

Methodological notes

The calculation of complete mortality life tables for districts and administrative districts of municipalities with extended powers (AD MEP) is based on age-period specific rates (m_x). The input death probabilities are derived from them using the indirect method. The life tables have been processed as complete (one-year age intervals), separately for males and females, and for a five-year period to exclude random fluctuations. Nevertheless, **low numbers of deaths and used (in time and spatially uniform) methodology for smoothing the function of probability of death (life table function) can stand behind the fluctuations in time series**. The complete life tables are published only for the districts. For AD MEP only life expectancies at selected ages are presented.

Complete life tables indicators

Number of deaths (D_x) states the absolute number of deaths by age (x) in a given area during the reference period.

Number of inhabitants (P_x) states the mid-population (sum of stocks in given years) in a given area by age (x). In case of districts, it is the population as at 1 July obtained by population balance. In case of AD MEP, it is an average of two successive end-population (in 2011 it is the average of stocks as at 1 January and 31 December, both based on the final results of the 2011 Population and Housing Census).

Death probability (q_x) expresses the probability that an individual at the exact age of x will die in a given period, i.e. before the exact age of $x+1$. The input death probabilities are derived from the age-specific mortality rates (m_x):

$$q_x = 1 - e^{-m_x}$$

Table number of survivors (l_x) is the hypothetical number of individuals alive at the exact age of x out of 100,000 live births (table root - l_0), given the mortality conditions of the reference period:

$$l_{x+1} = l_x \cdot (1 - q_x)$$

Table number of deaths (d_x) is the hypothetical number of individuals who die at the age of x :

$$d_x = l_x - l_{x+1}$$

Table number of person-years (L_x) expresses the hypothetical number of individuals alive at the completed age of x . It is calculated as the average of two subsequent table numbers of survivors, except for the age of 0. Table number of person-years at the age of 0 is derived from the frequency distribution of infant deaths during the analysed period by the year of born. The coefficient α informs what is the ratio of infant deaths born in the given year from all infant deaths in analysed year. In the life tables for the districts and the AD MEP the fixed value of α at 0.86 is applied, which corresponds to the long-term average of this indicator for the whole Czech Republic.

$$L_x = \frac{l_x + l_{x+1}}{2} \qquad L_0 = l_0 - \alpha \cdot d_0 \quad ; \quad \alpha = 0,86$$

Auxiliary indicator (T_x) expresses the number of years of life to be lived by the table generation (not of an individual) at a given age. It is the accumulation of L_x from the highest age of the table ($\omega-1$) to the age of x :

$$T_x = T_{x+1} + L_x \qquad T_x = \sum_{\omega-1}^x L_x$$

Life expectancy (e_x) indicates the expected remaining life duration with presumption of unchangeable mortality conditions of a given year.

$$e_x = \frac{T_x}{l_x}$$

It is a synthetic indicator displaying mortality conditions of a given year through all age groups.

Calculation of input death probability for the complete life table

Input data

- number of deaths (D) by age (x) and sex in given years (t)
- number of people (P) by age (x) and sex in given years (t)
- number of live-births (N^v) by sex in given years (t)

Method

1. Calculation of the age-specific mortality rates (m_x) for age $x \geq 1$ from empirical data.

$$m_x = \frac{\sum_t D_x}{\sum_t 1.7.t P_x}$$

2. Calculation of the death probabilities based on continuous function $q_x = 1 - e^{-m_x}$. The death probability at the age of 0 is equal to infant mortality rate (number of deaths under 1 year of age per live births).

$$q_0 = \frac{\sum_t D_0}{\sum_t N^v}$$

3. In order to exclude random, fluctuations the death probabilities are equalized at the age of 4 and more.

$$q_x^{equal} = [105 \cdot q_x + 90 \cdot (q_{x-1} + q_{x+1}) + 45 \cdot (q_{x-2} + q_{x+2}) - 30 \cdot (q_{x-3} + q_{x+3})] / 315$$

4. After equalization the function (for the older ages) is further adjusted by Gompertz-Makeham formula $\log p_x = a + b \cdot c^x$ with under mentioned input:

$$\ln p_x^{equal} = \ln(1 - q_x^{equal})$$

The Czech Statistical Office uses King-Hardy method of extrapolation where d is the interval length and x_0 is the start age of the first interval (here $x_0 = 60$ and $d = 8$).

$$R_1 = \sum_{i=x_0}^{x_0+d-1} \ln p_i^{equal} \quad R_2 = \sum_{i=x_0+d}^{x_0+2d-1} \ln p_i^{equal} \quad R_3 = \sum_{i=x_0+2d}^{x_0+3d-1} \ln p_i^{equal}$$

5. Calculation of constants included in Gompertz-Makeham formula according to:

$$c^d = \frac{R_3 - R_2}{R_2 - R_1} \quad c = \sqrt[d]{c^d} \quad b = \frac{(c-1) \cdot (R_2 - R_1)}{c^{x_0} \cdot (c^d - 1)^2} \quad a = \left[R_1 - \frac{(R_2 - R_1)}{(c^d - 1)} \right] / d$$

6. Calculation of model probability of survival $r_x = \exp(a + b \cdot c^x)$ for the age $x \geq 71$. Find the age y ($y \geq 75$) where $|p_x^{equal} - r_x|$ gains its minimum. From y onwards the death probability is best expressed by function q_x^{GM} that complements r_x up to 1. The transition is smoothed for the ages $z = (y - 4), \dots, (y + 4)$:

$$q_z^{GM} = 1 - \left[\left(1 - \frac{z - y + 5}{10}\right) \cdot p_z^{equal} + \frac{z - y + 5}{10} \cdot r_z \right]$$

Summary: The input death probabilities for the mortality life tables are:

at the age of 0	infant mortality rate
at the age of 1, 2, 3	probabilities based on age-specific mortality rates
at the age of 4 to $(y - 5)$	equalized probabilities
at the age of $(y - 4)$ to $\omega - 1$	equalized and adjusted probabilities

