Control Charts for Processes with an Inherent Between-Sample Variation

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Abstract

A number of processes to which statistical control is applied are subject to various effects that cause random changes in the mean value. The removal of these fluctuations is either technologically impossible or economically disadvantageous under current conditions. The frequent occurrence of signals in the Shewhart chart due to these fluctuations is then undesirable and therefore the conventional control limits need to be extended. Several approaches to the design of the control charts with extended limits are presented in the paper and applied on the data from a real production process. The methods assume samples of size greater than 1. The performance of the charts is examined using the operating characteristic and average run length. The study reveals that in many cases, reducing the risk of false alarms is insufficient.

Keywords

Statistical process control, modified chart, acceptance chart, variance component chart, average run length

JEL code

C44, C83, L15

INTRODUCTION

Control charts, the main tool of statistical process control (SPC), have been used for more than 80 years. The idea behind control charts is to separate the variation due to assignable causes from the random variation that is inherent to a process. Apart from the Shewhart chart introduced in 1931, the CUSUM chart based on cumulative sums or the EWMA chart using exponentially weighted moving averages belong to the best known ones. All these charts are based on the assumption that unless some special causes exist in a process, its parameters are constant.

The extensive study of real production processes in Germany performed by Kaiser et Nowack (2000; cf. Michálek, 2001) revealed that only 2% of processes met the assumption of constant parameters. Within the Six Sigma approach, the goal is no longer to maintain a constant mean value; it is allowed to move around the target as long as the process output conforms to specification. This applies to processes in which the variation within a sample taken from the process is very small as compared with the allowable variation given by the specification limits. When samples are taken from a process, the differences between their averages are greater than would correspond to the within-sample variation on which the conventional Shewhart control limits are based, which results in frequent alarms. Bringing such

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processes to the state when the process mean is constant could be technically or economically impossible and, therefore, common changes of the process mean are considered a part of the inherent process variation.

To allow for the process mean's changes that are still acceptable, the control limits have to be extended. Several methods have been introduced in the literature but the information about the properties of the resulting charts is missing. The aim of the paper is to examine and discuss the effectiveness of some selected control charts. The attention is paid to $\overline{X}$ -charts. As with the conventional $\overline{X}$ chart, the within-sample variation is monitored using the *R*-chart.

1 OVERVIEW OF CONTROL CHARTS WITH EXTENDED LIMITS

Limiting to cases when samples of size greater than 1 are taken from a process, two main approaches can be distinguished. The first approach uses specification limits *USL* and *LSL*, the other is based on the inherent process variability.

The charts based on the specification limits were introduced and discussed long ago (Rissik, 1943; Hill, 1956; Freund, 1957). The centre line of the conventional control chart is replaced by bounds μ_U and μ_L for the true process mean and the usual 3-sigma limits are drawn outwards from the interval (μ_L, μ_U). The resulting charts are called modified or acceptance control charts and they differ in how the interval limits are determined. These charts were presented by many authors without any criticism (Duncan, 1986; Montgomery, 2009; Mitra, 2008; Wadsworth et al., 2002). They are also included in the standard ISO 7870-3:2012. On the other hand, Bissell (1994) and Wheeler (2004) remark that this approach is contrary to the philosophy of continuous improvement because there is no incentive for reducing variability.

The charts based on the inherent process variation appear to be less referred to in the literature. None of them is mentioned in the books listed above, with the exception of Bissell (1994), who introduces some of the methods from Section 4. Methods of constructing control limits can be divided into two groups.

The first group includes methods in which the standard deviation representing the inherent variation is estimated using sample averages. Cryer et Ryan (1990) advocate the use of the overall standard error, Wheeler et Chambers (1986), Woodall et Thomas (1995), Laubscher (1996) and Bissell (1994) use moving ranges (or their squares) of sample averages.

The methods in the second group are based on the ANOVA model with random effects and variance components, which represent the within-sample and between-sample variability. The variance component chart (Laubscher, 1996; Woodall et Thomas, 1995; Wetherill et Brown, 1991) employs 3-sigma limits; the standard deviation of sample averages is derived using the ANOVA model. Dietrich et Schulze (2010) suggest an approach that is similar to the one using the specification limits, however, the bounds for the mean are based on the between-sample variance component.

2 PROCESS CAPABILITY

As mentioned above, the extended control limits are generally used in situations where the within-sample variation is considerably smaller than the allowable range *USL* – *LSL*. Such processes are called highly capable. Process capability reflects the ability of a process where no assignable causes are present to function in such a manner that its output, represented by a quality characteristic distribution, lies almost completely within specification limits *USL* and *LSL*. The concept of process capability was introduced in the '80s (Sullivan, 1984; Kane, 1986). The most common indices C_p and C_{pk} are given by the formulas (see e.g. Kotz et Johnson, 2002):

$$C_p = \frac{USL - LSL}{6\sigma} \qquad C_{pk} = \min(C_{pU}, C_{pL}), \tag{1}$$

where

$$C_{pU} = \frac{USL - \mu}{3\sigma} \qquad C_{pL} = \frac{\mu - LSL}{3\sigma} \ . \tag{2}$$

It is assumed that the process output is normally distributed with constant μ and σ over time, where σ is the measure of the within-sample variability. The assumption of the constant parameters is verified by control charts. While the C_p index measures only the process ability to meet the specification limits and its construction assumes μ as the midpoint of these limits (usually a target), C_{pk} accounts for the real process location. The difference between C_{pk} and C_p represents the potential improvement to be attained by centering the process. The generally accepted minimum value for C_p is 1.33. In the Six Sigma methodology, C_p of 2 is considered the aim of a process improvement; even if the mean of such process shifts by 1.5σ from the midpoint, no serious problems arise since the expected fraction nonconforming is as low as 3.4 ppm (C_{pk} equals 1.5 in this case). Referring to the shift of 1.5 σ relates to the performance of the conventional Shewhart chart – the shifts smaller than 1.5σ may be detected quite late by this chart.

It should be noted that the overall performance of a process with an inherent between-sample variation is evaluated using the performance indices P_p and P_{pk} recommended by the Automotive Industry Action Group (AIAG, 2005). These indices take into account the total process variation. However, the construction of the charts in Section 4.2 is based on the short-time process behaviour and therefore the capability index C_{pk} is considered when the charts are designed.

3 PERFORMANCE OF CONTROL CHARTS

The performance of control charts is evaluated using an operating characteristic (OC). The OC curve describes the relationship between the probability β of not detecting a shift from the reference value μ_0 to $\mu = \mu_0 + k\sigma$ on the first subsequent sample. Considering the conventional $\overline{X}$-chart (control chart for averages) we can write:

$$\beta = P(LCL \le \overline{x} \le UCL \mid \mu = \mu_0 + k\sigma) , \tag{3}$$

where the magnitude of the shift is expressed in k-multiples of σ. Usually the normal distribution N(μ, σ^2/n) of sample averages with known parameters is assumed. When evaluating the performance of charts with extended control limits, the magnitude of a shift will be expressed in k-multiples of total standard deviation σ_x and the distribution $N(\mu, \sigma_{\overline{x}}^2)$ of sample averages will be considered.

In SPC the average run length (ARL) is widely used. It is the expected value of the number of samples taken until the first point exceeds a control limit. If the values of a plotted characteristic can be considered independent, run lengths have the geometric distribution G(1– β) and

$$ARL = \frac{1}{1 - \beta} \ . \tag{4}$$

In a certain control chart, ARL depends on the shift magnitude $k\sigma$ ($k\sigma_x$). The average run length $ARL(0)$ for $k = 0$ is an important characteristic; it should be as large as possible since frequent false alarms may lead to overcontrol, which results in a larger variation of the process output, or at least they discourage operators. $ARL(0)$ of the conventional $\overline{X}$-chart is 370.4, which means that the false alarm (type I error) can be expected after 370 samples on average. Conversely, ARL for a given $k > 0$ should be as small as possible so that the shift of $k\sigma$ ($k\sigma_x$) can be detected quickly.

4 CONTROL CHARTS BASED ON SPECIFICATION LIMITS

The bounds μ_U and μ_L for the true process mean are based on the specified proportion of units exceeding limits *USL* and *LSL*. Formulas for calculation of control limits assume that the distribution from which the sample comes is normal with the current value of μ and variance σ^2, i.e. only the within-sample variation is taken into account.

4.1 Modified and acceptance control charts

The aim of the modified control chart is to determine whether the process mean is within interval (μ_L, μ_U) such that the fraction nonconforming does not exceed the chosen value p_A. The bounds for the process mean are given by the formulas:

$$\mu_U = USL - u_{1-p_A}\sigma \qquad \mu_L = LSL + u_{1-p_A}\sigma \,, \tag{5}$$

where σ denotes the within-sample standard deviation, which is often estimated using the average of sample ranges, $\hat{\sigma} = \overline{R} / d_2$. Values of d_2 can be found in ISO 7870-2:2013 or any book dealing with Shewhart control charts. The control limits are drawn outwards from the interval (μ_L , μ_U) and are positioned at:

$$UCL = USL - \left(u_{1-p_A} - \frac{u_{1-\alpha}}{\sqrt{n}} \right)\hat{\sigma} \qquad LCL = LSL + \left(u_{1-p_A} - \frac{u_{1-\alpha}}{\sqrt{n}} \right)\hat{\sigma} \;, \tag{6}$$

where α is the type I error risk.

As Hill (1956, p. 16) points out, the control limits determined by (6) "accept a sample mean nearer to the tolerance when there is less information, than when there is more information", which is an undesirable feature. He suggests that the other possibility should be used; the bounds for the mean are:

$$\mu_U = USL - u_{1-p_R}\sigma \qquad \mu_L = LSL + u_{1-p_R}\sigma \,, \tag{7}$$

and for process fraction nonconforming p_R to be rejected with probability 1 – β the control limits are:

$$UCL = USL - u_{1-p_R}\hat{\sigma} - \frac{u_{1-\beta}}{\sqrt{n}}\hat{\sigma} \qquad LCL = LSL + u_{1-p_R}\hat{\sigma} + \frac{u_{1-\beta}}{\sqrt{n}}\hat{\sigma} \,. \tag{8}$$

In this case the control limits lie within the interval (μ_L, μ_U) and they are nearer to the specification limits for larger sample sizes. The latter chart is sometimes classified as a variant of the acceptance chart (Montgomery, 2013; Mitra, 2008).

The acceptance control chart (Freund, 1957) is based on both risks α and β related to p_A and p_R and therefore the sample size must meet the condition:

$$n = \frac{u_{1-\alpha} + u_{1-\beta}}{u_{1-p_A} - u_{1-p_R}} \,, \tag{9}$$

which follows from equating either the upper or the lower control limits in (6) and (8).

4.2 Choice of parameters

In the following considerations only a predetermined sample size n is assumed.

The choice of p_A, p_R, α and β or directly percentiles u_{1-p_A}, u_{1-p_R}, $u_{1-\alpha}$ and $u_{1-\beta}$ will affect the chart performance. The values $u_{1-\alpha}$ = 3 and $u_{1-\beta}$ = 1.65 corresponding to the risks of 0.00135 and 0.05 are mostly used and will be applied here, too. The choice of u_{1-p_A}, u_{1-p_R} requires some attention.

So that the modified limits (6) are wider than the conventional 3-sigma limits, the following inequality must apply:

$$USL - LSL > 2u_{1-p_A}. \tag{10}$$

The choice of p_A can be based on the value of process capability index C_{pk}. Using relations (1) and (2) and the minimum acceptable value 1.33 of C_{pk}, we get $u_{1-p_A} = 4$ and $USL - LSL > 8\sigma$.

Some authors use $u_{1-p_A} = 3$ or $u_{1-p_A} = 4.5$ (Jarošová et Noskievičová, 2015, p. 81).

As for u_{1-p_R}, the control limits (8) are wider than the conventional 3-sigma limits if the condition

$$USL - LSL > 2\left(u_{1-p_R} + \frac{4.65}{\sqrt{n}} \right)\sigma \tag{11}$$

is met. For $u_{1-p_R} = 2.33$ corresponding to the fraction nonconforming of 0.01 (see e.g. Montgomery, 2013, p. 442) and $n = 5$ we get $USL - LSL > 9\sigma$, approximately.

It should be emphasized that the requirement of wider control limits itself does not guarantee the expected properties of a control chart, namely a sufficiently high value of $ARL(0)$ and a reasonably low value of ARL for a shift that should be detected. ARL depends on the variability of sample averages, which is affected both by within-sample and by between-sample variation.

5 CONTROL CHARTS BASED ON THE INHERENT VARIABILITY

The between-sample variation as a part of the inherent variation of a process is taken into account when constructing control limits. In most cases 3-sigma limits are used like in the Shewhart chart, i.e.:

$$UCL = \bar{\bar{x}} + 3\sigma_{\bar{x}} \qquad LCL = \bar{\bar{x}} - 3\sigma_{\bar{x}}, \tag{12}$$

where $\bar{\bar{x}}$ denotes the total average and $\sigma_{\bar{x}}$ the standard deviation of sample averages. Two main approaches to estimate $\sigma_{\bar{x}}$ exist: the first approach is based on sample averages, the second approach uses the ANOVA model and variance components.

Table 1 Estimation of $\sigma_{\bar{x}}$ using sample averages

	Standard error estimate	References
Overall standard error	$\frac{s_{\bar{x}}}{c_4} = \frac{1}{c_4}\sqrt{\frac{\sum_{j=1}^{m}(\bar{x}_j - \bar{\bar{x}})^2}{m-1}}$	Cryer et Ryan (1990)
Average moving range	$\frac{\overline{MR}}{d_2} = \frac{1}{d_2}\frac{\sum_{j=2}^{m}\lvert \bar{x}_j - \bar{x}_{j-1} \rvert}{m-1}$	Wheeler et Chambers (1986), Woodall et Thomas (1995)
Median moving range	$\frac{MeR}{d_4} = \frac{1}{d_4} Me \lvert \bar{x}_j - \bar{x}_{j-1} \rvert$	Laubscher (1996)
Square root of MSSD	$\frac{1}{c_4}\sqrt{\frac{1}{2}MSSD} = \frac{1}{c_4}\sqrt{\frac{1}{2}\frac{\sum_{j=2}^{m}(\bar{x}_j - \bar{x}_{j-1})^2}{m-1}}$	Bissell (1994)

Source: Own construction

5.1 Charts based on sample averages

Several methods of estimating $\sigma_{\bar{x}}$ together with references are listed in Table 1, where m is the number of samples, $\bar{x}_j$ (j = 1, 2, ..., m) are sample averages and $\bar{\bar{x}}$ is the total average. Unbiasing constants c_4, d_2, and d_4 can be found for example in Wheeler (2004, p. 416), c_4 and d_2 are also available in ISO 7870-2:2013. Constant c_4 depends on the number of samples m (differently from its use in the conventional Shewhart charts, where the sample size is cardinal), constants d_2 a d_4 relate to the use of differences between two adjacent observations and therefore correspond to the "sample size" of 2. Consequently, d_2 = 1.128 and d_4 = 0.9539 are always used in these calculations. Woodall et Montgomery (2000) examined the bias of various estimates when a shift in the mean is present and concluded that the estimates based on moving ranges are preferable. Moreover, the use of the median can reduce or possibly eliminate the bias.

5.2 Charts based on variance components

Variance component chart (Laubscher, 1996; Woodall et Thomas, 1995; Wetherill et Brown, 1991) is based on the model:

$$x_{ij} = \mu_0 + a_j + \varepsilon_{ij}, \tag{13}$$

where μ_0 denotes the grand process mean, $a_j \sim \mathrm{N}(0, \sigma_A^2)$ is the random effect of sample j, and $\varepsilon_{ij} \sim \mathrm{N}(0, \sigma^2)$ represents the within-sample variation. Under the assumption that a_j and ε_{ij} are independent, we can write:

$$\sigma_x^2 = \sigma_A^2 + \sigma^2, \tag{14}$$

and

$$\sigma_{\bar{x}}^2 = \sigma_A^2 + \frac{\sigma^2}{n}. \tag{15}$$

Control limits are given by (12).

ANOVA is used to estimate variance components σ_A^2 and σ^2:

$$\hat{\sigma}_A^2 = \frac{MSA - MSE}{n} \qquad \hat{\sigma}^2 = MSE, \tag{16}$$

where

$$MSA = \frac{n\sum_{j=1}^{m}(\bar{x}_j - \bar{\bar{x}})^2}{(m-1)} \qquad MSE = \frac{\sum_{j=1}^{m}\sum_{i=1}^{n}(x_{ij} - \bar{x}_j)^2}{m(n-1)}, \tag{17}$$

measure the between-sample and within-sample variability, respectively.

Another approach is recommended by Dietrich et Schulze (2010). It is also based on the random-effect model (13) but the control limits are constructed differently: the bounds for the process mean are distant by ±Δ from the centre line and the common 3-sigma limits are drawn outwards from them:

$$UCL = \bar{\bar{x}} + \Delta + \frac{3\hat{\sigma}}{\sqrt{n}} \qquad LCL = \bar{\bar{x}} - \Delta - \frac{3\hat{\sigma}}{\sqrt{n}}. \tag{18}$$

The authors suggest to choose $\Delta = 1.5\,\hat{\sigma}_A$, where $\hat{\sigma}_A^2$ and $\hat{\sigma}^2$ are given by (16).

6 EXAMPLE

The process in which steel frames are moulded has specification limits USL = 35.1 mm and LSL = 34.9 mm. Samples of size 5 are taken from the process and the $\overline{X}$-chart and R-chart with centre lines determined by $\overline{\overline{x}}$ = 35.0645 and $\overline{R}$ = 0.008 are drawn (Figure 1). The estimated within-sample standard deviation $\hat{\sigma} = \overline{R} / d_2 = 0.0035$ results in $\hat{C}_p$ = 9.46, indicating a highly capable process. Although more than half of the points lie outside the conventional control limits UCL_S and LCL_S, most of them are not considered to signal an assignable cause and therefore the control limits should be extended. All the methods described above were used and the resulting limits together with their distance are shown in Table 2. Three columns on the right contain the values of standard deviations used in the calculations. The variance components from Section 5.2 are displayed in the ANOVA table (Table 3).

Table 2 Extended control limits

	LCL	*UCL*	*UCL – LCL*	$\hat{\sigma}_{\bar{x}}$	$\hat{\sigma}$	$\hat{\sigma}_A$
Cryer	35.0386	35.0905	0.0519	0.0086	-	-
Wheeler	35.0451	35.0840	0.0389	0.0065	-	-
Laubscher	35.0485	35.0806	0.0321	0.0053	-	-
Bissell	35.0440	35.0850	0.0410	0.0068	-	-
Woodall	35.0389	35.0902	0.0513	0.0086	0.0036	0.0084
Dietrich	35.0471	35.0820	0.0348	-	0.0036	0.0084
Modif.	34.9094	35.0906	0.1813	-	0.0035	-
Accept.	34.9108	35.0892	0.1784	-	0.0035	-

Source: Own construction

Table 3 ANOVA, Variance component analysis

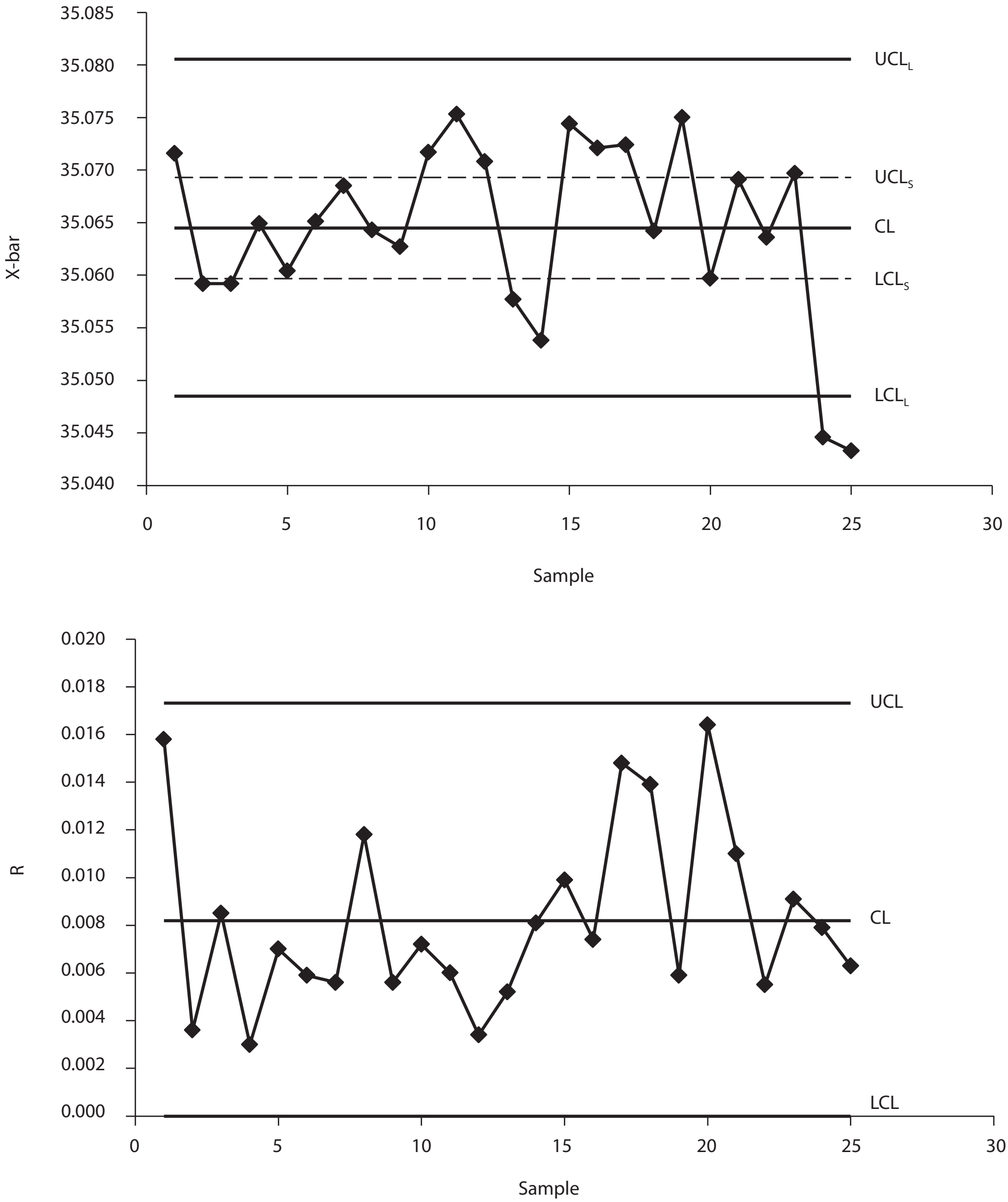
Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value	Var. Comp.
Between groups	0.008785	24	0.000366	28.46	0.0000	0.000071
Within groups	0.001286	100	0.000013			0.000013
Total (Corr.)	0.010071	124				

Source: Statgraphics

Different estimates of $\hat{\sigma}_{\bar{x}}$ obtained by various methods are due to the apparent shift before the last two samples (Figure 1). It should be noted that after the retrospective analysis, such points are usually omitted and the control limits revised, in which case the differences between the various constructions would be much smaller. The narrowest pair of the extended control limits (UCL_L and LCL_L), i.e. the one obtained using the median moving range according to Laubscher (1996), is drawn in Figure 1.

The distance of the modified and acceptance limits is several times larger, thus confirming the criticism of Bissell (1994) and Wheeler (2004).

Figure 1 $\overline{X}$ and R control chart



Source: Own construction

7 COMPARISON OF SELECTED CHARTS

To compare the performance of control charts, only the case with known parameters σ_A and σ is considered. The performance of the charts based on different estimates of $\sigma^2_{\bar{x}}$ is not examined – it would require carrying out some simulations.

To calculate the OC curves, the sample size of 5 and the within-sample $\sigma = 1$ were chosen. Several scenarios defined by the different ratio of σ_A/σ were applied. Mean's shifts were expressed as multiples of σ_x, where $\sigma_x = \sqrt{\sigma_A^2 + \sigma^2}$. In addition, percentiles $u_{1-p_A} = 4$, $u_{1-p_R} = 2.33$ and two values of C_p, namely C_p = 1.67 and C_p = 2.67 were used to determine the modified and acceptance control limits. The control limits according to Woodall (Eq. 12) and Dietrich et Schulze (Eq. 18) for the chosen σ depend only on the ratio σ_A/σ. Values of ARL for the selected charts are given in Table 4 to 6. Comparing charts, we focus on the $ARL(0)$ and $ARL(1.5\sigma_x)$, when the process mean shifts by $1.5\sigma_x$ from the reference value.

Table 4 *ARL* for the modified control chart

	$ARL(k\sigma_x)$							
	$USL - LSL = 10\sigma$ (C_p = 1.67)				$USL - LSL = 16\sigma$ (C_p = 2.67)			
k	$\sigma_A = 0.5\sigma$	$\sigma_A = 1\sigma$	$\sigma_A = 1.5\sigma$	$\sigma_A = 2\sigma$	$\sigma_A = 0.5\sigma$	$\sigma_A = 1\sigma$	$\sigma_A = 1.5\sigma$	$\sigma_A = 2\sigma$
0	2 075.8	30.7	7.4	3.9	5.6E+14	9.2E+05	1554.4	109.3
0.5	253.5	14.2	5.1	3.1	2.0E+12	8.6E+04	432.6	48.9
1	29.3	5.0	2.7	2.0	6.6E+09	5.9E+03	84.1	15.4
1.5	6.2	2.4	1.7	1.4	4.3E+07	6.1E+02	21.7	6.0
2	2.3	1.5	1.3	1.2	5.5E+05	9.2E+01	7.5	3.0

Source: Own construction

Table 5 *ARL* for the acceptance control chart

	$ARL(k\sigma_x)$							
	$USL - LSL = 10\sigma$ (C_p = 1.67)				$USL - LSL = 16\sigma$ (C_p = 2.67)			
k	$\sigma_A = 0.5\sigma$	$\sigma_A = 1\sigma$	$\sigma_A = 1.5\sigma$	$\sigma_A = 2\sigma$	$\sigma_A = 0.5\sigma$	$\sigma_A = 1\sigma$	$\sigma_A = 1.5\sigma$	$\sigma_A = 2\sigma$
0	256.0	13.0	4.6	2.9	5.3E+12	1.5E+05	619.5	62.4
0.5	49.6	7.2	3.5	2.4	2.9E+10	1.8E+04	197.1	30.5
1	9.0	3.1	2.1	1.7	1.6E+08	1.5E+03	44.1	10.6
1.5	2.9	1.8	1.4	1.3	1.7E+06	2.0E+02	13.0	4.5
2	1.5	1.3	1.2	1.1	3.5E+04	3.7E+01	5.1	2.4

Source: Own construction

The performance of both control charts based on specification limits is similar. For the process with C_p = 1.67, $ARL(0)$ is acceptably high only for $\sigma_A < \sigma$. Values of σ_A comparable with σ or higher result in frequent false alarms. On the other hand, $ARL(1.5\,\sigma_x)$ for the process with C_p = 2.67 and $\sigma_A < 2\sigma$ is unacceptably high, which means that such shifts may be detected quite late.

Due to the use of 3-sigma limits, the varcomp chart by Woodall keeps $ARL(0)$ at the same level regardless of σ_A, but ARL increases with σ_A (Table 6). Differently from the Shewhart chart, $ARL(1.5\,\sigma_x)$ is much longer for σ_A equal to or greater than σ. The chart by Dietrich et Schulze reveals relatively fast shifts of $1.5\,\sigma_x$ and greater regardless of σ_A, but for σ_A equal to or greater than σ, $ARL(0)$ is too small and hence the risk of false alarm is high.

Table 6 *ARL* of the control charts based on variance components

	$ARL(k\sigma_x)$							
	Woodall				*Dietrich, Schulze*			
k	$\sigma_A = 0.5\sigma$	$\sigma_A = 1\sigma$	$\sigma_A = 1.5\sigma$	$\sigma_A = 2\sigma$	$\sigma_A = 0.5\sigma$	$\sigma_A = 1\sigma$	$\sigma_A = 1.5\sigma$	$\sigma_A = 2\sigma$
0	370.4	370.4	370.4	370.4	549.3	105.4	46.0	29.3
0.5	65.8	106.3	127.4	137.9	89.3	38.1	22.3	16.2
1	11.0	22.9	31.0	35.5	13.6	10.4	7.9	6.5
1.5	3.2	7.0	9.8	11.6	3.7	3.9	3.5	3.2
2	1.6	3.0	4.1	4.8	1.7	2.0	2.0	1.9

Source: Own construction

CONCLUSION

It appears that the modified and acceptance control charts generally do not perform well when the process mean fluctuates randomly. This is likely why Wheeler claims that modified limits "can never work as well as the data-based three-sigma limits" (Wheeler, 2004, p. 21) or "It will only encourage alternating periods of benign neglect and intense panic." (Wheeler, 2004, p. 346). Although the performance of these charts is influenced by the extent of the between-sample variation, this is not taken into account when the bounds for the mean are chosen.

The approach by Dietrich et Schulze (2010) is similar to the modified chart, but the bounds for the mean are derived from the between-sample variation. However, the recommended choice of $1.5\,\sigma_A$ from the centre line results in small values of $ARL(0)$ for larger σ_A.

The varcomp chart (and similarly it can be said about the other charts based on sample averages) detects a mean's shift slower than the previous chart for σ_A equal to or greater than σ, but regardless of σ_A, it retains the desired value of $ARL(0)$.

The properties of the control charts were examined under the assumption of the normal distribution of sample averages with known parameters. As with the conventional Shewhart chart, the values of ARL will be influenced by departures from normality and by the fact that the parameters are usually estimated. The frequently chosen sample size of 5 was considered here. With exception of $ARL(0)$ with the varcomp chart, both $ARL(0)$ and ARL depend on the sample size. For most control charts, the increasing sample size leads, as with the Shewhart chart, to the narrower control limits and hence to lower values of ARL. The opposite is true with the acceptance chart; the control limits become wider and the values of ARL are higher when the sample size increases.

Based on the study, the charts using the specification limits and the chart according to Dietrich and Schulze (2010) are not advisable for use primarily because of the possibility of frequent false signals. The use of the two former charts could be taken into account in the statistical control of processes with a trend (Bissell, 1994; Jarošová and Noskievičová, 2015).

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