Quality Indicators of Development Dynamics at All Levels of the Economy

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Abstract

The paper answers one of the typical problems of economic theory – how it is in practice possible to measure and to interpret the quality of economic time series on all economic levels. The task is on the macroeconomic level solved by weighted geometric aggregation of input factors (labour and capital) into summary input factor (SIF) – the method is similar to the Cobb-Douglas production function. The papers shows differences of our approach to the approach of growth accounting – our approach is based on more general condition and covers not only situations of growth of economic indicators but also situations of their falls or stagnation. The approach allows also to distinguish the compensation of input factors. So, the methodology presented in the paper can be used in many practical applications, for instance it enables us to count clearly intensive and extensive parameters of economic growth.

Keywords	***JEL code***
Dynamic indicators, economic growth, intensive and extensive factors of change of indicators	*C22, C43*

INTRODUCTION

The question as to which factors cause the development dynamics of an economic unit (a firm, region, state, etc.) is one of the most discussed in the economics. Generally speaking, dynamics may be due to extensive or intensive factors; however, the effect of those factors needs to be properly quantified. This article summarises the knowledge from research in the quantification of the given factors, while following on the publications of Hrach, Mihola (2006), Mihola (2007a), Mihola (2007b), Hájek, Mihola (2008a), Hájek, Mihola (2008b), Hájek, Mihola (2009). The research is based on the crucial business criterion of the market economy, i.e. profit, while respecting the limits of the factors of production. In this context, the manner of achieving profit is not immaterial. The instruments included in the text are applicable to

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businesses as well as to the national economy and other sciences. The correct answer as to the method of generating profit and GDP has a significant impact on the management of large business groups, on seeking a forward-looking direction of national economies and transnational units, as well as on tackling the problems of tendering processes, outsourcing and other economic activities.

A crucial feature of the knowledge society is the application of new knowledge or the innovative application of existing knowledge. Schumpeter's economic analysis stresses the key role of dynamic processes based on permanent innovative efforts of businesses. However, innovation in all stages of business activities develops only in the environment which, owing to good education, fosters science and research as well as quality of human resources, and improves the use of innate human capacities. The innovation processes are also associated with the development of communication technologies, the management level, and a more efficient strategy and motivation. Such developments typically entail the use of qualitative or intensive factors of development in particular, as opposed to extensive expansion of the existing production.

In solving practical strategic tasks of the national economy and businesses, it is essential to use proper dynamic indicators that reflect the factor of time, without which neither a serious tendering process nor the increasingly popular outsourcing can exist. Before we derive the appropriate indicators of an innovative or, more generally, qualitative or intensive development, we will give one general illustrative example, which will help us with finding an appropriate basic correlation, on which the entire solution will be based.

1 INITIAL ILLUSTRATIVE EXAMPLE

Suppose we run a successful firm, which supplies the market with production, for which, over the given initial period (referred to as index *0*), it gains total revenue[4] TR_0, on which it spends total costs TC_0 over the same period. The difference between the two quantities defines the economic profit.

$$EP_0 = TR_0 - TC_0. \tag{1}$$

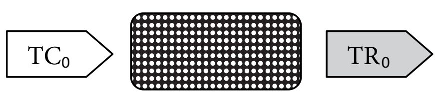
Then the total revenue / total cost ratio defines efficiency Ef_0, which expresses the portion of total revenue per CZK 1 of the total cost invested, that is:

$$Ef_0 = TR_0 \;/\; TC_0. \tag{2}$$

The economic profit / total cost ratio defines the cost profitability, i.e. the portion of profit per CZK 1 of the total cost. Then the correlation between efficiency and profitability can also be derived:

$$Ef_0 = (EP_0 + TC_0) / TC_0 = EP_0 / TC_0 + 1. \tag{3}$$

The following schema shows this initial situation.

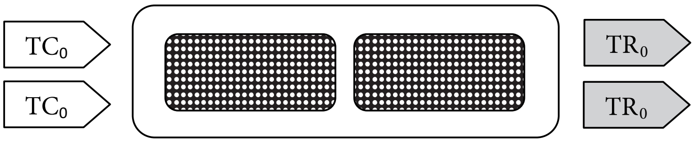


Suppose the market demand for the goods we produce doubles, with no other competing producer operating in the market. The production might be doubled in the two following specific ways: either we will build another production facility next to our existing one, or we will double the output of our existing facility solely through intensive factors of development.

In the first scenario, all inputs need to be doubled. We will need double our land. As the existing production method has worked well, we will build double production capacity of the same quality without

[4] We will initially describe outputs and inputs using microeconomic symbols, flow variables, TR as the total revenue and TC as the total cost. In both cases, the domain of definition includes positive rational numbers. $TR \geq 0$ and $TC \geq 0$. If $TR \leq TC$, the economic profit will be negative $EP \leq 0$.

any improvements. To operate such a capacity, we will also need double the number of our employees with the same skills. We could even only use our existing staff if reorganised into two shifts. Thus we will double both our capital and labour. The following schema illustrates this purely extensive way of production expansion.



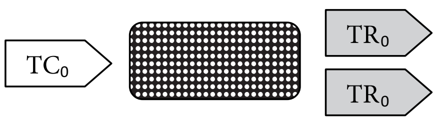
For the purely extensive development, the achieved economic profit and efficiency (referred to as index e) can be expressed by the total revenue and total cost commensurate with the initial situation before our production was doubled, as follows:

$$EP_e = 2 \times TR_0 - 2 \times TC_0 = 2 \times EP_0, \tag{4}$$

$$Ef_e = 2 \times TR_0 / 2 \times TC_0 = Ef_0. \tag{5}$$

That said, with the purely extensive development, the economic profit has doubled. Likewise, the total revenue and total cost have also doubled. However, the economic efficiency Ef_0 has not changed compared to the initial situation.

The second scenario includes the same inputs as the initial situation (referred to as index *0*). We will double our production solely through innovations based on intensive factors. Hence we will do with the same land, and will consequently have the same number of employees and the same amount of capital, which we may innovatively change, however. Another admissible variant is the one of deploying a fewer number of higher skilled employees, who are paid better, however, and thus the total production costs will not change. Only the production will double.



In the purely intensive development, the economic profit (referred to as index *i*) has more than doubled, as shown in the correlations below. In this scenario, the economic profit equals that of the purely extensive variant increased by the amount of total cost in the initial variant. The economic efficiency (referred to as index *i*) has exactly doubled:

$$EP_i = 2 \times TR_0 - TC_0 = 2 \times EP_0 + TC_0 = EP_e + TC_0, \tag{6}$$

$$Ef_i = 2 \times TR_0 / TC_0 = 2 \times Ef_0. \tag{7}$$

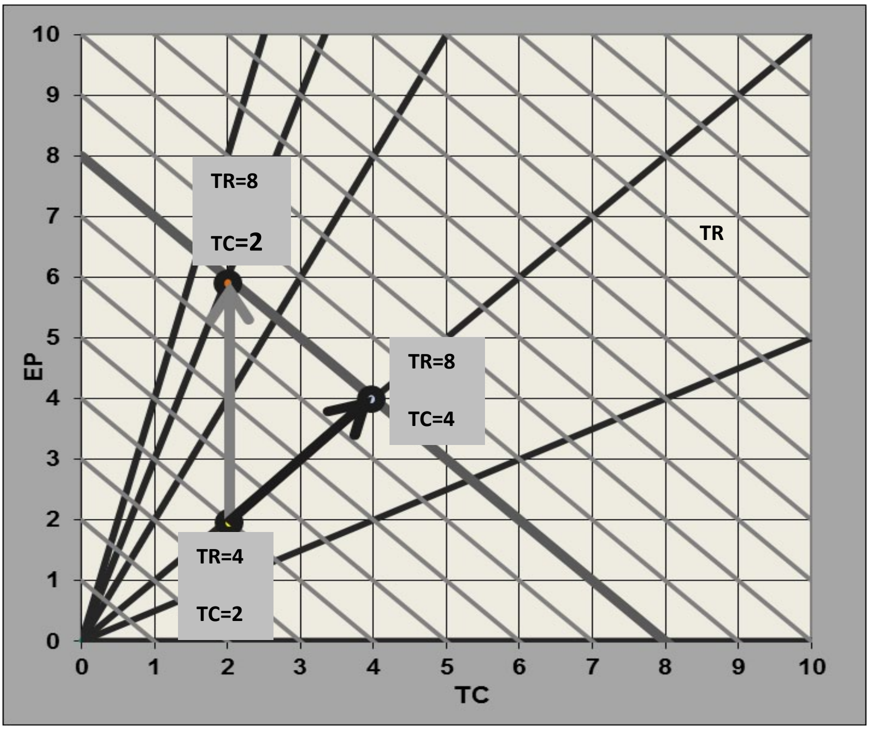
As the economic profit has increased in both variants, a more appropriate indicator of the economic development intensity is the efficiency, which remained unchanged in the purely extensive development, and increased as much as the output in the purely intensive development. This can be used very well in distinguishing the level of economic development intensity.

2 GENERALISATION OF THE INITIAL EXAMPLE

In effect, pure developments occur only rarely. Mixed developments, involving both components, are usual. The mixed development may also involve the compensation of individual factors, one of which may have an upside effect while the other may have a downside effect. The general expression of the level of development intensity or extensity must be applicable to any production increase as well as to its decrease or stagnation.

Any developments may be shown in Figure 1, with axis x denoting the total cost TC while axis y denoting the economic profit EP. In this chart, we can also easily draw the isoquants of constant total production (grey parallels) as well as those of constant efficiency (a bundle of straight lines with an intersection at the beginning of the coordinates). In Figure 1, this is selected as the initial point with coordinates TC = 2; EP = 2; and thus TR = 2 + 2 = 4 and Ef = 4 / 2 = 2.

Figure 1 Total cost, total income and economic profit



Source: Own construction

The light grey and dark grey arrows denote the above discussed special instances of doubling the production purely intensively and purely extensively. The purely intensive development, where the production increases at the constant total cost TC, is indicated by the light grey arrow. The purely extensive development, where the production increases at the constant efficiency Ef, is indicated by the dark grey arrow.

Figure 1 makes it clear that the required doubling of total revenue can be achieved in numerous mixed ways other than purely intensive or purely extensive ones. The highlighted grey isoquant for TR = 8 can also be reached from the initial point through the development at constant economic profit EP = 2 (this would be an arrow parallel to axis x), meaning a decline in efficiency Ef. We might also reach TR = 8 at the total cost decline to, for example, TC = 1. In that event, the total revenue would only double through an efficiency improvement, which will also cover the extensive diminution of TC.

Figure 1 also allows for analysing how a certain economic profit, e.g. EP = 3, may be reached if we again use the chosen initial point, where TC = 2; EP = 2, as the basis; that said, TR = 2 + 2 = 4 and

Ef = 4 / 2 = 2. This increase, as aptly illustrated by Figure 1, can be reached purely extensively on the dark grey line as well as purely intensively on the light grey line or in another mixed way. Economic profit EP = 3 can also be achieved at the constant total revenue TR = 4 if TC falls to 1. Efficiency again has to improve to the extent that it leads to an increase in economic profit and covers the extensive diminution of total costs TC.

Given this, it is evident that Figure 1 enables us to show and describe changes in movement from one point to another. If these points pertain to successive periods, we will be able to record a development or a trajectory of development of an economic unit. In any period, we will be able to analyse the development in terms of all the 4 monitored quantities TR, TC, EP, EF and the links among them, including the intensity level achieved.

If we need to express the share of the effect of economic profit EP or of total cost in the total revenue achieved, we can do so by using an additive expression derived, for example, from the correlation (1):

$$TR = EP + TC. \tag{8}$$

Then we only need to divide the expression (8) by quantity TR, and if the quotients are to be expressed as percentages, the linear equation must be multiplied by 100:

$$100 = 100 \times EP / TR + 100 \times TC / TR. \tag{9}$$

In the above considered scenario, the economic profit EP and the total cost TC in the initial situation make up 50% of the total revenue TR. In the purely extensive development, these shares remain unchanged, whereas if the production is doubled in a purely intensive manner, the share of profit in total revenue increases to 75% and the share of total cost in total revenue makes up for 25%.

If we wish to calculate the shares of the effect of a multiplicative link, such as the effect of efficiency and total cost on the total revenue, we can modify expression (2) as follows:

$$TR = Ef \times TC, \tag{10}$$

and subsequently convert expression (10) into a linear additive link using a logarithm. Thus we can also express the share of the effect of the qualitative magnitude in the form of efficiency Ef in the quantitative magnitude in the form of total cost TC.

The inputs and outputs of an economic unit may be of more than just the flow nature at the company level. In the national economy, the output may be expressed, for example, as gross domestic product (GDP) while inputs may be represented by functions of state such as labour L and capital K, which can be aggregated into a summary input[5] of factors SIF.

3 DYNAMIC PROBLEM

If the timeline of flow quantities such as TR, TC, EP, Ef and, where appropriate, the GDP, or of the functions of state such as the number of employees, essential means or the population constitutes what is known as the static problem, the changes in those quantities, measured by the dynamic characteristics of absolute or relative accrual (change rate) or index (change coefficient), constitute the dynamic problem.[6] In both events, we can express the extent to which the development is based on extensive or intensive factors of development at the levels of business, region or national economy.

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[5] More details available, for example, in Hájek and Mihola (2009, p. 745), where summary inputs are referred to as symbol N.

[6] Details of the definition of static and dynamic tasks available, for example, in Hájek, Mihola (2009, p. 745), or Mihola (2007b, p. 448).

If τ denotes the initial moment of a monitored period and T denotes the final moment, the number of monitored periods is:

$$m = \mathrm{T} - \tau. \tag{11}$$

Then the development of each quantity over a timeline can be observed by means of one of the three following dynamic characteristics used for any characteristic of the relevant system, with the characteristic being referred to as A (a general denomination of a characteristic, which may be TR, NC, L, K, etc.):

- absolute accrual $\Delta(\mathrm{A}) = \mathrm{A}_\mathrm{T} - \mathrm{A}_\tau$, (12)
- growth rate $\mathrm{G(A)} = \frac{\mathrm{A}_\mathrm{T} - \mathrm{A}_\tau}{\mathrm{A}_\tau} = \frac{\Delta(\mathrm{A})}{\mathrm{A}_\tau} = \mathrm{I(A)} - 1$, (13)
- change coefficient; (chain) index $\mathrm{I(A)} = \frac{\mathrm{A}_\mathrm{T}}{\mathrm{A}_\tau} = \mathrm{G(A)} + 1$. (14)

If $m = 1$, then we have dynamic characteristics of two successive periods. In addition to dynamic characteristics, we can also observe efficiency Ef, i.e. the correlation between input x and output[7] y over the given period of time. The expression of efficiency as a ratio does not necessarily require the same units of input and output quantities. The output quantity will be generally referred to as y (e.g. TR, GDP, etc.) and the input quantity as x (e.g. TC, capital K, labour L, SIF, etc.). This definition, which describes the given system by monitoring the changes in outputs, inputs and interrelations, corresponds to the cybernetic concept of the task. It provides us with information on efficiency,[8] i.e. the units of outputs per unit of inputs at time t:

$$\mathrm{Ef}_t = \frac{\mathrm{y}_t}{\mathrm{x}_t}. \tag{15}$$

An inverted value interprets the cost requirements, and specifies how many inputs are required per unit of outputs. Expressions (13), (14) and (15) can be used to derive the following correlations among the specified homogeneous dynamic characteristics:[9]

$$\mathrm{G}(y) = \mathrm{G}(x) + \mathrm{G(Ef)} + \mathrm{G}(x) \times \mathrm{G(Ef)}, \tag{16}$$

$$\mathrm{I}(y) = \mathrm{I}(x) \times \mathrm{I(Ef)}. \tag{17}$$

After the derivation of universal correlations for the unambiguous classification of developments according to the shares of qualitative and quantitative (or extensive and intensive) factors, we need to describe these development types first. The detailed derivations of this typology, which is used as the basis for the derivation of universal dynamic characteristics to analyse the intensity of any development, are included in Mihola (2007a). In brief, this typology is evident from Table 1.

4 DYNAMIC PARAMETERS OF INTENSITY AND EXTENSITY

The derivation of the correlations expressing the share of the effect of intensive factors on the development of outputs can be based on both the partly additive expression (16) and the purely multiplicative

7 The domain of definition for inputs as well as outputs includes positive rational numbers: $x \in \langle 0,\infty)$; $y \in \langle 0,\infty)$ then $\mathrm{I}(x) \in \langle 0,\infty)$; $\mathrm{I}(y) \in \langle 0,\infty)$; $\mathrm{G}(x) \in \langle -1,\infty)$; $\mathrm{G}(y) \in \langle -1,\infty)$.

8 This is how numerous authors define efficiency, e.g. Klacek (2006, p. 291), says: *"In general, we can define the total productivity of the factors of production as the ratio between the output of a production process and the summary of inputs of the factors of production. SP(t) = Q(t) / N(t), where Q is the product and N is the summary input"*.

9 For details of the correlations, sorts and types of aggregations between a static task and a dynamic task, see Mihola (1979) and Mihola (2005).

expression (17). The existing theoretical analyses as well as numerous practical applications that allow for the easy interpretation of results and further generalisation, e.g. into multiple factors, indicate that a logarithmically calculated correlation[10] (17) is more appropriate as the basis for further computations. If expression (16) is used, we must either omit[11] the multiplicative part of that expression, i.e. G(x) × G(Ef), or split that term 'somehow'. This problem even increases if we consider more than 2 factors because the number of multiplicative terms and their extent increase rapidly.

Literature specifies certain solutions that are only applicable to positive accruals (e.g. Cyhelský, Matějka, 1978, Toms, Hájek, 1966, Toms, 1983, Toms, 1988) of both factors. However, a dynamic task also needs to reflect the instances of declines in the individual factors as well as in the output. Furthermore, both considered factors may have a downside effect on outputs. If one factor has an upside effect while the other has a downside effect, the effects will partly compensate each other, or the mutual compensation may even lead to zero output growth. The following expressions were derived (the derivation is detailed in Mihola, 2007a) to truly express all situations that may occur in a dynamic task.

The derivation result is a correlation for a dynamic parameter of intensity:

$$i = \frac{\ln \mathrm{I(Ef)}}{\left|\ln \mathrm{I(Ef)}\right| + \left|\ln \mathrm{I}(x)\right|}, \tag{18}$$

and a supplementary correlation for extensity:

$$e = \frac{\ln \mathrm{I}(x)}{\left|\ln \mathrm{I(Ef)}\right| + \left|\ln \mathrm{I}(x)\right|}. \tag{19}$$

For the *purely intensive* development, expressions (18) and (19) generate $i = 1$ and $e = 0$ (or 100% and 0%, as appropriate), while for the *purely extensive* development, expressions (18) and (19) generate $i = 0$ and $e = 1$. Even in all the other instances, the given pair of dynamic parameters provides clear information on the type of development in the given sub-period or total period.

Adding up expressions (18) and (19) will derive the general correlation between the parameters of intensity and extensity:

$$i \times \mathrm{sgn}[\mathrm{G(Ef)}] + e \times \mathrm{sgn}[\mathrm{G(x)}] = 1, \quad \text{or} \quad |i| + |e| = 1. \tag{20}$$

The sum of both parameters in quadrant I, where both factors contribute to growth, equals 1. In quadrant III, the sum is –1, with both factors having a downside effect. In compensation quadrants II and IV, the sum of dynamic parameters of intensity and extensity equals 0. This can be used as guidance in the types of development. The sum of both dynamic parameters tells us whether it is quadrant I or III, or whether it is compensation. The fact that the sum of absolute values of both parameters equals 1 is used for designing well-arranged bar charts, for instance, which clearly express the shares of the effects of both factors.

The overview of values of the derived dynamic parameters for basic developments is included in Table 1.

[10] Even though growth rates in economic calculations are often very low numbers, it is not always the case. This is particularly relevant in use of short time intervals and in a deeper hierarchical structure of the economy, e.g. at the enterprise level. An uncontrolled omission of this multiplicative term is a similar operation as a not quite correct omission of the powers of fluents used by Newton in his derivations. See e.g. Seife (2005, p. 133).

[11] However, use of this procedure for the growing quantities is nothing new at all. As long ago as in 1978, this expression was proposed in Cyhelský, Matějka (1978, p. 302).

Table 1 Values of intensity and extensity parameters for basic developments

	Names – basic developments	Characteristics	Occurrence	Output development	Type	Parameter value	
						Of intensity i (%)	Of extensity e (%)
1	Purely intensive growth	Growth in output y only influenced by Ef developments	Axis y	Growth	Net developments – effect of only one parameter	100	0
2	Purely non-intensive decline	Decline in output y only influenced by Ef developments		Decline		–100	0
3	Purely extensive growth	Growth in output y only influenced by x	Axis x	Growth		0	100
4	Purely non-extensive development	Decline in output y only influenced by x		Decline		0	–100
5	Combined intensive & extensive growth	The same effect of Ef and x on growth in output y	Symmetry axis of quadrants I and III	Growth	Consonant effect	50	50
6	Combined non-intensive & non-extensive decline	The same effect of Ef and x on decline in output y		Decline		–50	–50
7	Intensive compensation	Stagnation of output y by growth in Ef and decline in x	Zero growth hyperbola	Stagnation	Compensation	50	–50
8	Extensive compensation	Stagnation of output y by decline in Ef and growth in x				–50	50

Source: Own construction

Derived dynamic parameters can be used wherever we consider the effect that the development of the relevant absolute and relative quantities had on the result achieved. For example, the effect and inertia, i.e. steady motion, that a speed change (i.e. acceleration) had during accelerated linear motion over a distance achieved. These parameters can be used wherever any outputs and inputs variable over time exist and where the effectiveness or efficiency measurable by changes in effectiveness or efficiency usually varies.

The advantage of those parameters is that they can be compared in respect of time. That said, they are comparable without further modifications even though they have been calculated for timelines of different lengths. This is due to the automatic averaging because no root extraction (averaging) is necessary for base indices, as shown in expression (18) (where a base index for m years is considered):

$$i = \frac{\ln \mathrm{I}^{1/m}(x)}{\left|\ln \mathrm{I}^{1/m}(\mathrm{u})\right| + \left|\ln \mathrm{I}^{1/m}(x)\right|} = \frac{(1/m)\ \ln \mathrm{I}(x)}{(1/m)\left|\ln \mathrm{I}(u)\right| + (1/m)\left|\ln \mathrm{I}(x)\right|}\ . \tag{21}$$

Derived dynamic parameters are not limited in space, and allow for easily comparing different countries, sectors, businesses, etc., due also to the fact that it is a dimensionless quantity. This is because definition expressions (18) and (19) only include dynamic characteristics, i.e. indices. It is an advantage of any dynamic parameter because these are independent of a scale or the units of characteristics used in static tasks.

Correlations (18) or (19) operate with growths as well as declines in any combination, including compensations, at any type of output development. The correlations also work with the limit states of net developments without problems. Also, there is no need to adopt any special simplifying assumptions or to check whether or not an unacceptable distortion has occurred during an approximate calculation, if any. The calculation is transparent, repeatable any time, and will always yield the same result.

The result obtained has a clear interpretation and constant information substantiality. The parameter of intensity i indicates the proportion at which the intensive (qualitative) factor, which makes itself felt as a change in efficiency, i.e. a change in the share of outputs and inputs over the given period of time, has contributed to the final development of outputs. The parameter of extensity e gives additional information on the proportion at which the extensive (quantitative) factor, i.e. the inflow of qualitatively unchanged inputs over the given period of time, has contributed to the final development of a product (outputs, effects).

A good interpretation of parameters leads to their easy application. Dynamic parameters aptly complement the existing characteristics with a fairly new perspective. The effort to express a share of influence or of the consequent contributions is evident in almost any economic analysis. The primary advantage of the solution presented here is that it comprehensively and systematically addresses all situations, including declines, decreases in one of the factors, and consequently in compensations. However, one should avoid any isolated assessment of those parameters irrespective of the distance from the point of stagnation, where all isoquants converge. Naturally, in assessing the developments which are very close to stagnation, the relevance of the assessment as to how intensively this was achieved disappears. For the same reason, it would be easy to manipulate the sizes of dynamic parameters.

5 MACROECONOMIC INTERPRETATION

Most practical applications have been subject to experiments using a classical macroeconomic task, where input y constitutes the GDP in constant prices and inputs are expressed by functions of state, namely labour L and capital K. Timelines and relevant dynamic characteristics of those quantities are also exogenous quantities of growth accounting.[12] A practical use of the growth accounting correlation is the specification of the residual quantity, which is the growth rate of the summary productivity of factors[13] G(SPF);[14] e.g. Mihola (2007, p. 111), specifies the correlations:[15]

$$G(Y) = G(SPF) + v_L \times G(L) + (1 - v_L) \times G(K), \tag{22}$$

$$G(SPF) = G(Y) - v_L \times G(L) - (1 - v_L) \times G(K). \tag{23}$$

Here the expression is derived, under special assumptions, from an additive identity of national economy,[16] as part of the reflections on the development of what is known as potential output. This includes weight vL as the labour elasticity of output, and weight vK as the capital elasticity of output. Assuming that the return to scale is constant, the sum of those weights equals 1:

$$v_L + v_K = 1. \tag{24}$$

In the expression (22), these weights are used in a weighted aggregation of the rates of growth of labour and capital. The assumption of the additive aggregation in a static task is not realistic just because one cannot imagine an economy without either of these factors, i.e. completely without labour or without

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[12] An analogous expression is derived in numerous studies and textbooks, e.g. Mihola (2007a, p. 108), or Hájek, Mihola (2009, p. 746). Today, this correlation constitutes the backbone correlation of growth theories that are primarily concerned with long-term economic growth of potential output.

[13] Robert M. Solow (see Solow, 1957) examines what is known as steady state growth, where the capital and labour growth rates reach equilibrium. Output growth per capita is subject to technological progress, which he sees as an exogenous factor here. Further elaboration of this idea has shown that not only technological progress but also the collective effect of all intensive factors of growth is relevant.

[14] For example Denison (1967, see p. 15) used the SPF growth rate for an international comparison of 9 developed countries.

[15] The calculation of the total factor productivity using this correlation has been discussed in a number of studies, such as OECD (2003), OECD (2004); some of Czech authors include Hurník (2005), Dybczak et al. (2006), Hájek (2006), Ministry of Finance (2009); in Slovakia: Zimková, Barochovský (2007).

[16] It also includes average wages and capital profitability dependent on labour or capital. In tackling this problem, one should also consider the issues of investment efficiency and the ongoing substitution of labour by technology.

any capital. While these factors are substitutable, they are substitutable relatively rather than absolutely. Hence the likely outcome is a multiplicative aggregation of these factors in a static task, with which a hyperbola-shaped isoquant is commensurate.

The growth rate of the summary productivity of factors G(SPF), calculated from expression (23), makes it possible, if the output growth rate is known, to calculate[17] also the share of the effect of the development of intensive factors on GDP developments, which can by ascertained from expression (18). To be able to use modified expression (18), we initially need to aggregate both inputs in a static task, i.e. labour L and capital.[18] This quantity is referred to as the summary input of factors (SIF). Both additive[19] and multipli cative aggregation functions are used to this end in static as well as dynamic tasks.[20] We believe that the most appropriate form of aggregation is the weighted geometric aggregation,[21] which is used, for example, in the form of Cobb-Douglas with technical progress:[22]

$$Y = SPF \times L^{\alpha} \times K^{(1-\alpha)}, \tag{25}$$

thus $$SIF = L^{\alpha} \times K^{(1-\alpha)}, \tag{26}$$

which means $$Y = SPF \times SIF. \tag{27}$$

Expression (27) is a macroeconomic application of expression (10), and can be derived from expression (15). Given the properties of indices, expression (27) can be easily used to derive its own dynamic form, analogous to expression (17):

$$I(Y) = I(SIF) \times I(SPF). \tag{28}$$

By the logarithmic calculation of this expression, we will obtain the initial correlation for a macroeconomic modification of macroeconomic dynamic parameters of intensity and extensity.

The macroeconomic form of the dynamic parameter of intensity is:

$$i = \frac{\ln I(SPF)}{|\ln I(SPF)| + |\ln I(SIF)|}. \tag{29}$$

[17] In these events, literature usually uses the share of growth rates G(SPF) / G(GDP), which is approximately applicable to positive quantities only, where G(SPF) < G(GDP); otherwise the result is difficult to interpret.

[18] Unlike other authors, we consider the factors of labour and capital to be crucial factors variable in time and complementing each other. In the Czech Republic, e.g. Klacek, Vopravil (2008) – on the KLEM production function – deals with multiple factors.

[19] The additive aggregation of labour L and capital K in a static task can be ruled out because thus we would admit either the possibility of generating production solely on the basis of labour without any capital (and consequently without tools) or production solely on the basis of capital, i.e. completely without staff, and this is impossible even in the highest level of automation. As both scenarios are unrealistic, only a weighted or simple multiplicative aggregation or geometric mean comes into consideration.

[20] The additive aggregation of labour L and capital K in a dynamic task at the multiplicative link in a static task means the use of correlation (16), and this necessitates an omission of the multiplicative term of that expression, with this being unfair and possibly leading to serious inaccuracies. See, for example, Hájek, Mihola (2009, pp. 742–743).

[21] The sum of weights equalling 1 leads to a linear production-possibility frontier (PPF) in a 2 × 2 × 2 × 2 model. If these weights are identical, i.e. 0.5, it is a simple geometric mean, and the isoquants will be hyperbolae symmetric around the axis of the first quadrant. For asymmetric weights, the asymmetry of isoquants will primarily express the long-term prevailing substitution by technology. Thus the interpretation of weights will change vis-à-vis that in Hájek, Mihola (2009, p. 746).

[22] We believe that one of the most comprehensive studies of multiplicative type production functions with factors of labour, capital and technical process is the Barro and Sala-i-Martin book (1995), where p. 29 includes the Cobb–Douglas production function in the form of $Y = AK^{\alpha} \times L^{(1-\alpha)}$. The study also includes comparisons to the proposals by Leontief Y = F(K,L) = min(AK, BL) from 1941; Harod from 1939; Domar from 1946; Solow from 1969; and many more. In the Czech Republic, see article Hájková, Hurník (2007), for instance.

The macroeconomic form of the dynamic parameter of extensity is:

$$e = \frac{\ln \mathrm{I(SIF)}}{\left|\ln \mathrm{I(SPF)}\right| + \left|\ln \mathrm{I(SIF)}\right|} \,. \qquad (30)$$

The calculation of the share of the effect of intensive and extensive factors using these parameters has numerous advantages vis-à-vis the calculation of the share of effect on the basis of correlation (23):

- It is applicable not only to an increase of the effect of sub-factors but also to their decrease and mutual compensations, i.e. opposing effects, which may lead to the complete compensation into zero output growth as well as to a GDP decline;
- It is not affected by any errors arising from the omission of multiplicative terms of the additive link in respect of growth rates;
- It allows for a very illustrative spatial representation of the trajectories of development (in a chart) of the change coefficients I(SPF) and I(SIF), where the isoquants (contour lines) of the rates of GDP growth and dynamic parameters of intensity as well as extensity can be shown concurrently.

The dynamic parameters of intensity and extensity are applicable not only to the measurement of intensity of economic developments but also whenever we need to find out how the absolute component such as time and the qualitative component such as speed have contributed to the development of a quantity. An interesting application of the above dynamic parameters is that of the assessment of development or innovation cycles or the analysis of demand or supply curves, where the use of dynamic parameters of intensity and extensity proves to be more universal than normally used elasticity, which lacks standardised values.

6 EXAMPLE – DEVELOPMENT OF THE CZECH ECONOMY

The use of the aforementioned correlations will be illustrated in an example analysing the Czech Republic's economy from 1995 to 2010. The initial data constitutes the timelines of real GDP (in constant prices for 2000), number of workers who represent labour L, and net fixed capital (in constant prices for 2000) which represents capital K. The first step includes the calculation of the summary input of factors SIF, correlation (26) (weight α was set at 0.57 ± 0.021). The total factor productivity was calculated by direct computation according to correlation (15). Dynamic characteristics and then the dynamic parameters of intensity *i* and extensity *e*, correlations (18) and (19) are calculated from all the quantities monitored.

Table 2 contains the annual growth rates[23] of all key quantities and the dynamic parameters of intensity and extensity.

Table 2 Growth rates of macroeconomic aggregates and parameters of intensity and extensity in the Czech Republic (in %)

	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
G(GDP)	4.0	–0.7	–0.8	1.3	3.6	2.5	1.9	3.6	4.5	6.3	6.8	6.0	2.5	–4.2	1.6
G(L)	0.9	0.2	–1.5	–3.4	–0.2	0.5	0.6	–1.3	0.3	1.0	1.6	2.7	1.2	–1.2	–1.9
G(K)	2.9	2.0	2.0	1.5	1.7	1.8	1.3	1.8	1.6	1.6	1.7	2.3	1.8	1.8	1.6
G(SIF)	0.9	0.9	1.4	–1.6	0.4	0.8	–0.5	–1.1	1.7	1.3	1.8	2.7	0.1	0.2	–0.2
G(SPF)	3.1	–1.6	–2.1	2.9	3.3	1.6	2.4	4.8	2.7	5.0	4.9	3.1	2.4	–4.4	1.8
i	78	–65	–61	65	90	66	83	80	61	80	72	54	97	–95	89
e	22	35	39	–35	10	34	–17	–20	39	20	28	46	3	5	–11

Source: Czech Statistical Office (2011), ECFIN (2011), own calculation

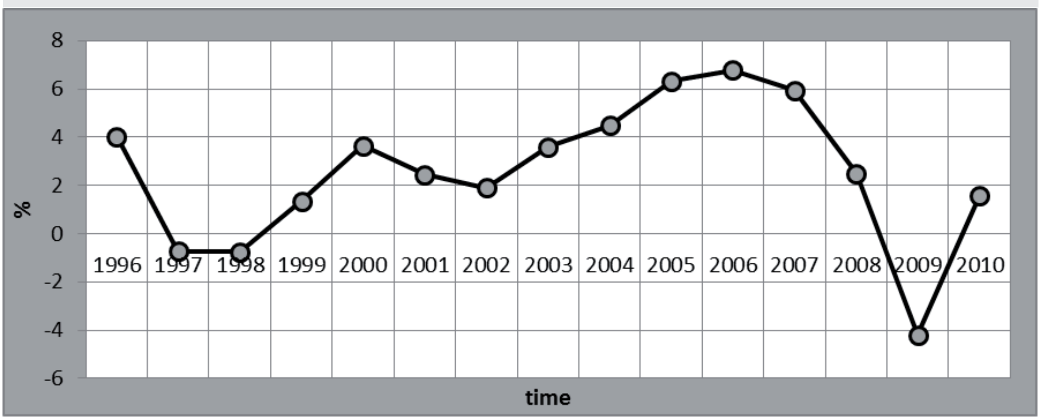
[23] Authors who calculated the SPF using growth accounting have arrived at similar results, e.g. Šindel (2009), slide 47, specifies the following G(SPF) for years 1996 to 2004: 2.9; –1.3; –1.2; 1.7; 3.5; 1.5; 0.4; 2.8; 3.7%.

The above growth rates of real GDP were generated with the effects of intensive and extensive factors shown in Figure 3. The height of each bar is 100%, the bar is divided into intensive and extensive effects, and each of those components may be positive or negative. We saw partial compensations of both effects in 1997 to 1999; in 2002; 2003; 2009 and 2010, one of the dynamic pair parameters was negative but they were not of the same size in the absolute value.

Table 1 and Figures 2 and 3 show the developments in the individual years of the analysed period. In 1997; 1998 and 2009, the real GDP declined. This decline occurred while the summary inputs were up by 0.9% in 1997, by 1.4% in 1998 and by 0.2% in 2010 but the SPF was down by 1.6% in 1997, by 2.1% in 1998 and by 4.4% in 2009. Thus the contribution of extensive factors was outweighed by the decline of intensive factors. The effects of extensive factors on economic growth were 35% in 1997; 39% in 1998 and 5% in 2009. By contrast, the downside effects of intensive factors on growth were 65% in 1997; 61% in 1998 and 95% in 2009.

As concerns the share of the effect of intensive factors, 1999 was an interesting year, as the increase in real GDP by 1.3% was achieved at the decline in summary inputs by 1.6%, and this decline was more than counterbalanced by a 2.9% SPF rise. In that year, the share of intensive factors in real GDP growth was 65% while extensive factors had a 35% downside effect. A similar situation, albeit more moderate, reoccurred in 2002; 2003 and 2010. In 2002, the real GDP went up by 1.9% at the moderate decline in summary inputs by 0.5%, which was more than counterbalanced by a 2.4% SPF rise. In that year, the share of intensive factors in real GDP growth was 83% while extensive factors had a 17% downside effect. In 2003, the effect of intensive factors was stronger because real GDP growth of 3.6% was generated at the decline in summary inputs by 1.1%. This decline was more than counterbalanced by a 4.8% SPF rise. In that year, the share of intensive factors in real GDP growth was 80% while extensive factors had a 20% downside effect. In 2010, the effect of intensive factors was even stronger because real GDP growth of 1.6% was generated at the decline in the summary inputs by 0.2%. This decline was easily counterbalanced by a 1.8% SPF rise. In that year, the share of intensive factors in real GDP growth was 89% while extensive factors had an 11% downside effect. The type of developments shown in the four years described was exceptional because real GDP growth was fuelled by such a strong increase in intensive (qualitative) factors that it outweighed the decrease in extensive factors.

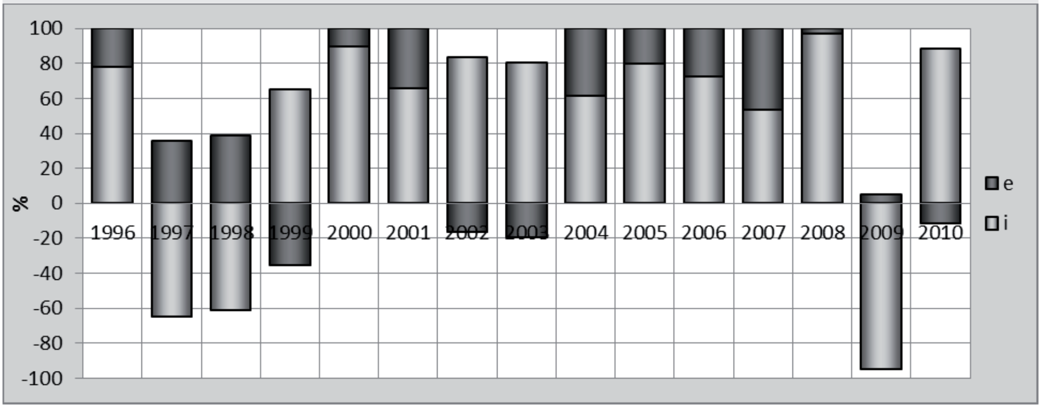
Figure 2 Rates of real GDP growth in the Czech Republic in 1995–2010 (in %)



Source: Czech Statistical Office (2011)

In all the other years, i.e. 1996; 2000; 2001, and in the last four years, 2004 to 2008, both factors, i.e. intensive and extensive, always had an upside effect. Intensive factors were always predominant, with their share being twice to eight times greater than that of extensive factors. Only in 2007, the predominance of the intensive factor was modest. During those years, the shares of extensive factors ranged between 1/8 and 1/2 while the corresponding intensive factors ranged between 7/8 and 1/2. The greatest intensity of 90% was achieved in 2000. The lowest positive intensity of 54% was achieved in 2007.

Figure 3 Shares of the effects of intensive and extensive factors in real GDP of the Czech Republic (in %)



Source: Czech Statistical Office (2011), ECFIN (2011), own calculation

The first period examined (1997–1998) saw a recession, arising from the instable political climate, making itself felt in highly restrictive monetary and fiscal policies. Uncoordinated interventions even led to a monetary crisis in 1997. Institutional barriers had the strongest impact on the banking sector, which found itself in a critical situation. The privatisation that was frequently unconsidered, and thus too spontaneous, led to instability, which delayed the required restructuring of enterprises and the launch of a more stable and more forward-looking innovative management. Investment stagnation was also accompanied by the poor inflow of foreign direct investments. There was still the aftermath of the strong past structural focus on heavy industry. The effect of high ecological investments was also evident.

Although the institutional environment was not yet refined in the subsequent period of 2000 to 2007, it improved significantly with the preparations for and the accession to the EU in 2004. The consequences of the growth-oriented economic policy and a more rational behaviour of the banking sector after its increased consolidation as well as the post-privatisation behaviour of enterprises had a positive effect. Domestic investments increased significantly, as did the inflow of foreign investments. Enterprises under strong foreign control were gaining ground, and exports were rising. However, the growth acceleration was not yet accompanied by the key long-lasting qualitative factors in HR improvement, and science and research development as a precondition of boosting the innovation process. The increasing openness of the economy had a positive effect on its performance but its dependence on and consequently its susceptibility to external environment increased somewhat as well. In addition, this vulnerability is boosted by the narrow portfolio of primary activities, particularly focused on the automotive industry, which is highly overgrown to the detriment of other transport alternatives as concerns ecology.

In 2008, the country lost its growth rate. The strong impact of intensive factors is due to the pre-crisis ousting of workforce rather than other factors. This became fully evident during the 2009 restriction, which was a result of the Czech economy reflecting the impacts. While 2010 was a year of adaptation to the new conditions, the adaptation is probably not based systematically, in a change of the structure of the economy. The negative extensity of 2010 was due to the post-crisis reduction of the economy in respect of both factors considered.

CONCLUSION

Development intensity is one of the major indicators of the quality of economic developments. At the macroeconomic level, it can be measured as the ratio between real GDP and summary input, which includes labour and capital. Its increase is a result of qualitative, i.e. intensive, factors of growth. To aggregate the factors of labour and capital in the summary input SIF, we used the weighted geometric aggregation.

To find out the shares of intensive (i.e. qualitative) and extensive factors in real GDP growth, we used the dynamic parameter of intensity and extensity. These parameters allow for measuring their shares if the factors have opposing effects as well as if the real GDP declines, are universally applicable, and are easy to compare in respect of time and space. This makes it possible to extend economic analyses with a new perspective. The application of the suggested methodology to the analysis of the Czech Republic's developments in 1995–2010 has shown that these parameters aptly complement conventional analysis tools.

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