# Rectifying Sampling Inspection by Variables or Attributes? Combined Inspection

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### *Abstract*

Acceptance sampling, one of the techniques used in quality control, is analysed in present paper. We shall study sampling inspection plans when the remainder of a rejected lot is inspected, i.e. rectifying plans. These plans were introduced by Dodge and Romig for inspection by attributes (each inspected item is classified as either good or defective). Analogous rectifying plans for inspection by variables with one specification limit for the quality characteristic were introduced by the author of this contribution. In present article we shall consider combined inspection (all items from the sample are inspected by variables, but remainder of a rejected lot is inspected only by attributes). We shall show that the combined inspection is the best in many situations. Using plans for combined inspection we can often achieve significant savings of the inspection cost under the same protection of producer and consumer.



| *Keywords* | *DOI* | *JEL code* |
|---|---|---|
| *Acceptance sampling, rectifying LTPD and AOQL plans, inspection by attributes, inspection by variables, cost of inspection* | *https://doi.org/10.54694/stat.2023.6* | *C40, L15, C83* |


## INTRODUCTION

The sampling inspection plans by attributes (each inspected item is classified as either good or defective) are acceptance plans ($n$, $c$), where $n$ is the number of items in the sample (the sample size), $c$ is the acceptance number. Using this acceptance plan we decide as follows – see e.g. Hald (1981): the lot is rejected when the number of defective items in the sample is greater than $c$. There are no assumptions for using these plans.

The rectifying plans by attributes were introduced in Dodge and Romig (1998). In this book are two types of inspection plans. For inspection of separate lots are used the LTPD plans (LTPD is the lot tolerance percent defective), for inspection of series lots from the same producer are used the AOQL plans (AOQL is average outgoing quality limit). The Dodge-Romig plans ($n$, $c$) minimize the mean number $I_a$ of items inspected per lot of process average quality $\bar{p}$, assuming that the remainder of a rejected lot is inspected under one of following conditions that protect the customer against receiving low-quality lots:

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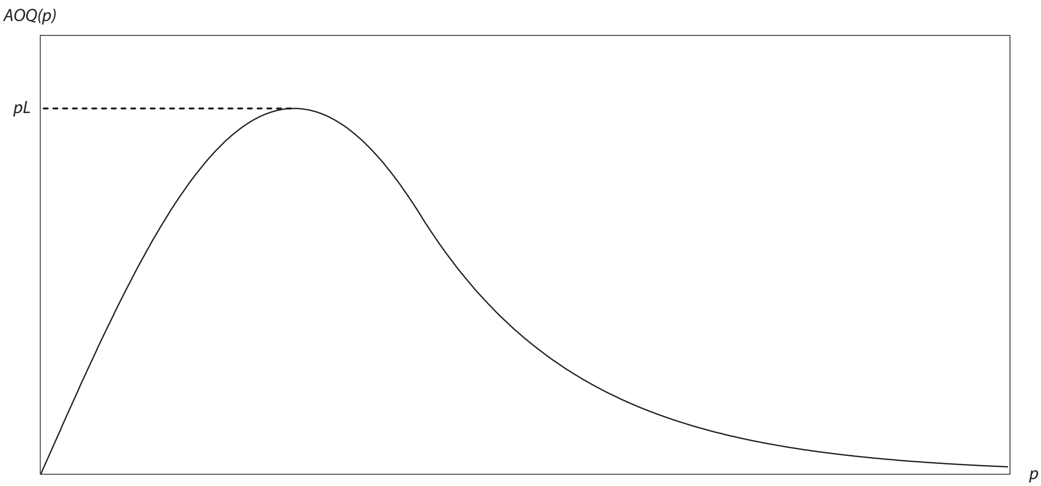
a) the average outgoing quality,[2] defined as the mean fraction defective after inspection when the fraction defective before inspection was $p$, is less or equal to $p_L$ for all values of input quality $p$, where $100p_L$ is average outgoing quality limit AOQL (the chosen parameter) – *the AOQL plans by attributes*,
b) the lots with the fraction defective $p \geq p_t$ ($100p_t$ is the lot tolerance percent defective LTPD, the chosen parameter) are accepted with probability which is less or equal to β, where β is consumer's risk (commonly β = 0.10) – *the LTPD plans by attributes*.

**Figure 1** Typical graph of the average outgoing quality *AOQ(p)*




**Source:** Own construction

The sampling inspection plans by variables are acceptance plans ($n$, $k$), where n is the number of items in the sample (the sample size), $k$ is the acceptance constant. Assumptions: Measurements of a single quality characteristic X are independent, identically distributed normal random variables with unknown parameters μ and $\sigma^2$. For the quality characteristic X is given either an upper specification limit U (the item is defective if its measurement exceeds U), or a lower specification limit L (the item is defective if its measurement is smaller than L). It is further assumed that the unknown parameter σ is estimated from the sample standard deviation s. Under the assumptions the lot is accepted when (see e.g. Klůfa, 2015):

$$\frac{U-\bar{x}}{s} \geq k, \quad \text{or} \quad \frac{\bar{x}-L}{s} \geq k,$$

where:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \; s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2}\,.$$

[2] The average outgoing quality AOQ is a function of the fraction defective before inspection $p$ – see Klůfa (2020). A typical graph of this function is in Figure 1.

Analogous rectifying plans by variables were introduced by the author of this contribution. These plans $(n, k)$ for inspection by variables minimize the mean number $I_v$ of items inspected per lot of process average quality $\bar{p}$, assuming that the remainder of a rejected lot is inspected under the same conditions which used Dodge-Romig for protection the consumer against receiving low-quality lots – see the condition a) and the condition b). In Klůfa (1994) are the rectifying LTPD plans by variables for inspection of separate lots. Calculation of these LTPD single sampling plans by variables using software Mathematica we can find in Klůfa (2010) using software R in Kasprikova and Klůfa (2011). In Klůfa (1997) are the rectifying AOQL plans by variables for inspection of series lots from the same producer. Calculation of the AOQL single sampling plans by variables we can find in Klůfa (2014).

Other papers concerning of Dodge-Romig rectifying plans are Chen and Chou (2001), Kasprikova and Klůfa (2015), Yazdi and Nezhad (2017), Klůfa (2018). Dodge-Romig LTPD sampling inspection plans by variables using EWMA statistics (the exponentially weighted moving average statistic) are in Kasprikova (2017). Dodge-Romig AOQL plans based on the EWMA statistic are in Kasprikova (2019). Other sampling inspection plans based on EWMA statics are in Wang (2016), Aslam, Azam and Jun (2015), Balamurali, Azam and Aslam (2014), Aslam, Azam and Jun (2018). Similar acceptance sampling plans we can find in Gogah and Al-Nnasser (2018), Yazdi, Nezhad., Shishebori and Mostafaeipour (2016), Wang and Lo (2016), Nezhad and Nesaee (2019). The AOQL plans for inspection by variables are also in Klůfa (2020), Chen (2016)

## 1 INSPECTION COSTS

Let us denote $L(p)$ the probability of accepting a submitted lot with fraction defective $p$. The function $L = L(p)$ is called the operating characteristic. The operating characteristic gives important information for producer and consumer. The function $L = L(p)$ is decreasing function of $p$ for each acceptance plan.

The number of inspected items when the lot with fraction defective $p$ is accepted ($n$ is the sample size, $N$ is the lot size) is:

$n$ with probability $L(p)$,

and the number of inspected items when the lot with fraction defective $p$ is rejected is:

$N$ with probability $1 - L(p)$.

Therefore, the mean number of items inspected per lot of process average $\bar{p}$ (the given parameter) is:

$$I = nL(\bar{p}) + N(1 - L(\bar{p})) = N - (N - n)\, L(\bar{p}) = n + (N - n)(1 - L(\bar{p})). \tag{1}$$

### 1.1 Cost of inspection by attributes

For inspection by attributes the operating characteristic of acceptance plan $(n, c)$ is (see e. g. Hald, 1981):

$$L(p; n, c) = \sum_{i=0}^{c} \frac{\binom{Np}{i}\binom{N - Np}{n - i}}{\binom{N}{n}}. \tag{2}$$

Therefore, according to (1) the mean number of items inspected per lot of process average quality $\bar{p}$ is $I_a = N - (N - n)\, L(\bar{p}; n, c) = n + (N - n)(1 - L(\bar{p}; n, c))$, i.e.:

$$I_a = N - (N-n)\sum_{i=0}^{c}\frac{\binom{N\bar{p}}{i}\binom{N-N\bar{p}}{n-i}}{\binom{N}{n}}. \tag{3}$$

Let us denote $c_a$ the cost of inspection of one item by attributes, $c_v$ the cost of inspection of the same item by variables. Usually is $c_v > c_a$ (when $c_v \le c_a$, the rectifying plans for inspection by variables are always more economical than the corresponding attribute sampling plans, since the sample size for inspection by variables is always less than the corresponding sample size for inspection by attributes). Under the notation:

$$I_a c_a = C_a, \tag{4}$$

is the mean cost of inspection by attributes per lot of process average quality $\bar{p}$, assuming that the remainder of a rejected lot is inspected.

### 1.2 Cost of inspection by variables

For inspection by variables the operating characteristic of acceptance plan $(n, k)$ is (see e. g. Kaspříková and Klůfa, 2011):

$$L(p; n, k) = \int_{k\sqrt{n}}^{\infty} g(t; n-1, u_{1-p}\sqrt{n})\, dt, \tag{5}$$

where: $g(t; n-1, u_{1-p}\sqrt{n})$ is probability density function of noncentral Student $t$-distribution with $(n-1)$ degrees of freedom and noncentrality parameter $\lambda = u_{1-p}\sqrt{n}$ ($u_{1-p}$ is quantile of standard normal distribution of order $1-p$). Therefore, according to (1) the mean number of items inspected per lot of process average quality $\bar{p}$ is $I_v = N - (N-n)\, L(\bar{p}; n, k) = n + (N-n)(1 - L(\bar{p}; n, k))$, i.e.:

$$I_v = N - (N-n). \int_{k\sqrt{n}}^{\infty} g(t; n-1, u_{1-\bar{p}}\sqrt{n})\, dt, \tag{6}$$

and

$$I_v c_v = C_v \tag{7}$$

is the mean cost of inspection by variables per lot of process average quality $\bar{p}$, assuming that the remainder of a rejected lot is inspected.

### 1.3 Cost of combined inspection by variables and attributes

For combined inspection by variables and attributes (all items from the sample are inspected by variables, but remainder of rejected lot is inspected only by attributes) the inspection cost, when the lot with fraction defective $p$ is accepted, is:

$nc_v$ with probability $L(p; n, k)$,

and the inspection cost, when the lot with fraction defective $p$ is rejected, is:

$nc_v + (N-n)\, c_a$ with probability $1 - L(p; n, k)$.

Therefore, the mean cost of combined inspection per lot of process average quality $\bar{p}$ is:

$$C_{va} = nc_v\, L(\bar{p}; n, k) + [nc_v + (N - n)\, c_a\,]\, [1 - L(\bar{p}; n, k)],$$

i.e.:

$$C_{va} = nc_v + (N - n)\, c_a(1 - L(\bar{p}; n, k)), \tag{8}$$

where the operating characteristic is in Formula (5). Instead of $C_{va}$ we can minimize $C_{va}/c_a$, i.e.:

$$I_{va} = nc_r + (N - n)(1 - L(\bar{p}; n, k)), \tag{9}$$

where:

$$c_r = \frac{c_v}{c_a}. \tag{10}$$

The new parameter $c_r$ is a ratio of the cost of inspection of one item by variables and the cost of inspection of the same item by attributes. Usually $c_r > 1$ (when $c_r \leq 1$, the acceptance plans for inspection by variables are always more economical than the corresponding attribute sampling plans). For determination of acceptance plan by variables and attributes (combined inspection) we must first estimate in each situation parameter $c_r$ from economical point of view.

## 2 COMPARISON OF THE INSPECTION COSTS

### 2.1 Inspection by variables versus inspection by attributes

For the comparison of the single sampling plans for inspection by variables with the corresponding Dodge-Romig plans for inspection by attributes from economical point of view we shall define the parameter $S$ by formula:

$$S = 100\left(1 - \frac{C_v}{C_a}\right). \tag{11}$$

When $S > 0$, acceptance plan for inspection by variables is more economical than the corresponding Dodge-Romig plan for inspection by attributes, when $S < 0$, acceptance by attributes is preferable. The parameter $S$ represents *the percentage of savings of inspection cost* when acceptance plan for inspection by variables is used instead of the corresponding plan for inspection by attributes. Using (10) the percentage of savings of inspection cost is:

$$S = 100\left(1 - \frac{I_v}{I_a}\, c_r\right). \tag{12}$$

### 2.2 Combined inspection versus inspection by attributes

For the comparison of the single sampling plans for combined inspection with the corresponding Dodge-Romig plans for inspection by attributes from economical point of view we shall similarly use the parameter:

$$S = 100\left(1 - \frac{C_{va}}{C_a}\right). \tag{13}$$

When $S > 0$, combined inspection is more economical than inspection by attributes, when $S < 0$, inspection by attributes is preferable. Since $C_{va} = I_{va}\, c_a$, the percentage of savings of inspection cost is:

$$S = 100\left(1 - \frac{I_{va}}{I_a}\right). \tag{14}$$

**2.3 Combined inspection versus inspection by variables**

If $c_r > 1$, the combined inspection is always more economical than the inspection by variables, as follows from the following mathematical theorem.

*Theorem:* Let us given $N$, $\bar{p}$ and $p_t(p_L)$. If $c_r > 1$, then the minimum mean cost of combined inspection per lot of process average quality $\bar{p}$ is less than the minimum mean cost of inspection by variables.

*Proof:* We must prove inequality:

$$\min_M C_{va} < \min_M C_v ,$$

where: $M$ is the set of plans $(n, k)$ for which one of conditions a) or b) applies - see Introduction. Since $C_{va} = I_{va}c_a$ and $C_v = I_v c_v$, according to (10) we must prove:

$$\min_M I_{va} < \min_M c_r I_v , \tag{15}$$

because:

$$\min_M I_{va} = \min_M \{nc_r + (N - n)(1 - L(\bar{p}; n, k))\}$$

and

$$\min_M c_r I_v = \min_M \{c_r[n + (N - n)(1 - L(\bar{p}; n, k))]\} = \min_M \{nc_r + (N - n)(1 - L(\bar{p}; n, k)) + (c_r - 1)(N - n)(1 - L(\bar{p}; n, k))\} ,$$

inequality (15) is evident.

*Illustration for the AOQL plans:* The AOQL was chosen 0.1%, i.e. $p_L = 0.001$. The process average fraction defective is $\bar{p} = 0.0008$ and $c_r = 2$ (the cost of inspection of one item by variables is twice the cost of inspection of one item by attributes). For inspection a lot with $N = 500$ items we shall look for the AOQL plan for inspection by attributes, the AOQL plan for inspection by variables and AOQL plan for combined inspection. These AOQL plans we shall compare from economical point of view.

For input parameters of acceptance sampling $p_L = 0.001$, $N = 500$, $\bar{p} = 0.0008$ we find the AOQL plan for inspection by attributes in Dodge and Romig (1998). We have:

$n = 210$, $c = 0$.

Under the same input parameters of acceptance sampling we can compute the corresponding AOQL plan for inspection by variables (see Klůfa, 2014):

$n = 75$, $k = 2.8265$.

Moreover, for $c_r = 2$ we can calculate according to (12) the percentage of savings of inspection cost:

$S = 8$.

It means that under the same protection of consumer the AOQL plan for inspection by variables (75, 2.8265) is more economical than the corresponding Dodge-Romig AOQL attribute sampling plan (210, 0). Since $S = 8$, it can be expected approximately 8% saving of the inspection cost.

Under input parameters of acceptance sampling $p_L = 0.001$, $N = 500$, $\bar{p} = 0.0008$ and moreover $c_r = 2$ we can compute the corresponding AOQL plan for combined inspection (see Klůfa, 2014):

$n = 49$, $k = 2.8561$,

and according to (14) the percentage of savings of inspection cost:

$S = 30$.

It means that under the same protection of consumer the AOQL plan for combined inspection (49, 2.8561) is more economical than the corresponding Dodge-Romig AOQL attribute sampling plan (210, 0). Since $S = 30$, it can be expected approximately 30% saving of the inspection cost.

Combined inspection is clearly the best in this situation – see also Table 1.

**Table 1** AOQL plans for inspection by attributes (upper row), variables (middle row) and combined inspection (lower row) and percentage of savings of inspection cost *S* (in %)

**$p_L = 0.001$, $c_r = 2.0$**

| $\bar{p}/N$ | 500 | 1 000 | 4 000 | 10 000 | 50 000 | 100 000 |
|---|---|---|---|---|---|---|
| | (210, 0) | (270, 0) | (340, 0) | (355, 0) | (830, 1) | (835, 1) |
| 0.0001 | (34, 2.8973)<br>S = 60 | (41, 2.8885)<br>S = 64 | (56, 2.8799)<br>S = 70 | (67 ,2.8776)<br>S = 76 | (88, 2.8778)<br>S = 80 | (97, 2.8788)<br>S = 80 |
| | (28, 2.9303)<br>S = 66 | (34, 2.9101)<br>S = 70 | (49, 2.8865)<br>S = 74 | (59, 2.8809)<br>S = 79 | (79, 2.8777)<br>S = 81 | (88, 2.8781)<br>S = 83 |
| | (210, 0) | (270, 0) | (340, 0) | (355, 0) | (830, 1) | (835, 1) |
| 0.0002 | (42, 2.8709)<br>S = 50 | (52, 2.8699)<br>S = 56 | (77, 2.8721)<br>S = 66 | (95, 2.8759)<br>S = 76 | (131, 2.8840)<br>S = 78 | (148, 2.8877)<br>S = 84 |
| | (33, 2.9012)<br>S = 58 | (43, 2.8841)<br>S = 63 | (65, 2.8751)<br>S = 71 | (82, 2.8756)<br>S = 80 | (115, 2.8811)<br>S = 81 | (131, 2.8844)<br>S = 85 |
| | (210, 0) | (270, 0) | (340, 0) | (775, 1) | (1 330, 2) | (1 350, 2) |
| 0.0003 | (48, 2.8579)<br>S = 42 | (63, 2.8600)<br>S = 48 | (98, 2.8715)<br>S = 64 | (125, 2.8798)<br>S = 68 | (180, 2.8934)<br>S=74 | (206, 2.8987)<br>S = 78 |
| | (37, 2.8857)<br>S = 52 | (49, 2.8738)<br>S = 57 | (80, 2.8718)<br>S = 69 | (105, 2.8769)<br>S = 73 | (156, 2.8888)<br>S = 78 | (181, 2.8941)<br>S = 80 |
| | (210, 0) | (270, 0) | (340, 0) | (775, 1) | (1 330, 2) | (1 350, 2) |
| 0.0004 | (54, 2.8482)<br>S = 34 | (72, 2.8549)<br>S = 42 | (119, 2.8732)<br>S = 60 | (158, 2.8853)<br>S = 64 | (240, 2.9038)<br>S = 72 | (280, 2.9107)<br>S = 78 |
| | (41, 2.8735)<br>S = 47 | (55, 2.8666)<br>S = 53 | (95, 2.8714)<br>S = 67 | (130, 2.8806)<br>S = 69 | (204,2.8978)<br>S = 76 | (241, 2.9047)<br>S = 81 |
| | (210, 0) | (270, 0) | (695, 1) | (775, 1) | (1 870, 3) | (2 480, 4) |
| 0.0006 | (65, 2.8353)<br>S = 20 | (91, 2.8486)<br>S = 28 | (170, 2.8794)<br>S = 46 | (244, 2.8986)<br>S = 54 | (417, 2.9262)<br>S = 66 | (512, 2.9361)<br>S = 68 |
| | (46, 2.8618)<br>S = 38 | (65, 2.8587)<br>S= 44 | (127, 2.8741)<br>S = 57 | (188, 2.8903)<br>S = 63 | (337, 2.9174)<br>S = 7 1 | (419, 2.9276)<br>S = 74 |
| | (210, 0) | (270, 0) | (695, 1) | (775, 1) | (2 420, 4) | (3 070, 5) |
| 0.0008 | (75, 2.8265)<br>S = 8 | (111, 2.8447)<br>S = 14 | (231, 2.8859)<br>S = 30 | (363, 2.9117)<br>S = 42 | (749, 2.9495)<br>S = 52 | (1 018, 2.9635)<br>S = 58 |
| | (49, 2.8561)<br>S = 30 | (73, 2.8545)<br>S = 36 | (159, 2.8780)<br>S = 47 | (258, 2.9004)<br>S = 56 | (562, 2.9383)<br>S = 62 | (749, 2.9517)<br>S = 67 |
| | (210, 0) | (270, 0) | (695, 1) | (775, 1) | (2 420, 4) | (3 070, 5) |
| 0.0010 | (75, 2.8265)<br>S = 8 | (111, 2.8447)<br>S = 14 | (231, 2.8859)<br>S = 30 | (363, 2.9117)<br>S = 42 | (749, 2.9495)<br>S = 52 | (1 018, 2.9635)<br>S = 58 |
| | (49, 2.8561)<br>S = 30 | (73, 2.8545)<br>S = 36 | (159, 2.8780)<br>S = 47 | (258, 2.9004)<br>S = 56 | (562, 2.9383)<br>S = 62 | (749, 2.9517)<br>S = 67 |

**Source:** Own calculation, Dodge and Romig (1998) – upper row

*Illustration for the LTPD plans:* The LTPD was chosen 2%, i.e. $p_t = 0.02$. The process average fraction defective is $\bar{p} = 0.004$ and $c_r = 2$ (the cost of inspection of one item by variables is twice the cost of inspection of one item by attributes). For inspection a lot with $N = 1000$ items we shall look for the LTPD plan for inspection by attributes, the LTPD plan for inspection by variables and LTPD plan for combined inspection. These LTPD plans we shall compare from economical point of view.

For input parameters of acceptance sampling $p_t = 0.02$, $N = 1000$, $\bar{p} = 0.004$ we find the LTPD plan for inspection by attributes in Dodge and Romig (1998). We have:

$n = 185$, $c = 1$.

Under the same input parameters of acceptance sampling we can compute the corresponding LTPD plan for inspection by variables (see Klůfa, 2010):

$n = 104$, $k = 2.2940$.

Moreover, for $c_r = 2$ we can calculate according to (12) the percentage of savings of inspection cost:

$S = 20$.

It means that under the same protection of consumer the LTPD plan for inspection by variables (104, 2.2940) is more economical than the corresponding Dodge-Romig LTPD attribute sampling plan (185, 1). Since $S = 20$, it can be expected approximately 20% saving of the inspection cost.

Under input parameters of acceptance sampling $p_t = 0.02$, $N = 1000$, $\bar{p} = 0.004$ and moreover $c_r = 2$ we can compute the corresponding LTPD plan for combined inspection (see Klůfa, 2010):

$n = 85$, $k = 2.3221$,

and according to (14) the percentage of savings of inspection cost:

$S = 31$.

It means that under the same protection of consumer the LTPD plan for combined inspection (85, 2.3221) is more economical than the corresponding Dodge-Romig LTPD attribute sampling plan (185, 1). Since $S = 31$, it can be expected approximately 31% saving of the inspection cost.

Combined inspection is clearly the best in this situation – see also Table 2.

**Table 2** LTPD plans for inspection by attributes (upper row), variables (middle row) and combined inspection (lower row) and percentage of savings of inspection cost S (in %)

**$p_t = 0.02$, $c_r = 2.0$**

| $\bar{p}/N$ | 500 | 1 000 | 4 000 | 10 000 | 50 000 | 100 000 |
|---|---|---|---|---|---|---|
| 0.0001 | (105, 0) | (115, 0) | (195, 1) | (265, 2) | (335, 3) | (335, 3) |
| | (43, 2.4478)<br>S = 28 | (49, 2.4192)<br>S = 46 | (62, 2.3736)<br>S = 46 | (70 ,2.3526)<br>S = 46 | (83, 2.3256)<br>S = 48 | (88, 2.3170)<br>S = 48 |
| | (36, 2.4909)<br>S = 37 | (43, 2.4478)<br>S = 51 | (56, 2.3925)<br>S = 50 | (64, 2.3679)<br>S = 50 | (77, 2.3371)<br>S = 52 | (83, 2.3256)<br>S = 52 |
| 0.0002 | (105, 0) | (115, 0) | (195, 1) | (265, 2) | (335, 3) | (335, 3) |
| | (57, 2.3891)<br>S = 22 | (68, 2.3574)<br>S = 46 | (87, 2.3186)<br>S = 52 | (100, 2.2992)<br>S = 48 | (120, 2.2760)<br>S = 54 | (129, 2.2675)<br>S = 66 |

**Table 2** (continuation)

$p_t = 0.02$, $c_r = 2.0$

| $\bar{p}/N$ | 500 | 1 000 | 4 000 | 10 000 | 50 000 | 100 000 |
|---|---|---|---|---|---|---|
| 0.0002 | (47, 2.4281)<br>S = 32 | (58, 2.3858)<br>S = 52 | (78, 2.3351)<br>S = 56 | (90, 2.3138)<br>S = 52 | (112, 2.2845)<br>S = 57 | (120, 2.2760)<br>S = 68 |
| 0.0003 | (105, 0) | (185, 1) | (330, 3) | (395, 4) | (520, 6) | (585, 7) |
| | (70, 2.3526)<br>S = 16 | (85, 2.3221)<br>S = 24 | (113, 2.2834)<br>S=34 | (131, 2.2657)<br>S = 36 | (160, 2.2441)<br>S = 38 | (172, 2.2369)<br>S = 40 |
| | (56, 2.3925)<br>S = 28 | (71, 2.3502)<br>S = 33 | (100, 2.2992)<br>S = 40 | (118, 2.2780)<br>S = 41 | (147, 2.2530)<br>S = 43 | (160, 2.2441)<br>S = 44 |
| 0.0004 | (105, 0) | (185, 1) | (330, 3) | (395, 4) | (520, 6) | (585, 7) |
| | (83, 2.3256)<br>S = 10 | (104, 2.2940)<br>S = 20 | (142, 2.2567)<br>S = 32 | (165, 2.2410)<br>S = 38 | (204, 2.2210)<br>S = 42 | (221, 2.2141)<br>S = 44 |
| | (64, 2.3679)<br>S = 24 | (85, 2.3221)<br>S = 31 | (124, 2.2721)<br>S = 40 | (148, 2.2523)<br>S = 43 | (188, 2.2284)<br>S = 46 | (204, 2.2210)<br>S = 47 |
| 0.0005 | (165, 1) | (245, 2) | (450, 5) | (520, 6) | (710, 9) | (770, 10) |
| | (97, 2.3033)<br>S –12 | (124, 2.2721)<br>S = 8 | (174, 2.2358)<br>S = 24 | (205, 2.2206)<br>S = 30 | (256, 2.2021)<br>S = 36 | (278, 2.1958)<br>S = 36 |
| | (72, 2.3479)<br>S = 8 | (99, 2.3005)<br>S = 21 | (150, 2.2508)<br>S = 33 | (182, 2.2315)<br>S = 37 | (234, 2.2093)<br>S = 40 | (256, 2.2021)<br>S = 41 |
| 0.0006 | (165, 1) | (245, 2) | (450, 5) | (520, 6) | (710, 9) | (770, 10) |
| | (110, 2.2868)<br>S | (145, 2.2544)<br>S = 2 | (211, 2.2181)<br>S = 22 | (251, 2.2037)<br>S = 34 | (318, 2.1861)<br>S = 44 | (346, 2.1804)<br>S = 50 |
| | (78, 2.3351)<br>S = 3 | (114, 2.2823)<br>S = 18 | (180, 2.2325)<br>S = 31 | (221, 2.2141)<br>S = 41 | (290, 2.1927)<br>S = 49 | (318, 2.1861)<br>S = 54 |
| 0.0007 | (165, 1) | (305, 3) | (510, 6) | (760, 10) | (1 060, 15) | (1 180, 17) |
| | (124, 2.2721)<br>S | (169, 2.2386)<br>S | (254, 2.2027)<br>S = 17 | (306, 2.1888)<br>S = 24 | (393, 2.1723)<br>S = 30 | (429, 2.1670)<br>S = 30 |
| | (83, 2.3256)<br>S | (128, 2.2684)<br>S = 12 | (215, 2.2164)<br>S = 18 | (268, 2.1986)<br>S = 32 | (356, 2.1785)<br>S = 35 | (393, 2.1723)<br>S = 36 |

**Source:** Own calculation, Dodge and Romig (1998) – upper row

## CONCLUSION

From the results of this paper (see Table 1 and Table 2) it follows that the combined inspection (all items from the sample are inspected by variables, but remainder of rejected lot is inspected only by attributes) is in many situations most economical. This conclusion is valid especially when we chose small values for the lot tolerance percent defective LTPD or the average outgoing quality limit AOQL and (see Table 1 and Table 2) the number of items in the lot $N$ is large and the process average fraction defective $\bar{p}$ is small. In this case it makes sense to estimate from economical point of view the parameter $c_r$, i.e. the ratio of the cost of inspection of one item by variables and the cost of inspection of the same item by attributes (without $c_r$ the acceptance plan for combined inspection cannot be determined). Using the acceptance plans for combined inspection instead of the corresponding acceptance plans for inspection by attributes or acceptance plans for inspection by variables we can achieve significant savings of the inspection cost (the combined inspection is always more economical than inspection by variables). Numerical investigations show that the percentage of savings of inspection cost is in many situations greater than 50%.

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