actually sold in say period t would have sold for instead in period $t + 1$.

Like the AC, one regression as (8) is estimated for each period. The regression in $t + 1$ is used to impute the price in $t + 1$ for each observed transacted dwelling in t. Likewise, the regression in t is used to impute the price in t for each observed transacted dwelling in $t + 1$. To construct the index, Hill (2013) recommended to use the regression estimated price in t instead of the observed price for each observed transacted dwelling in t. Such procedure is known as double imputation. From the hedonic imputation method geometric Laspeyres (GL), geometric Paasche (GP) and Tornqvist prices indices can be extracted.

Few European national statistical institutes use hedonic imputation methods. Hill et al. (2018) follow the German version of double imputation Tornqvist (DIT). From a set of regressions like (8) the GL, GP and DIT are estimated as follows:

$$GL = \frac{exp\sum_{c=1}^{C}\hat{\beta}_{(t,q)}\overline{z}_{(t-1,c)}}{exp\sum_{c=1}^{C}\hat{\beta}_{(t,q-1)}\overline{z}_{(t-1,c)}}, \tag{10}$$

$$GP = \frac{exp\sum_{c=1}^{C}\hat{\beta}_{(t,q)}\overline{z}_{(t,c)}}{exp\sum_{c=1}^{C}\hat{\beta}_{(t,q-1)}\overline{z}_{(t,c)}}, \tag{11}$$

$$DIT = \sqrt{GL * GP}. \tag{12}$$

1.2.2 Time-dummy approach

1.2.2.1 Simple time-dummy

The time-dummy approach consists in constructing price indices from the estimated parameter of a dummy time variable in a hedonic regression. Usually, the first period in the series is used as the base. The simple time-dummy model (TD) requires only one regression and it is the simples and most intuitive hedonic method. The typical TD regression is illustrated as follows:

$$\ln(p)_{(t,q),h} = \sum_{c=1}^{C}\beta_{t,c}z_{(t,q),h,c} + \gamma_{t,q}D_{t,q} + \varepsilon_{(t,q),h}, \tag{13}$$

where $D_{t,q}$ is a time dummy for each period and $\gamma_{t,q}$ is the prince index estimated for the period .

Despite its simplicity, there are some pitfalls in using TD (Hill, 2013). First, the TD does not allow the implicit price changes over time. As a result, the longer the series, the worst will the TD prince index estimations be. For national statistical institutes, TD is not recommended because it does not follow the temporal fixity criterium, as defined by Hill (2004). According to this criterium, once an index has already been disseminated by the national statistical institute it should remain unchanged when new data becomes available. Using the single regression TD approach, when new data is added, a new estimation of (13) changes all parameters $\gamma_{t,q}$ previously disseminated.

1.2.2.2 Rolling time-dummy method

The hedonic rolling time dummy method (RTD) consists in estimated hedonics time-dummy regressions for subperiods instead of only one regression for the whole period. The limiting case occurs when a regression is estimated for each pair of adjacent periods. Although when data points are scarce it is recommended to estimate a regression including more than one subperiod.

France and Portugal, for example, estimated price index from RTD with 2 quarter windows. In other words, both countries are using an adjacent period RTD. Other countries like Cyprus and Croatia estimated a 4 quarter windows RTD. The RTD price index is calculated from an RTD regression like (13) as showed below:

$$\frac{P_{q+1}}{P_q} = \frac{\exp\left(\hat{\gamma}_{q+1}\right)}{\exp\left(\hat{\gamma}_q\right)}. \tag{14}$$

2 THE DATA

2.1 Belo Horizonte, Brazil: an overview

Belo Horizonte, the capital of the State of Minas Gerais, is an important economic, politic and cultural center in Brazil. According to 2010 Brazilian Census (IBGE, 2010), Belo Horizonte had a population of almost 2.4 million and was the 6th most populous city in Brazil. The Metropolitan Area (MA) of Belo Horizonte, in turn, had a population of 5.4 million and was the 3th most populous MA in Brazil.

Belo Horizonte was a planned city, conceived to replace Ouro Preto as the capital of Minas Gerais, and was founded in 1897. Nowadays, the planned area corresponds to downtown and its nearby districts bounded by Contorno Avenue. Like Aguiar et al. (2014, 119) resumes: this planning created a center-periphery radial model for the city, which concentrated urban services and urban infrastructure in particular areas, and reinforced social disparities.

From an administrative point of view, the space of Belo Horizonte is divided into Districts (487), Planning Units (82), and Regionals (9). Following Villaça (1998), historically, the Central-South city's Regional (Regional Centro-Sul) concentrated the elite's neighborhoods. Nowadays, some bordering Central-South Regional districts in the West Regional (Regional Oeste) are also occupied by Belo Horizonte's elite. Nonetheless, a few elite's districts are in Pampulha Regional (Regional Pampulha), in the north of the city, surrounding Pampulha's lagoon.

2.2 Database

In Brazil all real estate transactions are subjected to the Real Estate Transfer Tax[2] (RETT) and its collection is in charge of municipalities. We used Belo Horizonte municipality's RETT as our dataset, covering the period from 2004 to 2015, collected by IPEAD/UFMG. The RETT dataset contains the value of transaction, type of building, area, age, quality of building finishing material, zoning and location (district). The type of building includes apartments, houses and commercial real estate. In this paper we analyzed only apartments market. Further we shall expand the analysis for house and commercial real estate markets.

Tables 1, 2 and 3 resume the data. There were 266 529 observations in our dataset for the whole period. The mean apartment price was around R$ 218 149 (approximately U$ 57 000) and the standard deviation was 235 011, indicating a high dispersion of this variable. Apartments sold in Belo Horizonte were fairly big and new, the mean area and age was 120 m^2 and 13 years, respectively. Most of apartments were

[2] Imposto de Transmissão Imobiliária Inter-Vivos (ITBI).

classified as normal in terms of quality of building finishing material and this variable classified the quality into 5 categories. Ordering from the top there were the following categories: luxury, high, normal, low and popular. Most observations where located in the Center-South (Centro-Sul) and West (Oeste) Regionals.

Table 1 Descriptive statistics for apartments in municipality of Belo Horizonte: 2004–2015

Year	Observation	Value (reais)			Area (m^2)			Age (years)		
		Mean	Median	Standard deviation	Mean	Median	Standard deviation	Mean	Median	Standard deviation
2004	17 767	90 682	60 122	91 427	122.0	102.9	70.6	12.4	8.0	11.7
2005	39 606	93 363	61 746	99 219	118.7	99.1	68.9	13.7	10.0	12.0
2006	37 614	107 572	70 196	117 326	120.4	102.1	69.2	14.3	10.0	12.2
2007	19 664	125 848	83 000	129 249	120.9	103.2	68.4	14.3	10.0	12.2
2008	19 224	155 117	100 000	156 457	121.4	101.7	70.6	14.5	10.0	12.5
2009	18 272	186 106	130 000	172 571	118.4	98.9	69.8	14.0	10.0	12.8
2010	21 177	227 110	160 764	201 441	114.4	94.4	68.5	12.1	8.0	13.1
2011	19 257	297 875	220 000	247 637	118.9	98.3	70.3	11.7	7.0	13.3
2012	18 408	362 735	274 412	284 955	123.8	107.0	70.9	11.8	6.0	13.5
2013	20 364	389 815	303 982	283 168	120.5	104.4	67.9	11.1	4.0	13.7
2014	19 516	418 895	330 000	291 744	120.6	105.7	66.9	10.1	2.0	13.3
2015	15 727	420 250	334 000	290 847	117.0	102.1	64.9	10.9	3.0	13.8
2004–2015	266 596	218 149	140 000	235 011	119.7	101.4	69.0	12.8	8.0	12.8

Source: IPEAD/UFMG, author's calculation

Table 2 Distribution of quality building finishing materials: Belo Horizonte, 2004–2015

Quality of building finishing materials	Mean	Standard deviation
Popular	0.02	0.15
Low	0.21	0.41
Normal	0.59	0.49
High	0.16	0.37
Luxury	0.02	0.13

Source: IPEAD/UFMG, author's calculation

Table 3 Apartments transactions – mean for regional – Belo Horizonte: 2004–2015

Regional	Mean	Standard deviation
Centro-Sul	0.21	0.31
Leste	0.09	0.28
Nordeste	0.10	0.29
Noroeste	0.11	0.31
Oeste	0.21	0.40
Pampulha	0.15	0.35
Venda Nova	0.05	0.21
Barreiro	0.04	0.18
Norte	0.04	0.18

Source: IPEAD/UFMG, author's calculation

3 QUALITY ADJUSTED PRICE INDICES FOR BELO HORIZONTE, BRAZIL

3.1 Hedonic prices indices for Belo Horizonte, Brazil

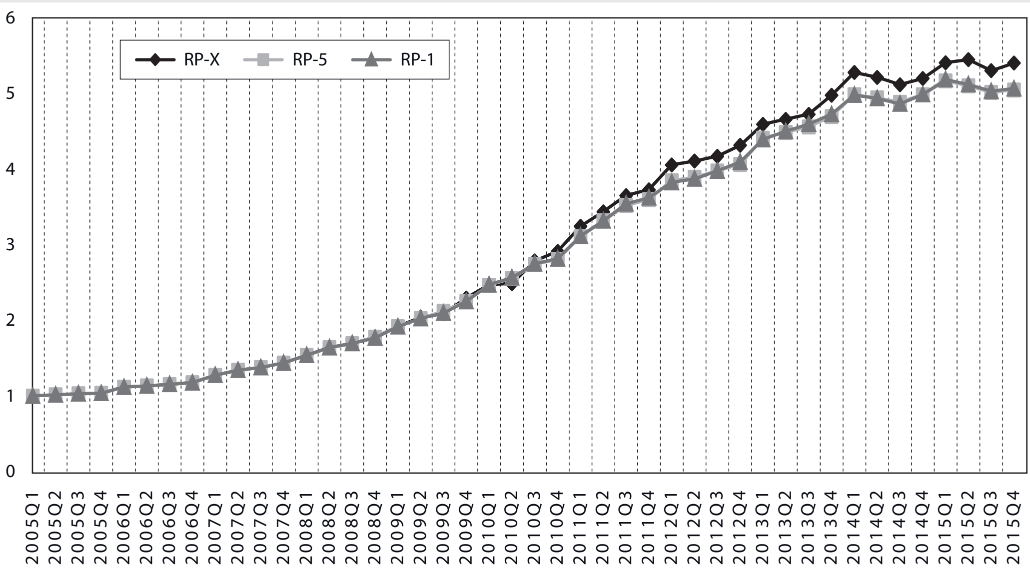
We estimated quarterly housing price indices for Belo Horizonte using the various methods discussed in Sections 1.2.1 and 1.2.2 and an UP's stratified median method (MIX-UP).[3] Since some methods were based on previous regression or mean characteristics, the 2004 data was used to compute reference baskets used in 2005. For this reason, results are represented for years 2005–2015. Following Hill et al. (2018) we estimated three forms of repricing model (RP): i) (RP-X, which uses shadow price from 2004 (no updating base year); ii) RP-5, which updates the shadows prices every five years; iii) RP-1, which updates the shadows prices annually.

The average characteristic indices (AC) were estimated with a base update every year. In AC case, the base is the one year lagged average characteristics. The double imputation indexes were calculated estimating, for some quarter set of observations, counterfactual housing's basket for previous and posterior periods. The double imputation Laspeyres (DIL), Paasche (DIP) and Tornqvist (DIT) were estimated as presented in Section 1.2.1.3. The rolling time dummy indices were estimated for 2 (RTD2) and 4 (RTD4) quarters window. Finally, we estimated an UP stratified median (MIX-UP) index, to compare the quality adjusted hedonic housing price index with a simpler and more intuitive price index. Table 6 resumes the quarter housing price indices estimated.

From the chosen period (2005–2015), the large appreciation of housing prices in Brazil was supported by estimated indices. Even so, the magnitude of the appreciation differs between methods. Using the DIP index, Belo Horizonte's house prices rose 383.6% in contrast with DIL which shows an increase of 435.7%.

RIP-X, which uses the shadow price from 2004, was apartheid from RP-5, which changes the base every five years, and RP-1, which updates shadow prices every year (Figure 1). From our Belo Horizonte's

Figure 1 Repricing Indices for apartments in Belo Horizonte (2005Q1 = 100)



Source: IPEAD/UFMG, author's calculation

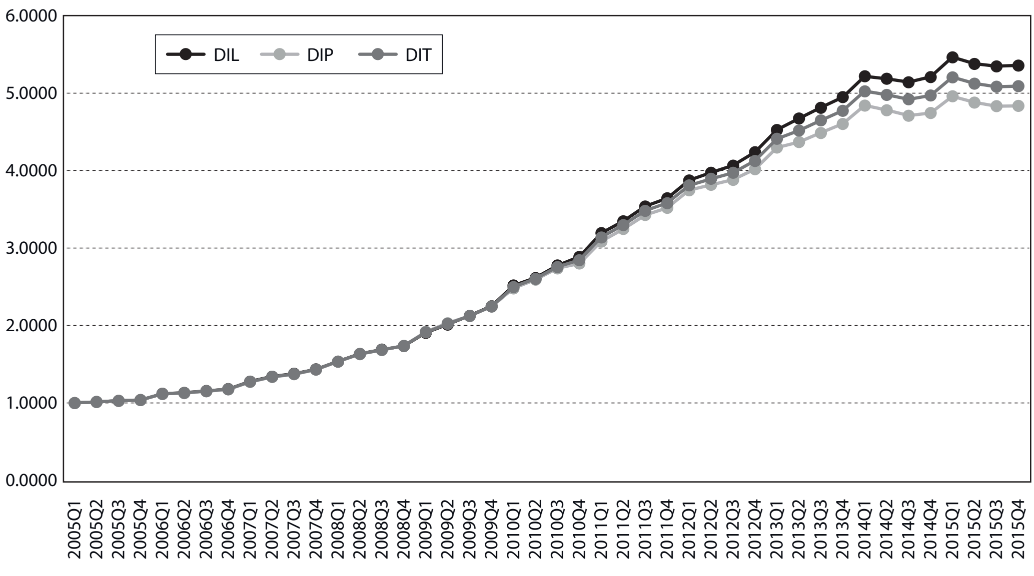
3 The control variables of hedonic price regression are resumed in Tables A1, A2, A3 and A4 in the Annex. We illustrated the results for control variables using a time-dummy regression (Formula 13) for the whole period in Table A5, also in the Annex. The results for the other regressions are available from the author.

database, the RIP-X seems not be an appropriate index due to its failure to control shadow prices change over time. RP-5 and RP-1 lines were close each other, highlighting the importance to update the base year from time to time. Since RP-1 is the more flexible RP index it will be used in the remainder of the paper.

The double imputation Laspeyres (DIL) and double imputation Paasche (DIP) showed evidence of drift (Figure 2), as Hill el al. (2018) also have noted for the Sydney data. For Belo Horizonte apartment's market, DIP estimated the smallest price variation while DIL estimated the biggest. In agreement with price index theory, DIL, as a Laspeyres index, tends to overestimate the price change and DIP, as a Paasche index, tends to underestimate it. Double imputation Tornqvist (DIT), in any case, does not exhibit a drift. DIT, as a Tornqvist-Geometric index, is a geometric mean of DIP and DIL. Bearing in mind that the Tornqvist indices are recognized as superlatives, the DIT becomes an attractive alternative method to compute housing index prices. Since DIP a DIL exhibited a drift behavior, the imputation methods will be reduced to DIT in the following analysis.

The rolling time dummies (RT) were estimated for 4 quarter window (RT4) and 2 quarter window (RT2) from our database. On the RT4 index the quarter base changes once a year, and on RT2 the base changes every quarter. RTs methods are attractive because the index corresponds to the estimated regression time dummy parameter. RT2 indices stayed above RT4 for the whole period. Both RT indices will be kept in the following analyses.

Figure 2 Double Imputation Indices for apartments in Belo Horizonte (2005Q1 = 100)

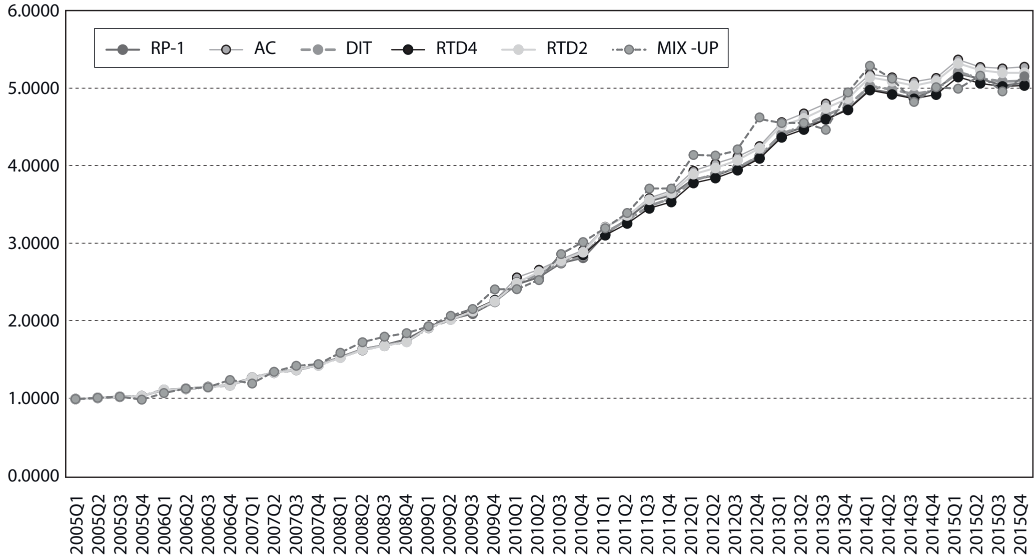


Source: IPEAD/UFMG, author's calculation

The Figure 3 compares the Belo Horizonte's housing price indices estimated by different hedonic methods and by stratified median.

DIT, RP1 and RT4 exhibited a very close behavior. AC and RT2 do the same, although the latter stayed above the former in the most recent quarters. Partly, the different behavior between AC and DIT is expected, since the first is a Laspeyres type of index and the second is a Tornqvist type. The MIX-UP line was more volatile than the other index lines due to the lack of characteristics control related to this method.

Figure 3 Estimate of Price Indices for apartments in Belo Horizonte (2005Q1 = 100)



Source: IPEAD/UFMG, author's calculation

Hill et al. (2018) recommended analyzing the volatility of indices in more detail. The authors present two volatility measures: the root mean squared error (RMSE) and mean absolute deviation (MAD). Also, they calculated the minimum (MIN) and maximum (MAX) value for each index. All these indicators are computed both on a year-by-year and quarter-by-quarter basis. The indicators volatility formulas are specified below:

$$RMSE = \sqrt{\frac{1}{T-1}\sum_{t-1}^{T-1}\left[ln\left(\frac{p_{(t+1,q)}}{p_{(t,q)}}\right) - \frac{1}{T-1}\left(\frac{p_{(T,q)}}{p_{(1,q)}}\right)\right]}, \quad (15)$$

$$MAD = \frac{1}{T-1}\sum_{t-1}^{T-1}\left|ln\left(\frac{p_{(t+1,q)}}{p_{(t,q)}}\right) - \frac{1}{T-1}ln\left(\frac{p_{(T,q)}}{p_{(1,q)}}\right)\right|, \quad (16)$$

$$MIN = Min_{1,\ldots,T-1}\left\{100\left[\frac{p_{(t+1,q)}}{p_{(t,q)}}\right]-1\right\}, \quad (17)$$

$$MAX = Max_{1,\ldots,T-1}\left\{100\left[\frac{p_{(t+1,q)}}{p_{(t,q)}}\right]-1\right\}. \quad (18)$$

The results are summarized in Table 4.

Table 4 Volatility of the House Price Indices in Belo Horizonte

	RP-X	RP-5	RP-1	AC	DIL	DIP	DIT	RTD4	RTD2	MIX-UP
Year-on-Year (Q1)										
RMSE	0.072	0.064	0.063	0.068	0.064	0.065	0.064	0.063	0.067	0.103
MAD	0.063	0.057	0.055	0.058	0.052	0.056	0.054	0.054	0.056	0.087
MIN	2.478	3.666	4.081	3.653	4.714	2.438	3.570	3.396	3.516	–5.605
MAX	31.089	28.572	29.046	32.496	32.032	29.225	30.621	29.619	30.013	32.880
Year-on-Year (Q2)										
RMSE	0.072	0.066	0.065	0.068	0.065	0.066	0.065	0.065	0.068	0.078
MAD	0.056	0.053	0.053	0.056	0.051	0.055	0.053	0.052	0.054	0.063
MIN	2.478	3.666	4.081	3.653	4.714	2.438	3.570	3.396	3.516	–5.605
MAX	31.089	28.572	29.046	32.496	32.032	29.225	30.621	29.619	30.013	32.880
Year-on-Year (Q3)										
RMSE	0.077	0.074	0.076	0.075	0.072	0.074	0.073	0.072	0.075	0.085
MAD	0.067	0.065	0.066	0.064	0.059	0.063	0.061	0.061	0.064	0.077
MIN	3.601	2.651	3.459	3.391	4.018	2.578	3.295	3.230	3.273	2.861
MAX	33.371	29.050	31.020	30.412	30.474	29.011	29.740	30.045	30.609	32.779
Year-on-Year (Q4)										
RMSE	0.075	0.075	0.076	0.077	0.075	0.077	0.076	0.075	0.078	0.090
MAD	0.065	0.065	0.066	0.065	0.062	0.065	0.064	0.062	0.065	0.080
MIN	3.961	1.250	1.535	2.799	2.847	1.961	2.403	2.360	2.115	1.320
MAX	29.141	28.151	28.631	29.721	29.755	29.023	29.388	30.185	30.102	30.513
Quater-on-Quarter										
RMSE	0.033	0.029	0.029	0.031	0.029	0.029	0.029	0.028	0.029	0.047
MAD	0.028	0.025	0.024	0.024	0.023	0.024	0.024	0.024	0.024	0.039
MIN	–2.665	–1.703	–1.703	–1.779	–1.531	–1.581	–1.556	–1.567	–1.661	–5.867
MAX	11.985	10.658	10.658	13.308	11.875	10.498	11.184	10.440	11.070	13.229

Source: IPEAD/UFMG, author's calculation

MIX-UP is more volatile than the other indices. As Hill et al. (2018) pointed out, it is expected for stratified median indices (like MIX-UP) to exhibit more volatility since they are not adjusted for changes in the quality of median over time. The hedonics quality adjusted indices exhibited relative low volatility, the magnitudes were between those which were estimated for Sidney and Tokyo by Hill et al. (2018). From volatility indicators perspectives our results suggest, for Belo Horizonte's housing market in 2005–2015, that the hedonic quality adjusted housing price indices were accurate, except for the MIX-UP cases.

3.2 Apartment price valuation in Belo Horizonte, Brazil: 2005–2015

We will illustrate the previous results measuring quarterly apartments prices rate of appreciation for Belo Horizonte. Table 5 summarizes the results for different methodologies.

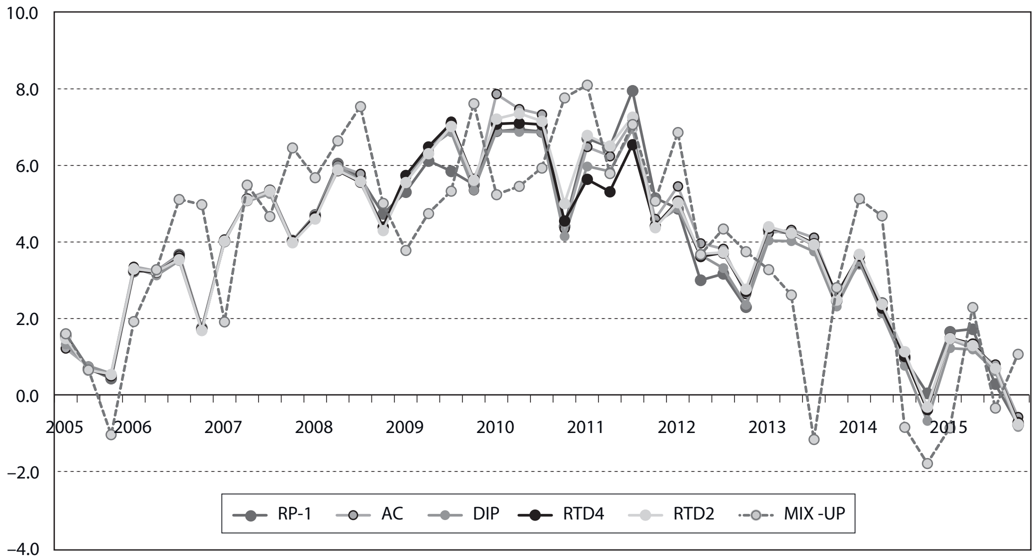
Table 5 Rate of appreciation (%) of apartment prices in Belo Horizonte: 2005–2015

Year	Quarter	RP-1	AC	DIP	RTD4	RTD2	MIX -UP
2005	Q2	1.59	1.22	1.32	1.51	1.48	1.60
2005	Q3	1.39	1.50	1.52	1.31	1.30	1.32
2005	Q4	0.59	0.85	0.92	0.88	0.97	–3.70
2006	Q1	7.86	7.86	7.65	7.73	7.75	8.52
2006	Q2	1.43	1.16	0.95	1.16	1.10	5.40
2006	Q3	1.84	2.04	2.03	1.98	1.90	1.53
2006	Q4	2.00	2.00	2.09	1.99	2.06	8.09
2007	Q1	8.33	8.25	8.25	8.30	8.24	–3.55
2007	Q2	5.14	5.23	4.95	5.08	5.07	12.59
2007	Q3	2.66	2.63	2.63	2.71	2.74	5.59
2007	Q4	4.33	4.27	4.37	4.24	4.15	1.49
2008	Q1	7.19	7.07	7.16	7.09	6.96	10.12
2008	Q2	6.64	6.65	6.44	6.28	6.56	8.50
2008	Q3	3.24	3.65	3.19	3.35	3.26	4.09
2008	Q4	4.34	2.71	3.33	3.57	3.12	2.53
2009	Q1	8.39	10.94	10.32	10.44	10.44	4.73
2009	Q2	5.66	5.59	5.80	5.57	5.49	7.02
2009	Q3	3.55	5.00	4.57	5.37	5.20	4.25
2009	Q4	7.31	5.47	5.70	5.97	6.15	11.70
2010	Q1	9.91	13.31	10.50	9.96	10.36	0.09
2010	Q2	3.68	3.87	4.53	5.45	5.62	4.89
2010	Q3	7.14	5.05	5.66	5.84	5.56	13.23
2010	Q4	2.46	4.13	2.29	2.38	3.85	5.38
2011	Q1	10.66	10.37	10.12	8.78	11.07	5.86
2011	Q2	6.45	4.35	5.27	4.87	4.76	6.13
2011	Q3	6.80	6.87	5.53	6.01	6.06	9.26
2011	Q4	2.25	2.63	2.69	2.38	2.34	0.02
2012	Q1	5.61	6.92	6.47	6.85	6.73	11.67
2012	Q2	1.19	2.40	1.88	1.70	2.04	–0.25
2012	Q3	2.75	2.21	1.68	2.66	2.49	1.97
2012	Q4	2.95	3.31	3.53	3.78	3.79	9.78
2013	Q1	7.30	7.24	6.97	6.67	6.96	–1.59
2013	Q2	2.44	2.46	1.63	2.32	2.01	0.02
2013	Q3	2.10	2.70	2.74	2.94	2.88	–1.91
2013	Q4	2.85	2.56	2.59	2.61	2.56	10.75
2014	Q1	5.39	5.14	5.16	5.38	5.61	6.94
2014	Q2	–0.92	–0.73	–1.23	–1.08	–0.95	–3.14
2014	Q3	–1.46	–1.12	–1.48	–1.14	–1.13	–5.87
2014	Q4	2.57	0.96	0.69	1.09	1.15	3.91
2015	Q1	3.93	4.60	4.55	4.59	4.50	–0.37
2015	Q2	–1.27	–1.78	–1.58	–1.57	–1.66	3.38
2015	Q3	–1.70	–0.32	–1.00	–0.81	–0.64	–3.90
2015	Q4	0.67	0.38	0.09	0.24	0.01	3.91

Source: IPEAD/UFMG, author's calculation

From the first quarter of 2007 to the first quarter of 2014 there was an intense apartment's price appreciation in Belo Horizonte. This situation is in line with Brazilian real estate outlook. This appreciation was contemporary with the expansion rate of housing credit in Brazil. Some institutional improvements like fiduciary alienation law's refinement in 2004 agreed with income growth and the decline interest rates helped the housing credit's growth (Aguiar, 2014). Cardoso and Leal (2009) highlighted the government politics and the restructuring (more market concentration) of real estate development's firm role in the real estate market expansion.

Figure 4 Apartment annual valuation rate (%) – triennial moving geometric average – Belo Horizonte: 2005–2015



Source: IPEAD/UFMG, author's calculation

Figure 4 shows the quarterly variation of Belo Horizonte's apartment prices. It's clear in the figure the great volatility of the median index (MIX-UP), as we have seen in the previous section. From 2005 to 2011 there was a significant housing prices growth path, from then there was a decline tendency. After the second quarter of 2014, the decline was more intense due to the Brazil's economic crises which began in this period.

CONCLUSION

Brazil does not have an official price index yet. In this paper we used a database from Belo Horizonte, a big Brazilian city, to test some hedonic quality adjusted price indices. The indices constructed were the same used by European Statistical Institutes as described by Hill et al. (2018). Our results suggested that the hedonic quality adjusted indices exhibited a good performance in volatility terms. However, some drift in double imputation Laspeyres and Paasche indices was detected. The former with a strong upper-ward bias relative to the other hedonic indices and the latter with a strong down-ward bias.

According to our analyses, the double imputation Tornqvis (DIT) and the repricing with an annual base update (RP-1) produced very similar magnitude's indices. The same could be said about the average characteristics (AC) and the 2-quarter rolling time dummy (RT2). However, the index price lines of the latter stayed above the former. The 4-quarter rolling time window, in its turn, exhibited an intermediate behavior, as compared with the previously listed indices.

In contrast to Hill et al. (2018) estimation for Sydney and Tokyo's evidence, the repricing method with no base update (RP-X) had an upper-ward bias relative to the other hedonic indices. The five years update base repricing method, as well as Hill et al. (2018) empirical evidence, exhibited a down-ward bias. Finally, our models result suggested that median indices are not the most appropriate to estimate housing price indices. The stratified median index (MIX-UP) used was more volatile than the hedonic indices. This is because this kind of index is imperfect in control for housing quality variation over time.

This paper emphasized the potentiality for constructed housing price indices in Brazil using hedonic quality adjusted price methods and for administrative data. The Real Estate Transfer Tax (RETT) emerges as a hopeful database once this tax is collected in the whole country. Further analysis could extend the hedonic price models to estimate price indices for the Brazilian smaller city context and its less frequent housing sales reality. The smaller number of observations imposes new challenges to estimated housing prices hedonic quality adjusted indices. In addition, future analysis could extend the types of real estate units, including houses and different types of commercial real estates.

References

AGUIAR, M. M., SIMÕES, R., GOLGHER, A. B. (2014). Housing Market Analysis using a hierarchical-spatial approach: the case of Belo Horizonte, Minas Gerais, Brazil [online]. *Regional Studies, Regional Science*, 1(1): 116–137. <https://doi.org/10.1080/21681376.2014.934391>

AGUIAR, M. M. (2014). Crédito e mercado imobiliário no Brasil: fatores institucionais e macroeconômicos (1964–2013). *XVI Seminário de Economia Mineria*, Diamantina, Anais, Diamantina: CEDEPLAR/UFMG.

ALBUQUERQUE, P., NADALIN, V., NETO, V., MONTENEGRO, M. (2018). Construção de índices de preços de imóveis para o Distrito Federal por meio de vendas repetidas e GWR [online]. *Nova Economia*, 28(1): 181–212. <https://dx.doi.org/10.1590/0103-6351/3366>.

CARDOSO, A. L., LEAL, J. A. (2009). Housing Markets in Brazil: recent trends and governmental responses to the 2008 crisis. *ISA International Housing Conference*, Glasgow: ISA.

COURT, A. (1939). *Hedonic price indexes with automotive examples*. The Dynamic of Automobile Demand, New York: General Motors, 98–119.

DIEWERT, W. E. (2009). The Paris OECD-IMF Workshop on Real Estate Price Indexes: conclusions and future directions. In: DIEWERT, E. E. et al. (eds.) *Price and Productivity Measurement*, Vol. 1 – Housing, Trafford Press.

GONZÁLEZ, M. A. S. (1997). Variação qualitativa e índice de preços na análise do comportamento recente dos aluguéis em Porto Alegre (1994–1997) [online]. *Análise Econômica*, 15(28). <https://doi.org/10.22456/2176-5456.10548>.

GOODMAN, A., THIBODEAU, T. (1995). Age-related heteroskedasticity in hedonic price equations. *Journal of Housing Research*, 6(1): 25–42.

GRILICHES, Z. (1961). Hedonic Price indexes for automobiles: an econometric of quality change. In: *The Price Statistics of the Federal Government*, General Series No 73, New York: Columbia Univ. Press for the National Bureau of Economic Research, 137–196.

GRILICHES, Z. (1971). Introduction: hedonic price indexes revisited. In: *Prices indexes and quality change: studies in new methods of measurement*, Cambridge: Harvard University Press.

GRILICHES, Z. (1958). The Demand for fertilizer: an econometric reinterpretation of a technical change [online]. *Journal of Farm Economics*, 40(3): 591–606. <https://doi.org/10.2307/1235370>.

HILL, R., MELSER, D. (2008). Hedonic imputation and the price index problem: an application to housing [online]. *Economic Inquiry*, 46(4): 593–609. <https://doi.org/10.1111/j.465-7295.2007.00110.x>.

HILL, R., SCHOLZ, M., SHIMIZU, C., STEURER, M. (2018). An evaluation of the methods countries to compute their official house prices indices [online]. *Economie et Statistique/ Economics and Statistics*, 500–501–502: 221–238. <https://doi.org/10.187/ecostat.2018500t.1953>.

HILL, R. (2004). Constructing price indexes across space and time: the case of the European Union [online]. *American Economic Review*, 4(5): 1379–1410. <https://doi.org/10.1257/0002828043052178>.

HILL, R. (2013). Price indexes for residential housing: a survey, evaluation and taxonomy [online]. *Journal of Economic Surveys*, 27(1): 879–914. <https://doi.org/10.1111/j.1467-6419.2012.00731.x>.

NADALIN, V., FURTADO, B. (2011). *Índices de preços para imóveis: uma revisão.* Boletim Regional, Urbano e Ambiental, 6: 21–32.

PAIXÃO, L. A. R. (2015a). *Índice de preços hedônicos para apartamentos: Belo Horizonte, 1995–2012*. Tese (Doutorado em Economia), Programa de Pós-Graduação em Economia, Universidade Federal do Rio de Janeiro.

PAIXÃO, L. A. R. (2015b). Índice de preços hedônicos para imóveis: uma análise para o município de Belo Horizonte [online]. *Economia Aplicada*, 19(1): 5–29. <https://doi.org/10.1590/1413-8050/ea36708>.
ROSEN, S. (1974). Hedonic price and implicit markets: products differentiation in pure competition [online]. *Journal of Political Economy*, 82(1): 35–55. <https://doi.org/10.2307/1830899>.
ROZENBAUM, S., MACEDO-SOARES, T. D. (2007). Proposta para a construção de um índice de preços de imóveis a partir dos lançamentos imobiliários de condomínios residenciais [online]. *Revista de Administração Pública*, 41(6). <https://doi.org/10.1590/S0034-76122007000600004>.
ROZENBAUM, S. (2009). *IMPA – Índice municipal de preço de apartamentos: proposta e metodologia*. Tese (Doutorado em Administração de Empresas), Programa de Pós-Graduação em Administração de Empresas, Pontifícia Universidade Católica do Rio de Janeiro.
SANTOS, F., SALAZAR, M. (2011). *Índice de preços imobiliários para o Brasil: estudos para discussão*. Rio de Janeiro: Instituto Brasileiro de Geografia e Estatística (IBGE).
SIMÕES, P. F. M. (2017). *Modelos longitudinais de efeitos mistos: aplicações ao cálculo de índices de preços de imóveis e à análise da perda não técnica de eletricidade*. Tese (Doutorado em Engenharia Elétrica), Programa de Pós-Graduação em Engenharia Elétrica, Pontifícia Universidade Católica do Rio de Janeiro.
TRIPLETT, J. (2004). *Handbook on hedonic and quality adjustments in price indexes: Special application to information technology products*. Paris: (OECD), Working Paper 2004/9.
VILLAÇA, F. (1988). *O espaço intraurbano no Brasil*. São Paulo: Studio Nobel.
WAUGH, F. (1928). Quality factors influencing vegetable prices [online]. *Journal of Farm Economics*, 10(2): 185–196. <https://doi.org/10.2307/1230278>.

ANNEX

Table A1 Belo Horizonte's apartment numeric variables

Variable	Description	Expected signal	Meaning
Area	Apartment's internal area	+	Consumers tend to prefer living in higher apartments
Age	Apartment's age	–	Proxy of depreciation
Age^2	Apartment's squared age	+	Repair and improvements reduce the depreciation's age effect. Also there is the vintage effect (Goodman and Thibodeau, 1995), when some old apartments are valued in the real estate market

Source: Own construction

Table A2 Belo Horizonte's apartment quality building finishing material dummy variable

Variable	Description	Expected signal
Luxury	Apartment's with the best kind of quality building finishing material classification	+
High	Apartment's with the second best kind of quality building finishing material classification	+
Normal	Apartment's with the mean kind of quality building finishing material classification	Basical category
Low	Apartment's with the second worst kind of quality building finishing material classification	–
Popular	Apartment's with the worst kind of quality finishing material classification	–

Source: Own construction

Table A3 Belo Horizonte's zoning dummy variables

Variable	Description	Expected signal	Meaning
ZAP	Zona de Adensamento Preferencial (preferential density zone)	Basic Category	Areas where the municipality encourages new bulding
ZPA	Zona de Proteção Ambiental (environmental protection zone)	–	Areas where new buildings are not allowed or encoraged due to natural or topographic conditions. Despite the new buildings restrictions, these areas are not valued in housing markets because of its lack of affordable natural conditions
ZAR	Zona de Adensamento Restrito (restricted density zone)	–	
ZA	Zona Adensada (dense zone)	+	High density areas where the municipality discourages new buildings. Commonly ZA's are in the most valuated Belo Horizonte's districts and it represents supply restrictions in a high demand context
ZE	Zona de Infraestrutura e Equipamentos Urbanos (urban's infrastructure equipment zone)	–	Areas with great urban equipment (like bus stations, cemeteries, waste treatment etc.). ZE represents poorly valued areas by the real estate's agents
ZEIS	Zona de Especial de Interesse Social (special social interest zone)	–	Original spontaneously occupations's areas (like informal slums), which were formalized by municipality. ZEIS represents poorly valued areas by the real estate's agents
ZHIP	Zona Hiper Central (over central zone)	+	Belo Horizonte's downtown. ZHIP represents valued land's location – where firms and families wish to be located
ZCBH	Zona Central de Belo Horizonte (Belo Horizonte's central zone)	+	ZCBH corresponds the districts besides Belo Horizonte downtown, into Contorno Avenue boundary. As ZHIP Its represents a valued land's location – where firms and families wish to be located
ZCVN	Zona Central de Venda Nova (Venda Nova's central zone)	+	Central area of Venda Nova's distanst suburb. ZCVN represents, in a minor magnitude, valued land's location on Belo Horizonte's extremely north location
ZCBA	Zona Central do Barreiro (Barreiro's central zone)	+	Central area of Barreiros's distant suburb. ZCBA represents, in a minor magnitude, valued land's location on Belo Horizonte's extremely west location

Source: Own construction

Table A4 Belo Horizonte's Planning Units (UP) dummy variables*

UP	Belo Horizonte's Regional
Sagrada Família	East
Floresta	East
Pompéia	East
Santa Efigênia	East
Santa Inês	East
Cabana	West
Jardim América	West
Barroca	West
Betânia	West
Buritis	West
Barro Preto	South-Center
Centro	South-Center
Savassi	South-Center
Prudente de Morais	South-Center
Serra	South-Center
São Bento	South-Center
Belvedere	South-Center
Anchieta	South-Center
Glória	Northwest
Padre Eustáquio	Northwest
Camargos	Northwest
PUC	Northwest
Abílio Machado	Northwest
Caiçara	Northwest
Pampulha	Pampulha
Santa Amélia	Pampulha
Ouro Preto	Pampulha
Jaraguá	Pampulha
Castelo	Pampulha
Cachoerinha	Northeast
Concórdia	Northeast
Cristiano Machado	Northeast
São Paulo	Northeast
Planalto	North
São Bernardo	North
Primeiro de Maio	North
Jaqueline/Tupi	North
Barreiro de Baixo	Barreiro
Cardoso	Barreiro
Europa	Venda Nova
Venda Nova	Venda Nova

Note: * Savassi is the basical category
Source: Own construction

Table A5 Hedonic price model for Belo Horizonte's apartment market: 2004–2015

TD regression				
Variable	**Estimated parameter**	**Standard deviation**	**t**	**P-value**
Constant	10.496	0.005	2 042.01	0.000
Area	0.007	0.000	705.04	0.000
Age	–0.018	0.000	–151.19	0.000
Age^2	0.000	0.000	53.89	0.000
Luxury	0.101	0.004	24.39	0.000
High	0.051	0.001	34.59	0.000
Low	–0.067	0.001	–50.69	0.000
Popular	–0.095	0.004	–25.79	0.000
ZPA	–0.054	0.009	–6.21	0.000
ZAR	–0.055	0.002	–33.83	0.000
ZA	0.139	0.002	60.23	0.000
ZHIP	0.072	0.006	12.74	0.000
ZCBH	0.443	0.004	123.34	0.000
ZCVN	0.062	0.020	3.18	0.002
ZCBA	0.200	0.013	15.62	0.000
ZE	–0.178	0.007	–25.5	0.000
ZEIS	–0.232	0.029	–10.61	0.000
Sagrada Família	–0.026	0.004	–6.367	0.000
Floresta	0.017	0.004	4.271	0.000
Pompéia	–0.130	0.007	–18.241	0.000
Santa Efigênia	–0.069	0.005	–14.192	0.000
Santa Inês	–0.090	0.007	–13.025	0.000
Cabana	–0.310	0.007	–47.263	0.000
Jardim América	–0.133	0.004	–35.616	0.000
Barroca	0.020	0.003	6.137	0.000
Betânia	–0.249	0.005	–46.833	0.000
Buritis	0.008	0.003	2.382	0.017
Barro Preto	–0.503	0.006	–81.412	0.000
Centro	–0.154	0.005	–30.452	0.000
Prudente de Morais	0.151	0.004	34.750	0.000
Serra	0.039	0.004	9.146	0.000
São Bento	0.205	0.008	26.220	0.000
Belvedere	0.450	0.010	43.427	0.000
Anchieta	0.141	0.003	41.652	0.000
Glória	–0.364	0.006	–64.096	0.000
Padre Eustáquio	–0.081	0.004	–20.465	0.000
Camargos	–0.323	0.005	–66.197	0.000
PUC	–0.117	0.005	–23.256	0.000
Abílio Machado	–0.244	0.006	–43.292	0.000
Caiçara	–0.062	0.004	–14.130	0.000
Pampulha	–0.186	0.009	–21.060	0.000
Santa Amélia	–0.189	0.004	–43.934	0.000
Ouro Preto	–0.093	0.004	–21.629	0.000
Jaraguá	–0.108	0.004	–26.144	0.000
Castelo	–0.139	0.004	–38.273	0.000
Cachoerinha	–0.233	0.005	–44.255	0.000
Concórdia	–0.202	0.008	–24.445	0.000
Cristiano Machado	–0.022	0.003	–6.313	0.000
São Paulo	–0.301	0.005	–62.567	0.000
Planalto	–0.279	0.005	–51.841	0.000
São Bernardo	–0.302	0.007	–46.206	0.000
Primeiro de Maio	–0.298	0.009	–32.703	0.000
Jaqueline/Tupi	–0.459	0.006	–82.517	0.000
Barreiro de Baixo	–0.348	0.004	–77.391	0.000
Cardoso	–0.498	0.006	–87.897	0.000
Europa	–0.309	0.006	–52.310	0.000
Venda Nova	–0.274	0.004	–63.328	0.000
Time fixed effect = yes				
Ajusted R2	0.924			
F	30 399.1			0.000

Source: IPEAD/UFMG, authors calculation