Proposition of the Quality Adjusted Geks-Type Price Index

Jacek Białek[1] | *University of Lodz, Lodz, Poland*
Natalia Pawelec[2] | *University of Lodz, Lodz, Poland*

Received 21.10.2024, **Accepted** (reviewed) 14.11.2024, **Published** 12.9.2025

Abstract

Scanner data from retailers like supermarkets, electronics stores, and online shops provides detailed transaction information at the barcode level (e.g., GTIN, EAN), allowing for the use of various price index formulas, including weighted ones. Due to high product turnover and seasonality, multilateral index methods are ideal, as they use a whole-time window and are transitive, avoiding chain drift. However, most commonly used multilateral indices (e.g., GEKS, CCDI, GK, TPD) fail the identity test, which requires the index to return to one when prices revert to their original levels. This paper proposes a new multilateral index inspired by GEKS but incorporating quality adjustments like the Geary-Khamis method. The index satisfies the identity test and other key axioms, demonstrating its robustness. Comparisons with the SPQ index and quality-adjusted indices (GEKS-AQU, GEKS-AQI) confirm its effectiveness, making it a highly useful tool for scanner data analysis in both theory and practice.

Keywords	***DOI***	***JEL code***
Scanner data, multilateral indices, the GEKS-index	*https://doi.org/10.54694/stat.2024.71*	*C43*

INTRODUCTION

The term "scanner data" refers to transaction data that specify turnover and numbers of items sold using a Global Trade Article Number (GTIN) code, European Article Number (EAN) code, Stock Keeping Unit code (SKU) or other barcode. Scanner data have numerous advantages over traditional survey data; for example, such data sets are much bigger than traditional ones and contain complete transaction records, i.e. information about prices and quantities, along with additional information about products, including the grammage, unit, label with description, and VAT. In other words, scanner data contain full information at the most detailed item level. Since the information about prices and quantities at the lowest COICOP (Classification of Individual Consumption by Purpose) level is available in scanner

[1] Department of Statistical Methods, University of Lodz, Lodz, Poland. E-mail: jacek.bialek@uni.lodz.pl. Also Statistics Poland, Department of Trade and Services, Warsaw, Poland. ORCID: <https://orcid.org/0000-0002-0952-5327>.
[2] Department of Economic Mechanisms, University of Lodz, Rewolucji 1905r 41/43, Lodz, 90-214, Poland. E-mail: natalia.pawelec@uni.lodz.pl. ORCID: <https://orcid.org/0009-0008-5289-6204>.

data, statistical offices can use any price index formula at this level of aggregation, including weighted formulas. The use of scanner data when assessing inflation improves the data collection process, reduces costs and better reflects changes that occur in consumer behaviour. The form of a sample scanner dataset is presented in Table 1.

Table 1 Sample scanner data frame obtained from a retail chain in Poland

Time	Prices	Quantities	retID	EAN	Label	Grammage	Unit	prodID
2022-12-31	10.47	8.48	26-617	590674717126	long grain rice	0.4	kg	1
2022-12-31	12.47	5.87	40-772	590674717126	long grain rice	0.4	kg	1
2022-12-31	11.40	15.65	70-001	590674717126	long grain rice	0.4	kg	1
2022-12-31	13.20	16.95	85-791	590674717126	long grain rice	0.4	kg	1
2022-12-31	11.47	85.41	01-460	590674717126	long grain rice	0.4	kg	1
2022-12-31	11.97	7.82	05-820	590674717126	long grain rice	0.4	kg	1

Source: Białek et al. (2022: 71)

Nevertheless, the choice of price index when using scanner data to measure inflation is not straightforward. More than 40% of EU countries have implemented or are in the process of implementing scanner data in their Consumer Price Index (CPI) calculations, but there are still no clear recommendations on the choice of price index formula. While there are guides and guidelines available for choosing this index (Eurostat, 2020), and we understand the advantages and disadvantages of each approach (ABS, 2016), European Union (EU) countries still differ in the final index formulas they use. The topic of price index selection will be discussed in more detail in Section 1 but let us just mention here that this paper proposes another multilateral index, the design of which is related to the GEKS and Geary-Khamis methods. Indeed, the main purpose of the paper is to propose a new multilateral price index and to study its properties, which on the one hand has a structure similar to the GEKS index, on the other hand it requires quality adjusting, which makes it similar to the Geary-Khamis method.

The paper is structured as follows: Section 1 reviews the literature on selecting a price index for scanner data, examining available price index formulas and key selection criteria. Section 2 discusses the concept of GEKS-type indices and quality-adjusted GEKS-type indices (GEKS-AQU, GEKS-AQI). Subsection 2.3 proposes a new type of index, the GEKS-GAQI index. Section 3 presents the results of an empirical comparison of the proposed index with popular multilateral indices. The last section provides the main conclusions of the study.

1 LITERATURE REVIEW

In general, the procedure of scanner data processing consists of the following steps: data cleaning ► extracting information from the product description ► classification of products into COICOP groups ► product matching over time ► product filtering ► data standardization ► imputing missing prices (optional) ► price index calculations. It is not the purpose of the paper to discuss these individual stages, but the interested reader is referred, for example, to Białek and Beręsewicz (2021). Similarly, although the construction of an IT environment for the procedure of scanner data is one of the major challenges, we will not devote attention to it but only mention that all the analysis was done in the *PriceIndices* R package (Białek, 2022). The paper focuses on the final stage of the scanner data procedure, namely the determination of the correct price index.

1.1 Criteria for selecting a price index

The three main approaches in evaluating (but also constructing) a price index are: the axiomatic approach, the economic approach and the stochastic approach (von der Lippe, 2007).

According to the axiomatic approach, desirable index properties (the so called "tests") are defined that an index may, or may not satisfy. A list of tests dedicated to bilateral indices, i.e., taking into account only the *current* and *base* period, can be found in Balk (1995) and von der Lippe (2007). A list of tests dedicated to multilateral indices, i.e., taking into account not only the *current* and *base* period but also all intermediate periods, can be found in ABS (2016), Eurostat (2018), and Białek (2023). Due to the high dynamics of scanner data, characterised by product churn, numerous appearances of new goods in supermarkets or the disappearance of goods previously sold, as well as the seasonality of products, multilateral indices are particularly recommended for determining changes in scanned prices (Eurostat, 2018). Thus, in the remainder of this paper, we will consider primarily multilateral indices. A key advantage of multilateral indices operating on the time window $[0, T]$ is their *transitivity*, which means that for any period $0 \leq s \leq t \leq T$, it holds that $P^{0,s}P^{s,t} = P^{0,t}$, where $P^{a,b}$ denotes a multilateral index that compares research period b to the base period a. By definition, *transitivity* eliminates the chain drift problem often associated with scanner data. Chain drift can be formalized as a violation of the *multi-period identity test*. This test expects that when all prices and quantities in the current period revert to their values from the base period, the index should indicate no price change and equal one. Thus, multilateral indices are free from chain drift within a given estimation time window. At this point it should be mentioned that one of the more restrictive tests against multilateral indices is the identity test, which is even a basic requirement for bilateral indices. The widely recognized and popular multilateral indices (GEKS, TPD or Geary-Khamis) do not meet this requirement. However, the identity test is a fairly natural axiom, since it expects the index to have a value of one when prices (for example, of seasonal goods) return to their initial values. In the following section, we will show that the proposed multilateral index formula satisfies the identity test.

The economic approach to the price index theory assumes that quantities are functions of prices (CPI Manual, 2004). This approach assumes that households optimise their purchase by maximising their utility under a budget constraint or by minimizing the cost of purchases for a given utility level. Multilateral methods involve different assumptions about the functional form of the utility function. For instance, the GEKS index is exact for a "flexible" functional form, i.e. it expresses the price differences experienced by optimising consumers without imposing restrictive assumptions about how they can substitute between products.

The main assumption of the stochastic approach is that prices or price changes can be estimated using an econometric model. The index formula is constructed based on appropriately estimated parameters from a panel regression equation, as is done at least for the TPD index (see Section 1.2). However, in the remainder of this paper, we will focus on the axiomatic approach, with particular attention to the identity test and the transitivity property.

1.2 Price indices considered for scanner data

As it was above-mentioned, the most promising group of indices in the context of scanner data appear to be multilateral indices. The multilateral price index is calculated for a given time window consisting of $T + 1$ consecutive months, which we number $0,1,2,\ldots,T$ (typically $T = 12$ or $T = 24$). Well-known and widely used multilateral methods include the GEKS method (Gini, 1931; Eltetö and Köves, 1964), the Geary-Khamis (GK) method (Geary, 1958; Khamis, 1972), the CCDI method (Caves et al., 1982) or the Time Product Dummy Methods (de Haan and Krsinich, 2018). Other GEKS-type indices can also be found in the literature (e.g., GEKS-W or GEKS-L indices, cf. Section 2.1), or indices based on hedonic regression that explain price behaviour (e.g., Time Dummy Hedonic index, TDH). In what is known as a dynamic approach, where the activity prior to calculating the index is to filter out relatively low sales, some EU countries determine the Jevons (1865) chain index. In this paper, however, we will limit our comparative analysis to multilateral indices only.

2 METHODS

2.1 The idea of GEKS-type indices

The popular GEKS method which is based on the Fisher price index and operates over the time interval $[0, T]$, can be expressed as follows:

$$P_{GEKS}^{0,t} = \prod_{\tau=0}^{T} \left(\frac{P_F^{\tau,t}}{P_F^{\tau,0}}\right)^{\frac{1}{T+1}}, \tag{1}$$

where $P_F^{a,b}$ denotes the Fisher (1922) price index that compares the research (current) period b to the base period a. As a rule, it is assumed that the base index in the GEKS formula is any *superlative* index (CPI Manual, 2004), which is also the Fisher index.

Accordingly, the literature also considers a GEKS index based on the superlative Törnqvist (1936) index and a GEKS index based on the superlative Walsh (1905) formula, which we will denote here as CCDI and GEKS-W, respectively. It is important to note that the GEKS, CCDI and GEKS-W indices do not satisfy the *identity test* (Eurostat, 2018).

Two new GEKS-type price indices have recently appeared in the literature that have broken the canons associated with assumptions towards the underlying index, located in the body of the GEKS formula. These GEKS-type indices are not based on a superlative price index at all, nor on an index that meets the *timere versal test*. The former (GEKS-L) is based on the Laspeyres (1871) price index, and the latter is based on the geometric Laspeyres index (CPI Manual, 2004). Please note that the GEKS-L and GEKS-GL indices satisfy the identity test (Białek, 2023b).

2.2. Quality-adjusted GEKS-type indices

Before we provide definitions of quality-adjusted GEKS-type indices, let us denote the sets of homogeneous products that belong to the same product group in months 0 and t as G_0 and G_t respectively, and let $G_{0,t}$ denote a set of matched products available in both periods 0 and t. Let p_i^τ and q_i^τ denote the price and quantity of the i – th product at the time τ and let $N_{0,t}$ be the number of elements of set $G_{0,t}$.

In the unit value concept, prices of homogeneous products are equal to the ratio of expenditure and quantity sold (Chessa et al., 2017). However, quantities of different products cannot be added together as in the case of homogeneous products. That is why the idea of quality-adjusted unit value assumes that prices p_i^τ of different products $i \in G_\tau$ in month τ are transformed into "quality-adjusted prices" p_i^τ / v_i and quantities q_i^τ are converted into "common units" $v_i q_i^\tau$ using a set of factors $v = \{v_i : i \in G_\tau\}$. Thus, the "classical" quality-adjusted unit value $QUV_{G_\tau}^\tau$ of a set of products G_τ in moth τ can be expressed as follows:

$$QUV_{G_\tau}^\tau = \frac{\sum_{i \in G_\tau} q_i^\tau p_i^\tau}{\sum_{i \in G_\tau} v_i q_i^\tau}, \tag{2}$$

and it is used in the definition of the Geary-Khamis (GK) index (Chessa et al., 2017).

In practice, consumer responses to price changes can be delayed or even accelerated as consumers not only react to current price changes but also use their own "forecasts" or concerns about future price increases. For example, the consumption of thermophilic (seasonal) fruit tends to be higher in summer because it is cheaper than in winter, when the season is almost over. For instance, some interesting study on "unconventional" consumer behaviour, such as stocking up and delayed quantity responses to price changes, and its impact on chain drift bias can be found in the paper by von Auer (2019). Since in practice we often observe prices and quantities that are not perfectly synchronised in time, the following form of the "asynchronous quality-adjusted unit value" was proposed (Białek, 2023a):

$$QUV_{G_\tau}^{\tau} = \frac{\sum_{i \in G_\tau} q_i^\tau p_i^\tau}{\sum_{i \in G_\tau} v_i q_i^\tau}, \tag{3}$$

which has more general form than (2) since we have $AQUV_{G_{\tau,\tau}}^{\tau,\tau} = QUV_{G_\tau}^{\tau}$. Let us define function $P^{\tau,s}(v, q^\tau, p^\tau, p^s)$ as follows:

$$P^{\tau,s}(v, q^\tau, p^\tau, p^s) = \frac{AQUV_{G_{\tau,s}}^{\tau,s}}{AQUV_{G_{\tau,\tau}}^{\tau,\tau}}. \tag{4}$$

Substituting Formula (4) into the GEKS formula and adopting the system of factors $v = \{v_i : i \in G_\tau\}$ corresponding to the augmented Lehr index (Lamboray, 2017; van Loon and Roels, 2018):

$$v_i = \frac{\sum_{\tau=0}^{T} p_i^\tau q_i^\tau}{\sum_{\tau=0}^{T} q_i^\tau}, \tag{5}$$

we obtain the GEKS-AQU index (Białek, 2023):

$$P_{GEKS-AQU}^{0,t} = \prod_{\tau=0}^{T} \left(\frac{P^{\tau,t}(v, q^\tau, p^\tau, p^t)}{P^{\tau,0}(v, q^\tau, p^\tau, p^0)}\right)^{\frac{1}{T+1}} = \prod_{\tau=0}^{T} \left(\frac{AQUV_{G_{\tau,t}}^{\tau,t}}{AQUV_{G_{\tau,0}}^{\tau,0}}\right)^{\frac{1}{T+1}}. \tag{6}$$

Białek (2023a) also considers the "asynchronous quality-adjusted price index" (AQI) which can be written as follows:

$$AQI_{G_{\tau,s}}^{\tau,s} = \frac{\sum_{i \in G_{\tau,s}} v_i q_i^\tau \frac{p_i^s}{p_i^\tau}}{\sum_{i \in G_{\tau,s}} v_i q_i^\tau}. \tag{7}$$

It can be treated as a weighted arithmetic mean of partial indices p_i^s / p_i^τ, where the weights are proportional to the relative share of each product's quality-adjusted quantities q_i^τ in the sum of all quality-adjusted quantities. Using AQI instead of AQUV in formula (6), we obtain the GEKS-AQI index (Białek, 2023a):

$$P_{GEKS-AQI}^{0,t} = \prod_{\tau=0}^{T} \left(\frac{AQI_{G_{\tau,t}}^{\tau,t}}{AQI_{G_{\tau,0}}^{\tau,0}}\right)^{\frac{1}{T+1}}. \tag{8}$$

Please note that the GEKS-AQU and GEKS-AQI indices meet transitivity and the identity test (Białek, 2023a).

2.3. The new quality-adjusted GEKS-type price index

Please note that the AQI defined in (7) can be viewed as a weighted version of the Carli (1804) price index. Currently, there is a trend in price statistics toward shifting from the Carli index to the Jevons (1865) index, which replaces the arithmetic mean of the sub-indices with the geometric mean. This shift serves as the foundation for our further considerations. Let us therefore define "the geometric version of asynchronous quality-adjusted price index" (GAQI) follows:

$$GAQI_{G_{\tau,s}}^{\tau,s} = \prod_{i \in G_{\tau,s}} \left(\frac{p_i^s}{p_i^\tau}\right)^{\frac{v_i q_i^\tau}{\sum_{k \in G_{\tau,s}} v_k q_k^\tau}} . \tag{9}$$

Using GAQI instead of AQUV in Formula (6), we obtain the proposed GEKS-GAQI index, i.e.

$$P_{GEKS-GAQI}^{0,t} = \prod_{\tau=0}^{T} \left(\frac{GAQI_{G_{\tau,t}}^{\tau,t}}{GAQI_{G_{\tau,0}}^{\tau,0}}\right)^{\frac{1}{T+1}} . \tag{10}$$

The properties of the index (10) will correspond to those of the GEKS-AQI and GEKS-AQU indices (Białek, 2023). It is noteworthy that the GEKS-GAQI index satisfies transitivity:

$$P_{GEKS-GAQI}^{0,s} P_{GEKS-GAQI}^{s,t} = \prod_{\tau=0}^{T} \left(\frac{GAQI_{G_{\tau,s}}^{\tau,s}}{GAQI_{G_{\tau,0}}^{\tau,0}}\right)^{\frac{1}{T+1}} \prod_{\tau=0}^{T} \left(\frac{GAQI_{G_{\tau,t}}^{\tau,t}}{GAQI_{G_{\tau,s}}^{\tau,s}}\right)^{\frac{1}{T+1}} =$$

$$= \prod_{\tau=0}^{T} \left(\frac{GAQI_{G_{\tau,t}}^{\tau,t}}{GAQI_{G_{\tau,0}}^{\tau,0}}\right)^{\frac{1}{T+1}} = P_{GEKS-GAQI}^{0,t} . \tag{11}$$

We will now show that the index (10) satisfies the identity test. Let us assume that $G_0 = G_t$ and that for any $i \in G_{0,t}$ we have $p_i^0 = p_i^t$. From this assumption, we can conclude that $G_{\tau,0} = G_{\tau,t}$ for any $\tau \in \{0,1,2,...,T\}$. Consequently, we obtain:

$$GAQI_{G_{\tau,t}}^{\tau,t} = \prod_{i \in G_{\tau,0}} \left(\frac{p_i^0}{p_i^\tau}\right)^{\frac{v_i q_i^\tau}{\sum_{k \in G_{\tau,0}} v_k q_k^\tau}} = GAQI_{G_{\tau,0}}^{\tau,0} . \tag{12}$$

From (10) and (12) we have:

$$P_{GEKS-GAQI}^{0,t} = \prod_{\tau=0}^{T} \left(\frac{GAQI_{G_{\tau,0}}^{\tau,0}}{GAQI_{G_{\tau,0}}^{\tau,0}}\right)^{\frac{1}{T+1}} = 1,$$

which means that the GEKS-GAQI index satisfies the identity test.

3 EMPIRICAL STUDY

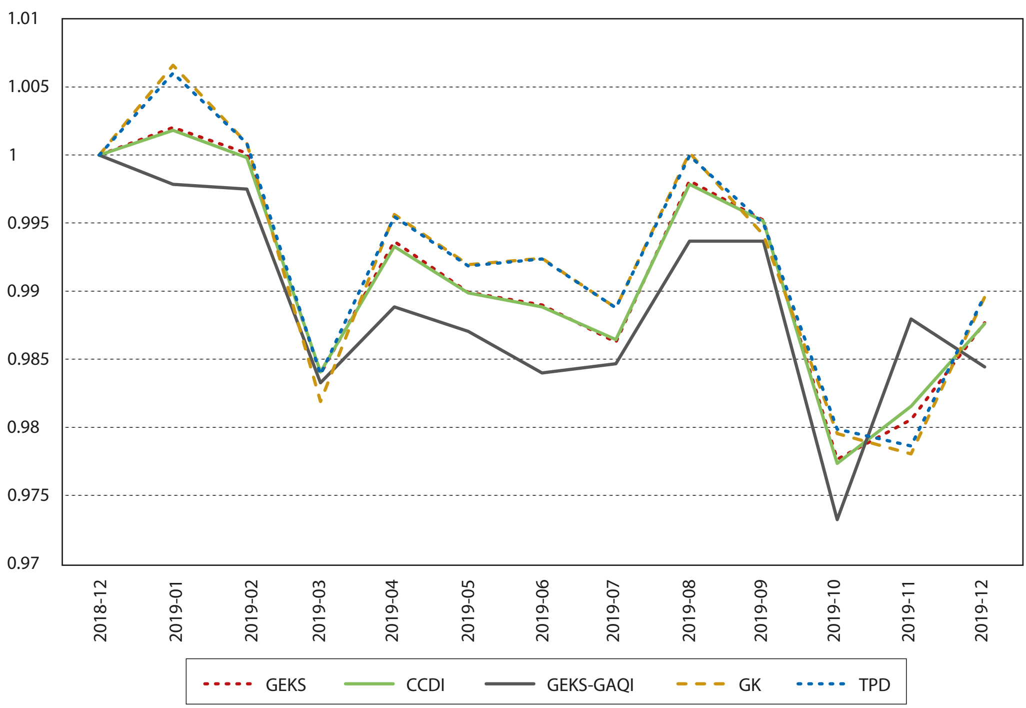
In this empirical study we use scanner data included in the R package *PriceIndices* (Białek, 2022). Two datasets are used: *milk* and *coffee*. The *milk* dataset contains scanner data on milk sales from a Polish supermarket, covering the period from December 2018 to December 2019. It consists of a data frame with 6 columns and 4 386 rows. The *coffee* dataset includes scanner data on coffee sales in a Polish supermarket from December 2018 to December 2019. Both datasets contain the following columns: *time* – dates of transactions (Year-Month-Day); *prices* – prices of sold products (PLN); *quantities* - quantities of sold products; *prodID* – unique product codes obtained after product matching (i.e. the *milk* dataset contains 68 different product IDs, and the coffee dataset contains 79 different product IDs); *retID* – unique codes identifying outlets/retailer sale points (the *milk* dataset contains 5 different retIDs, the *coffee* dataset contains 20 different retIDs); *description* – descriptions of sold milk/coffee products (the *milk* dataset contains 6 different product descriptions corresponding to subgroups of the milk group, the *coffee* dataset contains 3 different product descriptions corresponding to subgroups of the coffee group).

3.1 Comparing index values for *milk* and *coffee* harvests

First, we conducted a comparison of index values separately for the *milk* and *coffee* datasets. In the analysis, the indices were compared in two variants: at the lowest level of data aggregation (i.e. at the GTIN code) and at a broader level, using the COICOP 6 data aggregation.

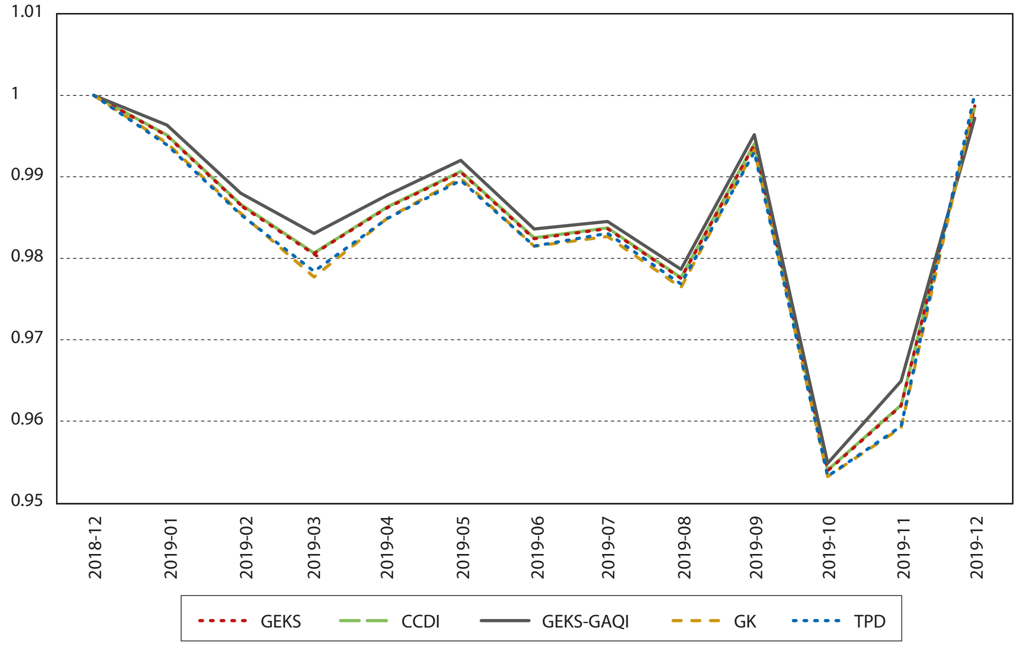
A comparison of the GEKS, the CCDI, Geary-Khamis (GK) and TPD indices for the *milk* dataset is shown in Figure 1 (at the GTIN level) and Figure 2 (at the COICOP 6 level). A comparison of the GEKS-L, GEKS-GL, GEKS-AQI, GEKS-AQU, and GEKS-GAQI indices for the *milk* dataset is shown in Figure 3 (at the GTIN level) and Figure 4 (at the COICOP 6 level). A similar analysis was conducted for the *coffee* dataset: Figure 5 and Figure 6 compare the GEKS, CCDI, GK, and TPD indices at the GTIN level, while Figure 7 and Figure 8 compare the GEKS-L, GEKS-GL, GEKS-AQI, GEKS-AQU, and GEKS-GAQI indices at the COICOP 6 data aggregation level. In addition, exact index values are presented for each dataset in Table 1 and Table 2.

Figure 1 Comparison of the GEKS, CCDI, GEKS-GAQI, TPD indices for the *milk* dataset at the GTIN level



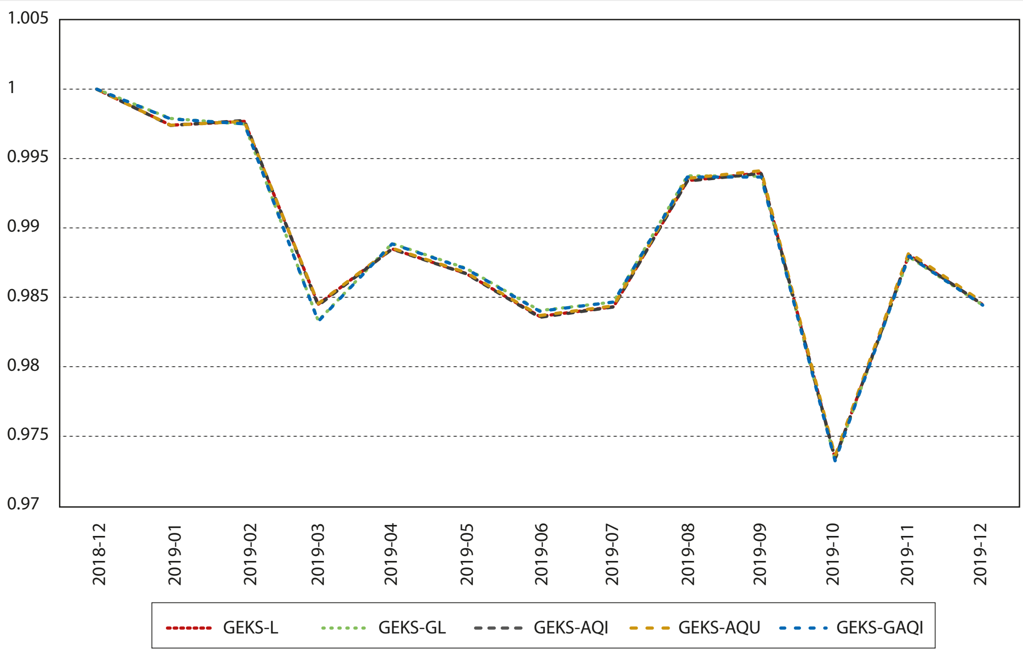
Source: Authorial computation

Figure 2 Comparison of the GEKS, CCDI, GEKS-GAQI, TPD indices for the *milk* dataset at the COICOP 6 level



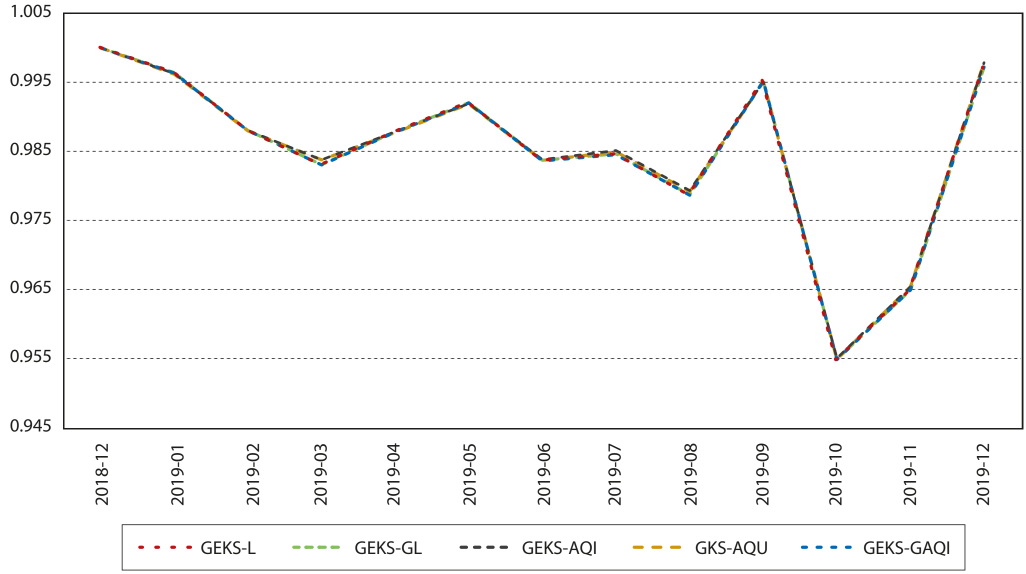
Source: Own computations

Figure 3 Comparison of the GEKS, GEKS-GL, GAQI, GEKS-AQU, GEKS-GAQI indices for the *milk* dataset at the GTIN level



Source: Own computations

Figure 4 Comparison of the GEKS, GEKS-GL, GAQI, GEKS-AQU, GEKS-GAQI indices for the *milk* dataset at the COICOP 6 level



Source: Own computations

At the GTIN level, smaller differences between index values are found for the indices presented in Figure 3 (GEKS-L, GEKS-GL, GEKS-AQI, GEKS-AQU, GEKS-GAQI). Similarly, at the COICOP 6 level, the differences between these indices are smaller than the differences between the other multilateral indices. A general observation is that smaller differences between the indices considered tend to occur at higher levels of data aggregation.

Table 2 Comparison of the GEKS-GAQI index and with the other indices for the *milk* dataset at two data aggregation levels

GTIN LEVEL

TIME	GEKS	CCDI	GEKS-L	GEKS-GL	GEKS-AQI	GEKS-AQU	GEKS-GAQI	GK	TPD
2019-01	1.0020172	1.0018004	0.99739	0.9978795	0.9973837	0.9973899	0.9978555	1.0065835	1.0060062
2019-02	1.000133	0.9997978	0.9976675	0.9974851	0.9977317	0.9976676	0.9974946	1.0008587	1.0008605
2019-03	0.9839258	0.9840643	0.9845048	0.9833189	0.9844226	0.9845519	0.9832747	0.9818819	0.9838784
2019-04	0.9936427	0.9932822	0.9884879	0.9888732	0.9884773	0.9885357	0.9888407	0.9956176	0.995459
2019-05	0.9899234	0.9898612	0.9867213	0.987091	0.9867022	0.9867849	0.9870584	0.9919439	0.9918611
2019-06	0.9889829	0.9888433	0.9836283	0.9840436	0.9835558	0.9836914	0.9839941	0.9924056	0.9923642
2019-07	0.9862652	0.9864494	0.9843188	0.9846598	0.9843093	0.9843833	0.9846664	0.9887818	0.9887632
2019-08	0.9981114	0.9978518	0.993411	0.9937479	0.9933716	0.9935633	0.9936708	1.0001467	0.9999378
2019-09	0.9952078	0.9951481	0.9939553	0.9937319	0.9939327	0.9941192	0.993664	0.9941353	0.9949736
2019-10	0.9776535	0.9773428	0.9733868	0.9732912	0.9733652	0.9735608	0.9732242	0.9795284	0.979855
2019-11	0.9805743	0.9815496	0.9880499	0.9879278	0.9880425	0.9882264	0.9879558	0.9780526	0.9786244
2019-12	0.9876664	0.9876167	0.9844756	0.984444	0.9844791	0.9846515	0.9844307	0.9895606	0.9896739

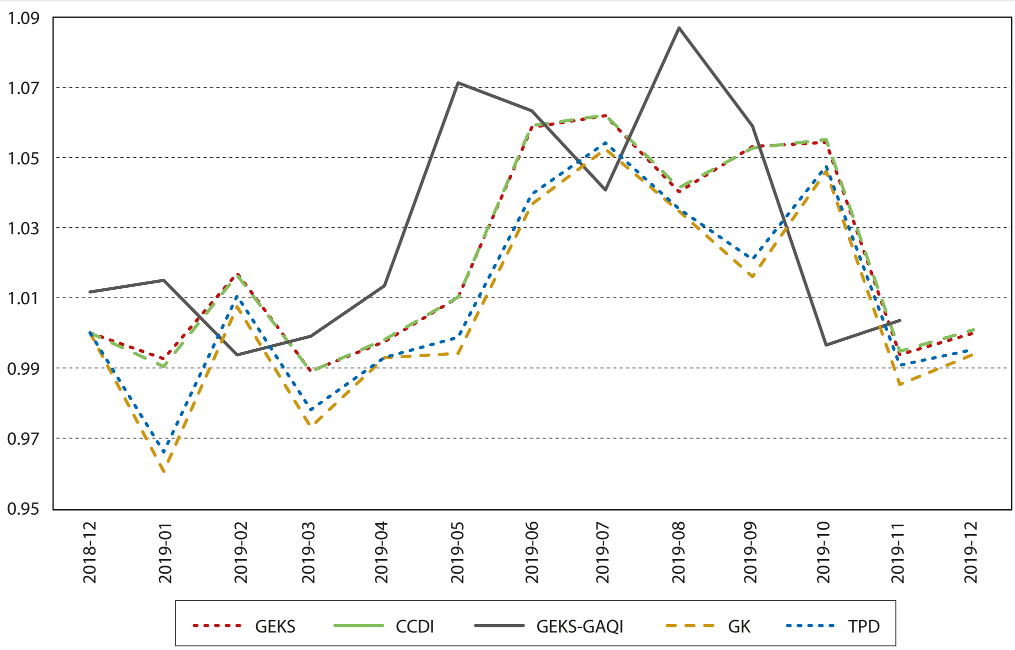
Table 2 (continuation)

COICOP 6 LEVEL									
TIME	GEKS	CCDI	GEKS-L	GEKS-GL	GEKS-AQI	GEKS-AQU	GEKS-GAQI	GK	TPD
2019-01	0.9951258	0.9950253	0.9962738	0.9964043	0.9961855	0.9962738	0.9963452	0.9941479	0.9939163
2019-02	0.9866407	0.9865364	0.9880469	0.988047	0.9879654	0.9880469	0.987992	0.9855307	0.9853641
2019-03	0.980645	0.9805819	0.9837793	0.9830692	0.9837639	0.9837793	0.9830445	0.9777051	0.9783779
2019-04	0.986313	0.9862242	0.9878349	0.9878334	0.9877271	0.9878349	0.9877601	0.9849386	0.9848847
2019-05	0.9906549	0.9905792	0.9919102	0.992044	0.991862	0.9919102	0.9920148	0.9898024	0.9895561
2019-06	0.9825183	0.9824019	0.9838127	0.9836925	0.9836782	0.9838127	0.9835999	0.9814701	0.9815032
2019-07	0.98374	0.9836404	0.9851054	0.9846292	0.9849333	0.9851054	0.9845176	0.9826784	0.9830589
2019-08	0.9776267	0.9775384	0.9793169	0.9787175	0.9791766	0.9793169	0.9786326	0.9764093	0.9768421
2019-09	0.9940356	0.9939354	0.9950924	0.9952164	0.9950268	0.9950924	0.9951712	0.9933037	0.9930228
2019-10	0.9540006	0.9539199	0.9550102	0.9548949	0.9549045	0.9550102	0.9548293	0.953199	0.9532513
2019-11	0.9619743	0.9618896	0.9653588	0.964897	0.9653985	0.9653588	0.9649294	0.9591934	0.9593704
2019-12	0.998605	0.9987082	0.9978269	0.9971573	0.9979069	0.9978269	0.9972056	0.9993562	1.000172

Source: Own calculations

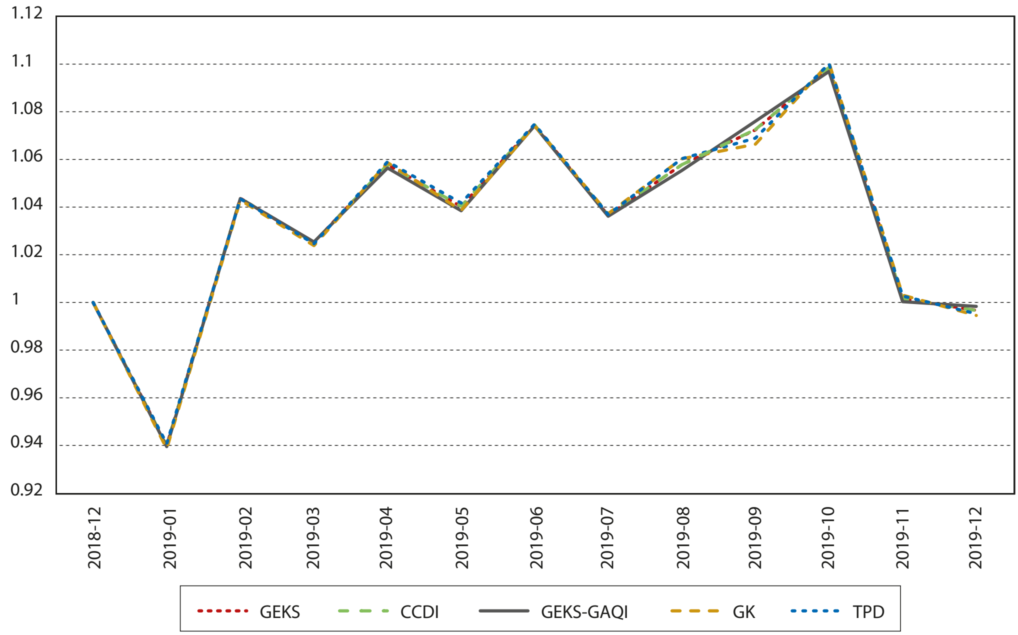
The differences between the indices (GEKS, CCDI, etc.) are relatively minor. The lines closely follow each other, indicating small variations.

Figure 5 Comparison of the GEKS, CCDI, GEKS-GAQI, TPD indices for the *coffee* dataset at the GTIN level



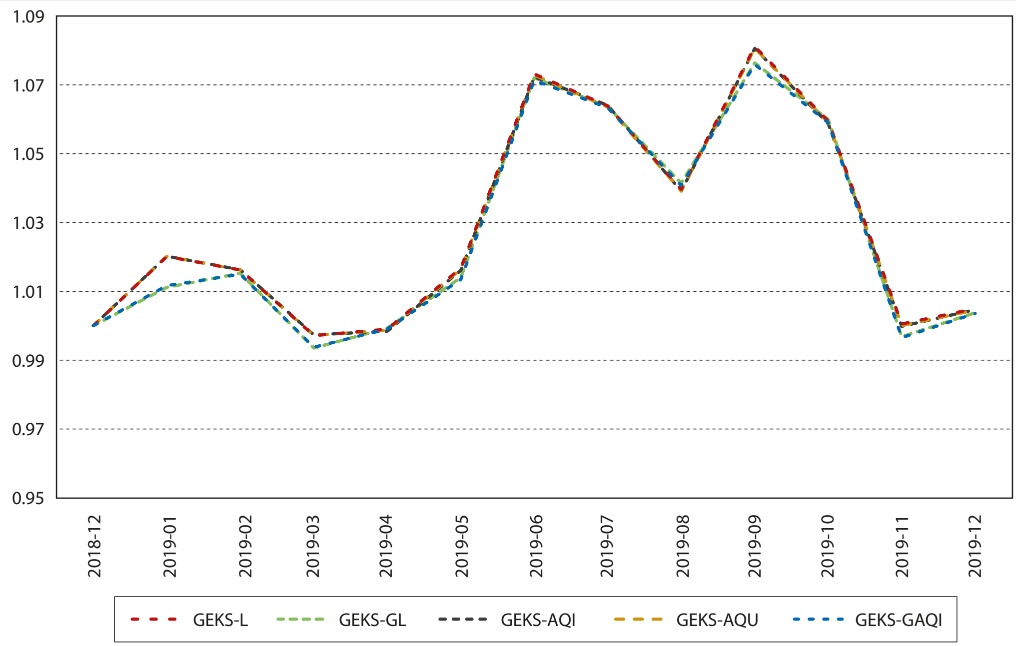
Source: Own computations

Figure 6 Comparison of the GEKS, CCDI, GEKS-GAQI, TPD indices for the *coffee* dataset at the COICOP 6 level



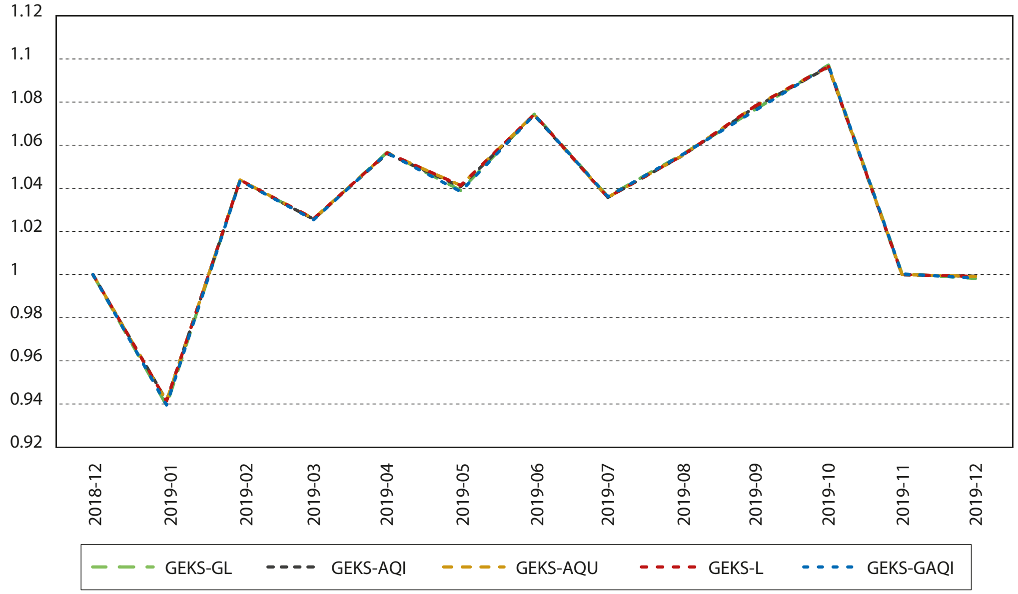
Source: Own calculations

Figure 7 Comparison of the GEKS, GEKS-GL, GAQI, GEKS-AQU, GEKS-GAQI indices for the *coffee* dataset at the GTIN level



Source: Own calculations

Figure 8 Comparison of the GEKS, GEKS-GL, GAQI, GEKS-AQU, GEKS-GAQI indices for the *coffee* dataset at the COICOP 6 level



Source: Own calculations

The largest differences for the *coffee* dataset were observed among the GEKS, CCDI, GEKS-GAQI, and TPD indices at the GTIN level. Moreover, at the COICOP 6 level, these differences decrease and mainly occur in the period 2019-07 to 2019-10. For GEKS, GEKS-GL,GAQI, GEKS-AQU, and GEKS-GAQI indices, the differences in values are also smaller at the COICOP level 6 compared to the GTIN level. Figure 7 and Figure 8 show greater stability in index values, indicating closer alignment between the considered indices. In contrast, Figure 5 and Figure 6 demonstrate larger fluctuations in index values and a more pronounced downward trend.

Table 3 Comparison of the GEKS-GAQI index with the other indices for the *coffee* dataset at two data aggregation levels

GTIN LEVEL									
TIME	GEKS	CCDI	GEKS-L	GEKS-GL	GEKS-AQI	GEKS-AQU	GEKS-GAQI	GK	TPD
2019-01	0.9926021	0.9903859	1.0201251	1.0111599	1.0201991	1.0201502	1.0116642	0.9605424	0.9660623
2019-02	1.0169978	1.0164447	1.0162029	1.0152675	1.0162685	1.0161897	1.0149682	1.0074853	1.010593
2019-03	0.9890729	0.9890071	0.9971429	0.9935341	0.9973624	0.997191	0.9937572	0.9731303	0.9780821
2019-04	0.9975123	0.998064	0.999012	0.9993254	0.9984175	0.9987502	0.9990596	0.9929061	0.9929982
2019-05	1.0101558	1.0101987	1.0167377	1.0139972	1.0160737	1.0164169	1.0134474	0.9941988	0.998733
2019-06	1.0585925	1.0591717	1.0731752	1.0720446	1.0723201	1.0728185	1.0713077	1.0366952	1.0395706
2019-07	1.0619259	1.0621074	1.0638811	1.0636037	1.064041	1.0635635	1.0633356	1.0524529	1.0541889
2019-08	1.0402685	1.0413992	1.0396131	1.041449	1.0391567	1.0391571	1.0407706	1.0346923	1.0353911
2019-09	1.0531954	1.0527549	1.0811057	1.0763807	1.0806033	1.0806315	1.0759171	1.0160246	1.0209771
2019-10	1.0543306	1.0551312	1.0595413	1.0599256	1.0587449	1.0590765	1.0590269	1.046137	1.0475339
2019-11	0.9937735	0.9947734	1.0005079	0.996838	0.9998289	1.000069	0.9965204	0.9852843	0.9907918
2019-12	0.9999563	1.0008778	1.004988	1.0037686	1.0046772	1.0045472	1.0035874	0.9939147	0.995287

Table 3 (continuation)

COICOP 6 level									
TIME	**GEKS**	**CCDI**	**GEKS-L**	**GEKS-GL**	**GEKS-AQI**	**GEKS-AQU**	**GEKS-GAQI**	**GK**	**TPD**
2019-01	0.9399788	0.9399845	0.9413398	0.9393232	0.9415082	0.9413398	0.9394949	0.9386217	0.9407134
2019-02	1.0436911	1.0436747	1.0438981	1.0438759	1.0435872	1.0438981	1.0435712	1.0429357	1.0434871
2019-03	1.025181	1.0251125	1.0259509	1.0256551	1.0257049	1.0259509	1.02541	1.0238247	1.0246086
2019-04	1.0578008	1.0578484	1.0564336	1.0567317	1.0560931	1.0564336	1.0564099	1.0586664	1.0590341
2019-05	1.0402501	1.0403372	1.0413519	1.0390375	1.0407758	1.0413519	1.0384942	1.0386484	1.0417217
2019-06	1.074446	1.0745063	1.0742963	1.0744018	1.0739492	1.0742963	1.0740775	1.0741201	1.0747576
2019-07	1.036507	1.0365144	1.0358323	1.0362459	1.0357696	1.0358323	1.0361837	1.0372845	1.0368635
2019-08	1.0577503	1.0577807	1.0549522	1.055459	1.0548623	1.0549522	1.055368	1.0602694	1.0602884
2019-09	1.0724209	1.0725615	1.0781131	1.0764596	1.0776213	1.0781131	1.0760158	1.0664161	1.0689335
2019-10	1.0985534	1.0985983	1.096479	1.0971887	1.0962252	1.096479	1.0969446	1.1004277	1.1001417
2019-11	1.0015007	1.0015133	0.9999942	1.0004513	0.9998789	0.9999942	1.0003346	1.0030366	1.002678
2019-12	0.9967013	0.9966486	0.9991691	0.9982211	0.9992552	0.9991691	0.998309	0.9945617	0.995269

Source: Own calculations

3.2 Distances of all indices from the benchmark GEKS-GAQI

Table 4 for the *milk* data set and Table 5 for the *coffee* dataset show the differences in distances between the target GEKS-GAQI index and the other indices.

Table 4 Distances between the benchmark GEKS-GAQI index and the other indices for the *milk* dataset at two data aggregation levels

Index	Distance	Underestimation	Overestimation
GTIN level			
GEKS	0.356	0.062	0.295
CCDI	0.336	0.053	0.282
GEKS-L	0.034	0.018	0.017
GEKS-GL	0.004	0.000	0.003
GEKS-AQI	0.035	0.019	0.016
GEKS-AQU	0.037	0.014	0.023
GK	0.550	0.094	0.456
TPD	0.540	0.078	0.462
COICOP 6 LEVEL			
Index	**Distance**	**Underestimation**	**Overestimation**
GEKS	0.141	0.130	0.012
CCDI	0.151	0.138	0.013
GEKS-L	0.032	0.002	0.030
GEKS-GL	0.006	0.001	0.005
GEKS-AQI	0.029	0.004	0.025
GEKS-AQU	0.032	0.002	0.030
GK	0.272	0.254	0.018
TPD	0.272	0.247	0.025

Source: Own calculations

For the *milk* dataset, the biggest differences in values were noted between the GEKS-GAQI and GK indices and between the GEKS-GAQI and TPD indices. The smallest differences were found between the GEKS-GAQI and GEKS-GL indices. This conclusion holds for both the GTIN and the COCIOP 6 levels.

Table 5 Distances between the benchmark GEKS-GAQI index and the other indices for the *coffee* dataset at two data aggregation levels

Index	Distance	Underestimation	Overestimation
GTIN level			
GEKS	0.659	0.642	0.017
CCDI	0.644	0.626	0.018
GEKS-L	0.259	0.010	0.249
GEKS-GL	0.045	0.006	0.039
GEKS-AQI	0.245	0.021	0.224
GEKS-AQU	0.242	0.016	0.226
GK	2.082	2.082	0.000
TPD	1.776	1.776	0.000
COICOP 6 level			
Index	Distance	Underestimation	Overestimation
GEKS	0.125	0.045	0.080
CCDI	0.127	0.045	0.082
GEKS-L	0.086	0.013	0.073
GEKS-GL	0.025	0.002	0.022
GEKS-AQI	0.081	0.021	0.060
GEKS-AQU	0.086	0.013	0.073
GK	0.259	0.137	0.122
TPD	0.249	0.092	0.157

Source: Own calculations

For the *coffee* dataset, the largest differences in distances are observed for the GK and TPD indices at both the GTIN and COICOP 6 levels. Conversely, the smallest differences occur between the GEKS-GAQI and GEKS-GL indices.

CONCLUSIONS

The overall conclusion of the study is not surprising and is in line with predictions: the benchmarking of indices is strongly dependent on the level of aggregation at which it is conducted. The COICOP 6 level provides more stable and consistent patterns, while the GTIN provides more volatility in index values. However, the following observation detected is interesting and quite surprising: for the milk collection, at the GTIN code level, the considered multilateral indices seem to generate higher values than the proposed GEKS-GAQI index. The opposite situation is observed at the COICOP 6 level using the same scanner dataset: here the multilateral indices under consideration seem to mainly generate lower values than the GEKS-GAQI index value. In contrast, for the c*offee* dataset, the situation is reversed: at the GTIN code level, the existing multilateral indices generate lower values than the proposed GEKS-GAQI index. Yet, at the COICOP 6 level and using the same scanner dataset, the existing multilateral indices generate higher values than the proposed GEKS-GAGI index.

Overall, our proposal for the GEKS-GAQI index appears to be methodologically and axiomatically correct. This index satisfies the restrictive *identity test*, which makes it, along with the GEKS-L and GEKS-GL indices, a rather unique multilateral index. In an empirical study, it turned out that indices satisfying the *identity test* (the GEKS-GAQI and GEKS-GL indices) generate the most similar values (see Tables 3 and 4). In addition, since the index does *quality adjusting*, it bridges the gap between the idea of GEKS-type indices and the Geary-Khamis method.

References

AUSTRALIAN BUREAU OF STATISTICS. (2016). *Making Greater Use of Transactions Data to Compile the Consumer Price Index*. Information Paper 6401.0.60.003, Canberra.

BALK, M. (1995). Axiomatic price index theory: a survey. *Int. Statist. Rev.*, 63: 69–95.

BIAŁEK, J., BERĘSEWICZ, M. (2021). Scanner data in inflation measurement: from raw data to price indices. *The Statistical Journal of the IAOS*, 37: 1315–1336.

BIAŁEK, J., PANEK, T., KŁOPOTEK, M. (eds.). (2022). *Nowoczesne technologie i nowe źródła danych w pomiarze inflacji*. Warsaw: GUS.

BIAŁEK, J. (2022). Scanner data processing in a newest version of the PriceIndices package. *Statistical Journal of the IAOS*, 38(4): 1369–1397.

BIAŁEK, J. (2023a). Quality adjusted GEKS-type indices for price comparisons based on scanner data. *Statistics in Transition New Series*, 24: 151–169.

BIAŁEK, J. (2023b). Improving quality of the scanner CPI: proposition of new multilateral methods [online]. Quality and Quantity, 57: 2893–2921. <https://doi.org/10.1007/s11135-022-01506-6>.

CARLI, G. (1804). Del valore e della proporzione de'metalli monetati. In: *Scrittori Classici Italiani di Economia Politica*, 13: 297–336.

CAVES, D. W., CHRISTENSEN, L. R. , DIEWERT, W. E. (1982), Multilateral comparisons of output, input, and productivity using superlative index numbers. *Economic Journal*, 92(365): 73–86.

CHESSA, A. G., VERBURG, J., WILLENBORG, L. (2017). A comparison of price index methods for scanner data. In: *15th Meeting of the Ottawa Group*, Eltville, 10–12.

DE HAAN, J., KRSINICH, F. (2018). Time dummy hedonic and quality-adjusted unit value indexes: Do they really differ? *Review of Income and Wealth,* 64(4): 757–776.

ELTETÖ, O., KÖVES, P. (1964). On a problem of index number computation relating to international comparison. *Statisztikai Szemle,* 42(10): 507–518.

EUROSTAT. (2018). *Practical guide for processing supermarket scanner data*. Harmonised Index of Consumer Prices.

FISHER, I. (1922). *The making of index numbers: a study of their varieties, tests, and reliability, Number 1*. Houghton Mifflin.

GEARY, R. C. (1958). A note on the comparison of exchange rates and purchasing power between countries. *Journal of the Royal Statistical Society: Series A*, 121(1): 97–99.

GINI, C. (1931). On the circular test of index numbers. *Metron,* 9(9): 3–24.

JEVONS, W. S. (1865). On the variation of prices and the value of the currency since 1782. *Journal of the Statistical Society of London,* 28(2): 294–320.

KHAMIS, S. H. (1972). A new system of index numbers for national and international purposes. *Journal of the Royal Statistical Society: Series A,* 135(1): 96–121.

INTERNATIONAL LABOUR OFFICE. (2004). *Consumer Price Index Manual: Theory and Practice*. Geneva.

LAMBORAY, C. (2017). The geary khamis index and the lehr index: how much do they differ. In: *15th Meeting of the Ottawa Group*, 10–12.

LASPEYRES, K. (1871). Die Berechnung einer Mittleren Waarenpreissteigerung. *Jahrbücher für Nationalökonomie und Statistik*, 16(1): 296–318.

TÖRNQVIST, L. (1936). The bank of Finland's consumption price index. *Bank of Finland Monthly Bulletin*, 1–8.

WALSH, C. M. (1901). *The Measurement of General Exchange Value*. Macmillan and Co.

VAN LOON, K. V., ROELS, D. (2018). Integrating big data in the Belgian CPI. In: *Meeting of the group of experts on consumer price indices*, Geneva, Switzerland, 8–9 May.

VON AUER, L. (2019). The nature of chain drift. In: *17th Meeting of the Ottawa Group on Price Indices*, Rio de Janerio, Brasil, 8–10 May.

VON DER LIPPE, P. (2007). *Index Theory and Price Statistics*. Berlin, Germany: Peter Lang.