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Explaining Implausible Results in Shadow Economy Estimation Using MIMIC Models

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Abstract

For decades, economists have been trying to estimate the magnitude of the shadow economy (SE), which is not directly observable. This paper explores how the MIMIC (Multiple Indicator Multiple Cause) model can yield estimates of the SE/GDP (the proportion of the SE to the gross domestic product) below 0%, above 100%, and other implausible results. The focus is on the new calibration methods by Dell'Anno (2022) and data on the Czech Republic (1993–2021). The paper concludes that one of the leading causes of implausible results is the misalignment between the SE definition implied by the MIMIC model and that used for the exogenous estimates applied for calibration. Therefore, the authors propose testing the alignment between the SE definitions, such as assessing trends in the latent variable (first-stage scores) resulting from the MIMIC model and the exogenous estimates or applying regression or correlation analysis.

Keywords	***DOI***	***JEL code***
MIMIC model, implausible results, shadow economy, calibration	*https://doi.org/10.54694/stat.2024.12*	*O17, C39, H26*

INTRODUCTION

This paper explains the calibration of shadow economy (SE) estimates derived from the MIMIC (Multiple Indicator Multiple Cause) models, focusing on the implausible results. MIMIC models belong to the most advanced and controversial SE estimation methods. They offer a complex depiction of the economic system, deriving SE estimates as latent variables through the interplay of observable causes and indicators. One of the main reasons for the controversy is that the MIMIC models do not estimate the SE directly. Instead, the latent variable has the form of series of dimensionless indices (first-stage scores) that has to be calibrated to fit an exogenous estimate or series of exogenous estimates.

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Implausible calibration results in this paper are defined as calibration results (i.e., the resulting proportion of the SE to the gross domestic product (SE/GDP)) that are economically improbable (such as the resulting SE/GDP being negative or over 100%), do not correspond to expectations based on economic theory and observed phenomena, significantly diverge from exogenous estimates used for calibration, or other estimates that are considered reliable. Finally, results are considered implausible when different calibration methods yield significantly different results.

One of the critical reasons for implausible results may be the misalignment between the SE definition implied by the MIMIC model and the SE definition applied for the exogenous estimates. However, there are other reasons that are connected to economic phenomena, which are briefly discussed in this paper. There is no universally accepted definition of the SE. Instead, the SE is defined by the author of each estimate, whether it is through causes and indicators included in the MIMIC model or formulations of questions in a survey-based estimate.

The misalignment might not necessarily lead to implausible results. However, the reliability of the results may still be dubious since it is unclear which SE definition the resulting SE/GDP represents. Given that the resulting SE/GDP is derived from the first-stage scores based on one SE definition and calibrated to fit exogenous estimates based on another SE definition, drawing any policy-making implications from them or using them for further research is not advisable.

Giles and Tedds (2002) developed the first widely used calibration method, which multiplicatively scales first-stage scores. This method was followed by Dell'Anno and Schneider (2003) and Bajada and Schneider (2005). The main common principle of these three methods (traditional methods) is the use of exactly one exogenous estimate (or its growth rate). Besides that, the methods significantly differ. Dell'Anno and Schneider (2003) shift the time path of the first-stage scores so that the resulting SE/GDP in the base year is equal to the exogenous estimate. In contrast, Bajada and Schneider (2005) add a constant to the first-stage scores (interpreted as growth rates) so that the resulting SE/GDP growth rate equals the exogenous estimate of the SE/GDP growth rate.

Past research (most of which uses the traditional methods) overlooks the need to check the alignment between the SE definition implied by the model and that utilized for the exogenous estimate. While some authors (see Giles and Tedds, 2002; Bajada and Schneider, 2005; and Hassan and Schneider, 2016) use exogenous estimates derived from a separately estimated currency demand model with similar variables to those in the MIMIC model, others (see Dell'Anno, 2007; Nchor, 2021; and Oyibo and Schneider, 2022) use estimates from different methods or variables without documenting the verification of the alignment. Some, see Buehn and Schneider (2008), employed the average of multiple exogenous estimates for the same period. However, this may be an issue when the available estimates are based on different methodologies or (implicit) SE definitions.

In order to battle some deficiencies of the traditional methods, Dell'Anno (2022) developed three new distinct calibration methods. In contrast to the traditional methods, the methods by Dell'Anno (2022) (the new methods) do not merely anchor first-stage scores but adjust estimated coefficients from the MIMIC model using the estimated factor of scale. The new methods also use a more extended time series of exogenous estimates. However, both the traditional and new methods might yield implausible results.

To the authors' knowledge, no research explains and solves the issue of implausible results. Moreover, this issue is rarely discussed in the literature on SE estimation. The only mention of the implausible results the authors were able to find is Feige (2016), who states that Maenhout (2016) attempted to replicate selected estimates from Schneider et al. (2010) and reached negative values of SE/GDP for certain countries based on data for 2006 (–242% for Australia, and –257% for Canada).

The following types of implausible calibration results will be examined and explained in this paper: an inverted trend of the estimated SE/GDP, implausibly low variability of the resulting SE/GDP (i.e., "too flat" time series of the resulting SE/GDP), significant differences between different new calibration methods (especially opposite trends), negative (or implausibly low) and implausibly high values of SE/GDP, and local trends of the resulting SE/GDP that do not correspond to reasonable expectations (e.g., the SE/GDP decreases during a crisis).

This paper aims to give a general framework for recognising the implausible results and explain the reasons for their occurrence from a statistical standpoint. Firstly, it defines and classifies implausible calibration results of the MIMIC model SE estimates. Secondly, it provides answers why implausible results occur in the traditional and new methods. Thirdly, it explains the assumptions the input data must meet for the new methods to yield plausible results and how to use them correctly.

This paper does not cover the economic theories regarding the SE causes and indicators, nor does it extensively discuss the economic phenomena that lead to implausible results. Although it gives several general examples of economic phenomena that may lead to implausible results, it does not explain the specific phenomena in the Czech Republic (1993–2021). However, the authors believe that analysing economic theories and phenomena that lead to implausible results is an important topic that may be the subject of future research.

The first part of this paper explains the structure of the MIMIC model and delineates the reasons for the implausible results observed in both traditional and new calibration methods. The second part briefly describes the MIMIC model estimation for the Czech Republic (1993–2021). Subsequently, the results of the model are calibrated using the new methods and three different types of exogenous estimates: two distinct national accounts-based estimates sourced by Eurostat and the Czech Statistical Office (CZSO), along with a survey-based estimate by Hanousek and Palda (2006).

1 METHODOLOGICAL FRAMEWORK AND CHALLENGES

1.1 Structure of the MIMIC model

The MIMIC model is one of the most advanced methods of the SE estimation since it allows for including multiple SE indicators and not just one – such as the currency demand or electricity consumption approaches - and exploits the economic theory on causal relationships between SE and its observable causes and indicators.

The MIMIC model consists of a structural model linking the latent variable to its causes and a measurement model linking the latent variable to its indicators. The structural model in Formula (1) describes the relationship between the latent variable and its causes:

$$SE_t = \sum_{i=1}^{C} \gamma_i X_{i,t} + \zeta_t \, , \tag{1}$$

where SE_t is the SE first-stage score at time t, γ_i are the structural coefficients, $X_{i,t}$ is the value of the cause i in time t for $i \in \{1, 2, \ldots, C\}$, ζ_t is the error component at time t and C is the total number of causes.

The measurement model is the equation system that links the indicators to the latent variable. The MIMIC model also allows for the addition of direct relationships between some causes and indicators, where the cause explains the indicator. For example, a model may have four causes: tax burden, unemployment rate, self-employment, and economic freedom, and two indicators: GDP, and cash outside of banks. The unemployment rate may directly influence the GDP, and not just through the SE. Formula (3) is an example of a measurement equation with a direct relationship.

$$Y_{1,t} = \lambda_1 SE_t + \varepsilon_{1,t} \, , \tag{2}$$

$$Y_{2,t} = \lambda_2 SE_t + \delta_1 X_{1,t} + \varepsilon_{2,t} \, , \tag{3}$$

$$\ldots$$

$$Y_{P,t} = \lambda_P SE_t + \varepsilon_{P,t} \, , \tag{4}$$

where $Y_{p,t}$ are values of the indicators at time t, P is the total number of indicators, λ_p are the measurement coefficients, δ_1 is the coefficient describing the direct relationship between the indicator Y_2 and the cause X_1, and $\varepsilon_{p,t}$ are the error components at time t for $p \in \{1, 2, \ldots, P\}$.

The MIMIC specification assumes that the error component in the structural equation (Formula 1) ζ_t is not correlated with causes, the error components in the measurement model $\varepsilon_{p,t}$ do not correlate either with causes or with the latent variable, and the error component in the structural equation ζ_t does not correlate with the error components in the measurement equations ε_p (Buehn and Schneider 2008).

The parameters of the MIMIC model are not just the structural and measurement coefficients (including intercepts) but also the latent variable (SE) variance, residual variances from the structural and measurement equations, and covariances between the causes. Unless either one of the measurement coefficients or the latent variable's variance is constrained,[3] the MIMIC procedure yields infinite solutions to the structural and measurement coefficients ($\gamma_1, \ldots, \gamma_C$ and $\lambda_1, \ldots, \lambda_P$).

The variance of the latent variable is usually constrained to 1 and the measurement coefficient to ± 1. The indicator whose coefficient is constrained is called the reference indicator, and it is usually the indicator that is expected to have the highest correlation with the SE. The latent variable, therefore, has the same unit of measure as the reference indicator. The sign of the constrained measurement coefficient is chosen based on the expected relationship between the SE and the reference indicator. This paper constrains one of the measurement coefficients since it is more common.

The MIMIC model is a model specification of a structural equation model (SEM).[4] In this paper, the estimation is done using the covariance-based approach, which minimises the distance between the sample covariance matrix that contains variances and covariances between the observed causes and indicators, and the covariance matrix predicted by the model. There are different algorithms to estimate the coefficients. In this paper, the maximum likelihood (ML) function is used.

1.2 The traditional methods: methodology and causes for implausible results

Calibration converts first-stage scores from the MIMIC model to SE/GDP values. The MIMIC models capture the economic phenomena influencing the SE. In contrast, calibration is a purely mathematical exercise that aims to make the MIMIC model output more economically interpretable. The calibration methods are not directly tied to economic theories but entail assumptions, elaborated later. Failing these may yield implausible results. Possible causes of not meeting the assumptions are changes in the economy's structure during the examined period (such as tax reforms or laws reducing the SE).

Firstly, the first-stage scores are calculated as:

$$\hat{SE}_t^{FS\_est} = \sum_{i=1}^{C} \hat{\gamma}_i X_{i,t}, \tag{5}$$

where $\hat{SE}_t^{FS\_est}$ is the first-stage latent score at time t, $\hat{\gamma}_i$ are the ML estimates of the structural coefficients from equation (1), $X_{i,t}$ is the value of the cause i in time t, and C is the total number of causes used in the final model. Giles and Tedds (2002) used standardised data for the estimation. However, they calculated the first-stage scores with raw data, while Dell'Anno and Schneider (2003), and Bajada and Schneider (2005) used deviations from means of the variables.

Traditional calibration methods use only one or two exogenous estimates. The exogenous estimate is a SE/GDP estimate obtained separately from the model. It may be, for example, an estimate from an official authority (such as the estimates by Eurostat or CZSO used in this paper), a survey

[3] Based o the authors' knowledge and the current literature, these are the only two types of parameters whose constraint can solve the identification issue.

[4] For more information on SEMs, see, for example, Bollen (1989).

(e.g., the estimate by Hanousek and Palda (2006) used in this paper), or a model-based estimate published. The fundamental principle of the traditional calibration methods is that the estimated SE/GDP, or its growth rate in the calibration method by Bajada and Schneider (2005), in the base period (i.e., the period for which the exogenous estimate is available) must be equal to the exogenous estimate.

The most widely used calibration method is the one by Giles and Tedds (2002), which multiplicatively scales the first-stage scores so that the resulting SE/GDP in the base year equals the exogenous estimate, as seen in Formula (6). Therefore, this method assumes a constant ratio of the SE to the first-stage scores. The SE/GDP is:

$$\hat{SE}_t = \frac{SE_{t*}^{exog}}{\hat{SE}_{t*}^{FS\_est}} \hat{SE}_t^{FS\_est} , \tag{6}$$

where $\hat{SE}_t$ is the resulting SE/GDP at time t, SE_{t*}^{exog} is the exogenous estimate, t^* is the base period, and $\hat{SE}_t^{FS\_est}$ is the first-stage score at time t.

The exogenous estimate's level determines the resulting SE/GDP's level, while the first-stage scores determine the time path. However, the standard deviation of the resulting SE/GDP is not equal to the standard deviation of the first-stage scores but is influenced by the $\frac{SE_{t*}^{exog}}{\hat{SE}_{t*}^{FS\_est}}$. The higher the SE_{t*}^{exog} than the $\hat{SE}_{t*}^{FS\_est}$, the higher the standard deviation of the resulting SE/GDP. Therefore, the resulting SE/GDP's implausibly low variability ("a too flat time series") occurs when the first-stage score in the base period is significantly higher than the exogenous estimate.

On the other hand, if the exogenous estimate is much higher than the first-stage score in the base period, the slope of the estimated SE/GDP may be very steep, and implausibly low or high values of the SE/GDP may occur. An inverted trend occurs when the first-stage score is negative in the base period, which might happen depending on the estimated coefficients or the values of the causes.

The method by Dell'Anno and Schneider (2003) calibrates the first-stage scores by adding a constant, which preserves absolute differences between the first-stage scores. Therefore, this method assumes a constant difference between the the SE and the corresponding first-stage score. This method can be applied only when the model is estimated in differences (i.e., the first-stage scores are interpreted as the first difference of the SE/GDP). The resulting SE/GDP is calculated as follows:

$$\hat{SE}_{t*} = SE_{t*}^{exog} , \tag{7}$$

$$\hat{SE}_t = \hat{SE}_{t-1} + \sum_{i=1}^{C} \hat{\gamma}_i \Delta X_{i,t} \text{ for } t > t_* , \tag{8}$$

$$\hat{SE}_t = \hat{SE}_{t+1} - \sum_{i=1}^{C} \hat{\gamma}_i \Delta X_{i,t+1} \text{ for } t < t_* . \tag{9}$$

The implausibly low variability and inverted trend are impossible since the first-stage scores are shifted, so the SE/GDP in the base period equals the exogenous estimate. However, implausibly high or low values of the resulting SE/GDP may be an issue depending on the difference between the first-stage score in the base period and the exogenous estimate.

The method by Bajada and Schneider (2005) calibrates the growth rate of the SE/GDP so that the growth rate in the base period is equal to the growth rate of the exogenous estimates. The model is specified and estimated in terms of growth rates. This method needs two consecutive exogenous estimates to calculate the growth rate. The constant $\tilde{g}_{t*}$ is calculated as:

$$\tilde{g}_{t*} = g_{t*}^{exog} - g_{t*}^{FS\_est}, \tag{10}$$

where g_{t*}^{exog} is the exogenous growth rate, and $g_{t*}^{FS\_est}$ is the base year's first-stage score growth rate. Then, the constant $\tilde{g}_{t*}$ is added to the growth rates of the first-stage scores. Therefore, this method assumes a constant difference between the growth rates of the first-stage scores and the SE. The SE/GDP growth rates are calculated as:

$$\hat{g}_t = \tilde{g}_{t*} + g_t^{FS\_est}, \tag{11}$$

where $\hat{g}_t$ is the growth rate of the estimated SE/GDP at time t. The growth rates are then employed to get the estimates of SE/GDP levels.

The inverted trend (i.e., the estimated SE/GDP growth rates having opposite signs to first-stage scores) can occur when the growth rate of the first-stage scores in the base period and the growth rate of the exogenous estimates have opposite signs. However, this does not mean all negative growth rates change into positives. If the first-stage score at the time t is less than the constant $\tilde{g}_{t*}$, the resulting SE/GDP at time t can still be negative. The opposite signs (as well as vastly different growth rates of the first-stage scores and exogenous estimates) indicate a misalignment between the SE definitions implied by the MIMIC model and the exogenous estimates.

The resulting SE/GDP variability heavily depends on the constant $\tilde{g}_{t*}$ since it accumulates exponentially. If the constant $\tilde{g}_{t*}$ is high, the estimated SE/GDP rises exponentially, and some values of the resulting SE/GDP may be implausibly high. Conversely, the variability may be implausibly low if the constant is negative.

The misalignment between the SE definition implied by the MIMIC model and the SE definition used for the exogenous estimate may cause issues even when applying the traditional methods. Suppose the exogenous estimate (or its growth rate) is available for more periods. In that case, the authors advise testing the alignment by comparing the trends, correlation, or regression with the exogenous estimate as the explained variable and the first-stage scores as the explanatory variable. If the tests show misalignment, it is not advisable to perform the calibration. Instead, the authors advise using a different exogenous estimate corresponding to the SE definition implied by the model or redeveloping the MIMIC model so that the causes and indicators correspond more to the SE definition used for the exogenous estimate. The authors recommend testing for robustness by calibrating with multiple base years if possible and seeing if the resulting SE/GDP levels and time paths differ significantly.

Suppose there is only one exogenous estimate available. In that case, the authors recommend carefully examining the methodology used for the exogenous estimate and checking if the causes and indicators in the MIMIC model reflect the same SE definition.

1.3 The new methods: detailed methodology and data issues leading to implausible results

The new calibration methods by Dell'Anno (2022) provide several substantial advantages over the traditional methods. Firstly, they do not merely anchor first-stage scores but adjust the estimated coefficients using an estimated scale factor. Secondly, they use a more extended time series of exogenous estimates with at least two observations. Thirdly, the results of these methods are not sensitive to the level of first-stage scores (i.e., the resulting SE/GDP is the same when a constant is added to the first-stage scores). Fourthly, they allow for thorough testing of the alignment of the SE definition implied by the MIMIC model with the definition used for the exogenous estimates.

Dell'Anno (2022) developed three calibration methods. However, Method 3 gives the same results as Method 1. Therefore, it will not be discussed in this paper. Since Dell'Anno (2022) developed the methods for raw data and this paper works with standardised data, the first-stage scores are calculated as follows. Formula (12) then replaces Formula (5):

$$\hat{SE}_t^{FS\_est} = \sum_{i=1}^{C} \hat{\gamma}_i \frac{\sigma_{Y_1}}{\sigma_{X_i}} X_{i,t} , \tag{12}$$

where $\sigma_{Y_{1,t}}$ is the standard deviation of the reference indicator, σ_{X_i} is the standard deviation of the i-th cause, $X_{i,t}$ are the non-standardised values of the causes.

Method 1 minimises the difference between the first-stage scores and exogenous estimates. Firstly, the exogenous estimates are regressed on the first-stage latent score:

$$SE_{t*}^{exog} = \rho_0 + \rho_1 \hat{SE}_{t*}^{FS\_est} + a_{t*} \text{ with } t* \in W , \tag{13}$$

where SE_{t*}^{exog} are the exogenous estimates of SE/GDP, ρ_0 and ρ_1 are auxiliary regression coefficients, a_{t*} is the error component, $W = \{w_1, \ldots, w_e\}$ are years for which exogenous estimates are available, and e is the total number of exogenous estimates. This method can only be used if it is possible to obtain $\hat{\rho}_0$ and $\hat{\rho}_1$, OLS (ordinary least squares) estimates of ρ_0 and ρ_1. Then $\hat{\rho}_1$, is used to rescale the structural coefficients as:

$$^{est\_1}\hat{\gamma}_i^* = \hat{\gamma}_i \hat{\rho}_1 , \tag{14}$$

where $^{est\_1}\hat{\gamma}_i^*$ is the rescaled coefficient for the i-th cause for Method 1. This method estimates the regression coefficients for years with available exogenous estimates and then uses them to alter the first-stage scores in the entire period. Therefore, this method assumes a constant linear relationship between the first-stage scores and the SE (meaning constant variability of the first-stage scores and constant correlation between the first-stage scores and the exogenous estimates).

The final step is to calculate the resulting SE/GDP using the estimated intercept from Formula (13) and rescaled coefficients as in the equations below:

$$^{est\_1}\hat{SE}_t = \hat{\rho}_0 + \sum_{i=1}^{C} {}^{est\_1}\hat{\gamma}_i^* \frac{\sigma_{Y_1}}{\sigma_{X_i}} X_{i,t} , \tag{15}$$

where $^{est\_1}\hat{SE}_t$ is the resulting SE/GDP from Method 1 at time t.

Method 2 minimises the difference between the means of the first-stage scores and exogenous estimates. This time, the reference indicator is regressed on the exogenous estimates:

$$Y_{1,t} = const + {}^{REG}\lambda_1 SE_{t*}^{exog} + e_{t*} \text{ with } t* \in W , \tag{16}$$

where $Y_{1,t*}$ is the reference indicator, const is the intercept, $^{REG}\lambda_1$ is the "coefficient of scale", the true value of the reference coefficient (in the MIMIC model, the reference coefficient was constrained to 1), and e_{t*} is the error component. This method can only be used if it is possible to obtain $\widehat{const}$ and $^{REG}\hat{\lambda}_1$, OLS estimates of *const* and $^{REG}\lambda_1$.

Then, the structural coefficients are rescaled as follows:

$$ {}^{est\_2}_{\square}\hat{\gamma}_i^{*} = \frac{\hat{\gamma}_i}{{}^{REG}_{\square}\hat{\lambda}_1}\,, \tag{17}$$

where ${}^{est\_2}_{\square}\hat{\gamma}_i^{*}$ is the rescaled coefficient for the i-th cause for Method 2.

This method assumes a constant linear relationship between the reference indicator and the SE (constant variability of the reference indicator and the first-stage scores and constant correlation between the reference indicator and the exogenous estimates).

The next step is to estimate the intercept that is later added to the rescaled first-stage scores as:

$$ {}^{\Delta\mu\_est}_{\square}\hat{\gamma}_0^{*} = Mean(SE_W^{exog}) - Mean\left(\frac{\hat{SE}_W^{FS\_est}}{{}^{REG}_{\square}\hat{\lambda}_1}\right), \tag{18}$$

where ${}^{\Delta\mu\_est}_{\square}\hat{\gamma}_0^{*}$ is the intercept, $Mean(SE_W^{exog})$ is the mean of exogenous estimates, $Mean(\hat{SE}_W^{FS\_est})$ is the mean of the first-stage scores in periods with exogenous SE/GDP estimates available.

The resulting SE/GDP is described as:

$$ {}^{est\_2}_{\square}\hat{SE}_t = {}^{\Delta\mu\_est}\hat{\gamma}_0 + \sum_{i=1}^{C} {}^{est\_2}_{\square}\hat{\gamma}_i^{*} \frac{\sigma_{Y_1}}{\sigma_{X_i}} X_{i,t}\,, \tag{19}$$

where ${}^{est\_2}_{\square}\hat{SE}_t$ is the resulting SE/GDP from Method 2 at time t.

The inverted trend refers to a scenario where the trend resulting from the calibration is opposite to that of the first-stage scores. This occurs when the OLS estimate of the auxiliary regression coefficient (ρ_1 in Formula (13) for Method 1 or ${}^{REG}_{\square}\lambda_1$ in Formula (16) for Method 2) is negative. Rescaling structural coefficients reverses the signs that no longer make economic sense. Notably, observations with available exogenous SE estimates determine the regression coefficients. Even though the global trends of the exogenous estimates and the first-stage scores (or the reference indicator) are accordant, the local trends for the observations with exogenous SE estimates available might differ, turning the regression coefficient negative.

Implausibly low variability of the estimated SE/GDP means that the estimated SE/GDP (from Formula (15) or (19)) has a standard deviation significantly lower than the first-stage scores. The authors have decomposed the standard deviation of the estimated SE/GDP (for the derivation of Formula (20) and (21) see Annex B). For Method 1, the standard deviation of SE/GDP is:

$$ \sigma_{est\_1_{\square}\hat{SE}} = \sigma_{SE_W^{exog}} \frac{\sigma_{\hat{SE}_{\square}^{FS\_est}}}{\sigma_{\hat{SE}_W^{FS\_est}}} \left| corr(\hat{SE}_W^{FS\_est}, SE_W^{exog}) \right|, \tag{20}$$

where $\sigma_{est\_1_{\square}\hat{SE}}$ is the standard deviation of the resulting SE/GDP according to Method 1, $\sigma_{SE_W^{exog}}$ is the standard deviation of the exogenous estimates, $\sigma_{\hat{SE}_{\square}^{FS\_est}}$ is the standard deviation of the first-stage scores, $corr(\hat{SE}_W^{FS\_est}, SE_W^{exog})$ is the correlation between first-stage scores and the exogenous estimates.

Therefore, implausibly low variability of the estimated SE/GDP may be caused by the following factors (or their combination): low correlation between the first-stage scores and the exogenous estimates, a low standard deviation of the exogenous estimates, or the standard deviation of the first-stage scores for observations with exogenous SE estimates available being significantly higher than the standard deviation of the first-stage scores for the whole period. If the standard deviation of the first-stage scores is invariant throughout the entire period, it does not influence the standard deviation of the result suggested by Method 1.

For Method 2, the standard deviation of SE/GDP is:

$$\sigma_{est\_2_{\hat{SE}}} = \sigma_{SE_W^{exog}} \frac{\hat{\sigma}_{\hat{SE}^{FS\_est}}}{\sigma_{Y_{1,W}}} \frac{1}{|\,corr(Y_{1,W}, SE_W^{exog})|}\,, \tag{21}$$

where $\sigma_{est\_2_{\hat{SE}}}$ is the standard deviation of the resulting SE/GDP according to Method 2, $\sigma_{Y_{1,W}}$ is the standard deviation of the reference indicator, and $corr(Y_{1,W}, SE_W^{exog})$ is the correlation between the reference indicator and the exogenous estimates.

Therefore, implausibly low variability of the estimated SE/GDP may be caused by the following factors (or their combination): high correlation between the first-stage scores and the exogenous estimates (only to a certain extent), a low standard deviation of the exogenous estimates, or the standard deviation of the reference indicator for observations with exogenous SE estimates available being significantly higher than the standard deviation of the first-stage scores.

There are two types of significant differences between the results of the two methods: significantly different variabilities, and opposite trends. The resulting SE/GDP of both methods are equal when (for the derivation, see Annex C):

$$\frac{1}{\sigma_{\hat{SE}_W^{FS\_est}}} |corr(\hat{SE}_W^{FS\_est}, SE_W^{exog})| = \frac{1}{\sigma_{Y_{1,W}}} \frac{1}{|corr(Y_{1,W}, SE_W^{exog})|}\,. \tag{22}$$

In the ideal case, the exogenous estimates' correlation with both the first-stage scores and the reference indicator is 1, and the standard deviation of the first-stage scores is equal to the standard deviation of the reference indicator. Yet, the equation can hold even if $\frac{1}{\sigma_{\hat{SE}_W^{FS\_est}}}$ is A times greater than $\frac{1}{\sigma_{Y_{1,W}}}$, and simultaneously $\frac{1}{|corr(Y_{1,W}, SE_W^{exog})|}$ is A times greater than $|corr(\hat{SE}_W^{FS\_est}, SE_W^{exog})|$.

The opposite trends of the resulting SE/GDP from Method 1 and Method 2 occur when there is an inverted trend in one method but not in the other. This is when exogenous estimates positively correlate with first-stage scores (the reference indicator) and negatively with the reference indicator (first-stage scores).

Implausibly low (high) values of SE/GDP may occur when the SE/GDP has a steep upward trend and there are low (high) exogenous values towards the end (beginning) of the time series. Alternatively, the same may occur when the SE/GDP has a steep downward trend and there are low (high) exogenous values towards the beginning (end) of the time series. The slope's steepness is determined by the resulting SE/GDP variability.

Local trends of the SE/GDP not corresponding to reasonable expectations (e.g., the SE/GDP is expected to rise during a crisis) may be caused by the characteristics of the exogenous estimates, characteristics of the first-stage scores, or interactions between the two. The local trend of the exogenous estimates itself may not correspond to expectations, which could result from the methodology and the SE definition implied by the exogenous estimates. If the cause are the first-stage scores, it may point to an incorrect model specification or an implied SE definition, according to which the local trend does not correspond to expectations. The local trend not corresponding to expectations may also be caused by the inverted trend.

1.4 Potential causes for the implausible results of the new methods

The auxiliary regressions in the new methods can be interpreted as testing the alignment of the SE definition implied by the model to the SE definition used for the exogenous estimates. The authors consider a good regression fit when R^2 is at least 0.6 and the estimated coefficient $\hat{\rho}_1$ for Method 1 or ${}^{REG}_{\square}\hat{\lambda}_1$ for Method 2 is significant at least at the 10% level. Suppose these conditions are not met. In that case, the first-stage scores do not explain the exogenous estimates well (Method 1), or the exogenous estimates do not explain the reference indicator well (Method 2).

Both could be signs that the SE definition implied by the model (either through the causes or the reference indicator) is not aligned with the SE definition used for the exogenous estimates. Some of the causes of implausible results in these methods can be traced to the poor fit of the auxiliary regressions.

However, implausible results could also result from structural economic breaks that happened after the period of the last exogenous estimate or before the period of the first exogenous estimate and change the relationship between the SE and the first-stage scores or the reference indicator. For example, a new policy makes involvement in the SE more difficult. In this case, the SE may decrease without a decrease in the first-stage scores or in the reference indicator.

Another issue are influential observations in the auxiliary regressions. An influential observation does not have to be influential from the economic standpoint (i.e., there was a significant event). There may be the following reasons for the occurrence of influential points.

Firstly, the SE/GDP may have been incorrectly measured in a particular period. Secondly, the methodology for the exogenous estimates may have changed. Thirdly, an extreme value of one or more causes may have occurred that did not influence the SE. Fourthly, the exogenous estimates capture something that the first-stage scores do not, and vice versa. One explanation for this may be a misalignment in the SE definitions. Another explanation may be a phenomenon captured by the model and not by the exogenous estimates but not directly connected to the SE definitions.

This may happen, for example, when increased unemployment in one sector leads to increased involvement in the SE for workforce that has formerly worked in that sector. The model captured this shock through the unemployment rate. However, the exogenous estimates were based on a survey of respondents who work in different sectors.

Another example may be that the exogenous estimates accurately captured an SE increase, but the model did not. For example, the model has three SE causes: tax burden, unemployment rate, and regulatory burden. However, the SE increase may have been caused by an immigration wave, resulting in many immigrants starting to work informally, at least initially. In this case, the first-stage scores would increase, unlike the exogenous estimates.

These phenomena that cause influential points in the auxiliary regressions may also cause an issue when applying traditional methods when the exogenous estimate is available for a period when there is an influential point either in the exogenous estimate or the first-stage scores. However, this paper will not further examine economic phenomena that may lead to implausible results. This paper primarily explores the statistical patterns that lead to implausible results and tries to derive a general framework for classifying and discovering them. Nevertheless, economic phenomena leading to implausible results are an important topic that may be the subject of future research.

2 MODEL BUILDING FOR CZECH CONDITIONS

This section briefly explains the MIMIC model estimation for the Czech Republic from 1993 to 2021. All the variables considered for the analysis have economic reasoning behind them. Nonetheless, this paper does not aim to explore economic theories regarding the SE causes and indicators. Instead, it aims to explain implausible results in the SE estimation using the MIMIC model, which is used only as a tool.

The authors divided causes into seven segments: taxation, job market, freedom, education, the economic situation of businesses, the economic situation of households, and others. The complete list of causes is in Table A1 in the Annex A. The list of causes and indicators used in the reported models is in Table 1, along with details on the data sources and adjustments.

Three types of indicators were used: ratio of value added (VA) per hour worked in the four sectors with the highest SE relative to VA per hour worked in the four sectors with the lowest SE, real household consumption per capita, and a monetary variable (either Cash outside of banks / M1 or Card payments value / M1). Since the first indicator is newly used in this paper, it is explained in more detail in this section than the other variables.

The four sectors with the highest and lowest SE were determined from the SE estimation by sector for the Czech Republic for 2018 from the CIRCABC database (Eurostat, 2023). The sectors with the highest SE are: T – Activities of households as employers, A – Agriculture, forestry and fishing, F – Construction, and I – Accommodation and food service activities. The four sectors with the lowest SE are: B – Mining and quarrying, K – Financial and insurance activities, P – Education, and Q – Human health and social work activities.

VA per hour worked in the four sectors with the highest SE relative to VA per hour worked in the four sectors with the lowest SE approximates undeclared work. In order to calculate VA per hour worked, the VA is divided only by the declared hours. Therefore, the declared VA per hour worked increases even though the same VA is produced with the same number of hours just because of increased hours worked informally (i.e., the VA was divided by a lower number of declared hours). The VA per hour worked of the four sectors with the highest SE is divided by the VA per hour worked in the four sectors with the lowest SE to control for technological progress and changes in labour productivity. This paper assumes that the four sectors with the highest SE are influenced by technological progress, and their labour productivity grows at the same rate to those with the lowest SE. The authors are aware of this indicator's deficiencies. However, they believe it still partially captures the SE as one of the three indicators in the model.

The authors also tried two, three, and five sectors with the highest and lowest SE. However, the variant with four sectors leads to the best fit of the models. It is also noteworthy that including too few sectors may make the indicator unstable and influenced too much by the development of individual sectors.

Household consumption per capita was preferred over GDP in this study due to its closer alignment with the SE, as GDP encompasses the non-SE-related components such as government procurement and international trade. The authors used the Card payments value / M1 as an alternative to Cash outside of banks / M1, supported by recent findings such as those of Marmora and Mason (2021), indicating a significant negative association between SE and electronic payments.

Table 2 contains the best model results and a model used for robustness check. Many other models were estimated but discarded due to one of the following reasons: model not converging, negative residual variance, rejected hypothesis of the chi-square test, RMSEA (Root Mean Square Error of Approximation) greater than 0.1, not economically justifiable signs of the coefficients, Akaike information criterion (AIC) or Bayesian information criterion (BIC) higher than the best model's.

Furthermore, three more modifications of the best model were estimated for robustness check. The modifications are changing the Fraser Institute Freedom index for the economic freedom index from the Heritage Foundation, changing the NPL ratio of loans to households for the NPL ratio of consumer loans, and changing the Card payments value / M1 for Cash outside of banks / M1. Only the results for the last modification are shown here because the changes in the model and calibration results are most apparent for that case.

While the coefficient at the Card payments value / M1 is negative, the coefficient at Cash outside of banks / M1 is positive. All the other coefficients for causes and indicators have the same signs in both models. However, in the robustness check, the coefficient for the lagged unemployment rate is insignificant. Moreover, the model used for the robustness check has an RMSEA slightly greater than 0.1, which is not considered a good fit according to the rule of thumb by Browne and Cudeck (1993). However, the hypothesis of the chi-square test is not rejected.

Table 1 Data description of causes and indicators used in the best model and the model used for the robustness check

Variable name	Data source	Group	Range of data available	Data adjustment
Causes				
Fraser Institute Freedom index	Fraser Institute database	Freedom	1995, 2000–2020	Linear interpolation, extrapolation by taking the last known difference between observations
Unemployment rate (%)	Czech Statistical Office statistics	Job market	1993–2021	–
Public social spending / GDP (%)	OECD database	Others	1993–2021	–
NPL ratio of loans to households (%)	Czech National Bank – the ARAD time series database	Economic situation of households	2002–2021	Linear extrapolation
Indicators				
VA per hour worked in the four sectors with the highest SE relative to VA per hour worked in the four sectors with the lowest SE	Czech Statistical Office statistics	NA	Values added: 1993–2021 Hours worked: 1995–2021	Hours worked: extrapolation by taking the last known difference between observations
Household consumption per capita	Czech Statistical Office statistics	NA	1993–2021	–
Card payments value / M1 (%)	Card payments value: European Central Bank data portal	NA	Cards payments value: 2000–2021 M1: 1993–2021	Cards payments value: exponential extrapolation
Cash outside of banks / M1 (%)	M1: Federal Reserve economic data	NA	Cash outside of banks: 2002–2021 M1: 1993–2021	Cash outside of banks: linear extrapolation

Notes: NA means not applicable. The indicators were not divided into groups.
Source: Own construction

Table 2 Selected MIMIC estimation results

	The best model	Robustness check
Causes		
Fraser Institute Freedom index	–0.19***	–0.29***
	(–3.05)	(–2.61)
Unemployment rate (%) (t – 1)	0.21***	0.02
	(3.2)	(0.85)
Public social spending/GDP (%) (t – 1)	–0.25***	–0.12**
	(–3.15)	(–2.34)
NPL ratio of loans to households (t – 1)	0.07***	0.08**
	(2.74)	(2.35)
Indicators		
VA per hour worked in the four sectors with the highest SE relative to VA per hour worked in the four sectors with the lowest SE	1.00	1.00
	(NA)	(NA)

Table 2		(continuation)
	The best model	**Robustness check**
Indicators		
Card payments value/M1 (%)	–1.91***	–
	(–3.3)	–
Real final consumption expenditure of households per capita	–3.35***	–2.3***
	(–3.13)	(–2.62)
Cash outside of banks/M1 (%)	–	2.09***
	–	(2.67)
Direct relationships		
Real final consumption expenditure of households per capita ~ Unemployment rate (%) (t – 1)	0.51***	–0.18***
	(5.11)	(–4.06)
Real final consumption expenditure of households per capita ~ Public social spending/GDP (%) (t – 1)	–0.67***	–0.11
	(–4.07)	(–1.26)
Fit measures		
Converged	YES	YES
Negative variance	NO	NO
Chi-square (model vs. saturated)	7.98	12.43
DF	9.00	9.00
p-value	0.54	0.19
Chi-square (baseline vs. saturated)	234.42	225.63
DF	15.00	15.00
p-value	0.00	0.00
RMSEA	0.00	0.12
AIC	27.07	36.63
BIC	43.06	52.61

Notes: The asterisks denote p-values of the estimated coefficients, *** p<0.01, ** p<0.05, * p<0.1. Z-scores are in parentheses. Since standardised data was used, all intercepts are zero and not reported.
Source: Own construction

3 DEMONSTRATION OF THE IMPLAUSIBLE RESULTS

3.1 Exogenous estimates

This paper uses three different exogenous estimates: a national accounts-based estimate by Eurostat (2005 and 2023), a national accounts-based estimate from the CZSO (Czech Statistical Office, 2023), and a survey-based estimate from Hanousek and Palda (2006). The objective was to use exogenous estimates that are not model-based. Some authors believe (see Kirchgässner, 2016; Dell'Anno, 2022) that national accounts or survey-based estimates are more reliable than econometric methods (such as the currency demand method and MIMIC models). However, estimates from national statistical offices and surveys are often available only for selected periods and might be published with a significant time lag. On the other hand, the MIMIC models can predict the SE/GDP for periods for which estimates from surveys or national statistical offices are not available.

The authors do not advise using the MIMIC estimates as exogenous estimates since these estimates are based on another exogenous estimate. In the authors' opinion, using an original exogenous estimate is always better than an estimate derived from it. A crucial weakness of the currency demand method is that it captures only the one-sided relationship between the SE and the amount of currency and does not consider SE components that are uncorrelated to using cash.

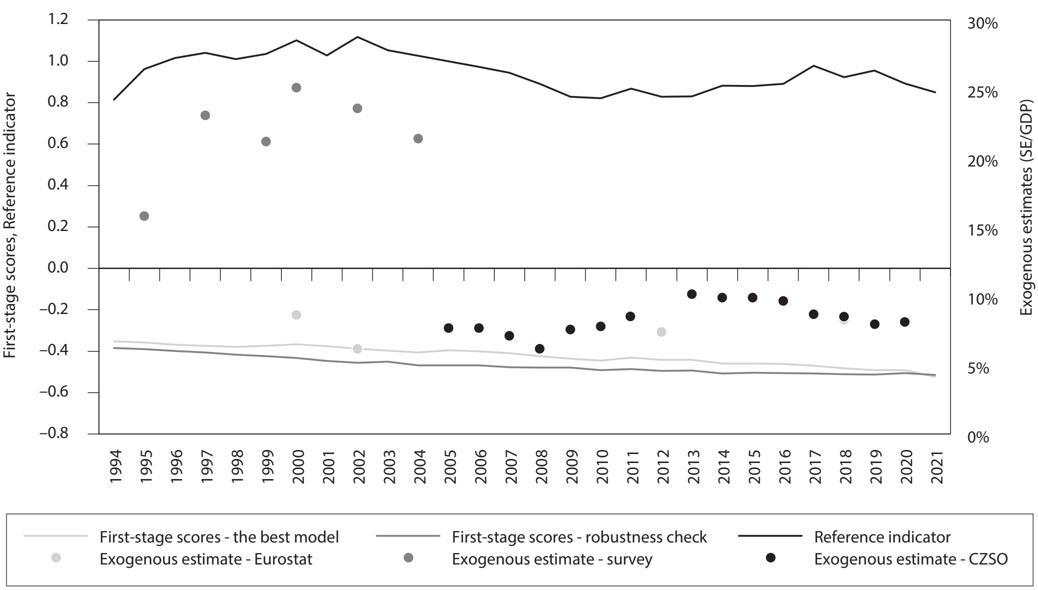
The national accounts-based method by Eurostat is based on adjusting the estimated GNP by non-exhaustiveness. There are the following seven types of non-exhaustiveness: N1 – Underground production, N2 – Illegal production, N3 – Informal production, N4 – Production of households for own final use, N5 – Statistically underground production, N6 – Producers deliberately misreporting, N7 – Deficiencies in the statistical system. The types of non-exhaustiveness are separately estimated using the labour input method, commodity flow method, special surveys, fiscal and other audit data, and other methods. Eurostat estimates are available for 2000, 2002, 2012, and 2018.

CZSO uses a similar national accounts-based method as Eurostat with slight differences in types of non-exhaustiveness; N4 is not part of the estimate, and N7 is defined more specifically as wages and salaries in kind (which are also part of the N7 as defined by Eurostat). For this paper, N2 and N7 were excluded by Eurostat and CZSO estimates since they are outside the scope of the SE according to most definitions.

Hanousek and Palda's (2006) estimate is based on a survey carried out in 2000, 2002, and 2004, wherein respondents were asked about their SE participation with response options categorized as never, sometimes, or often. The latter two responses were considered indicative of the SE involvement, while the former indicated non-participation. The respondents were asked about their informal activities in the current year and two and five years prior. Therefore, the estimates are available for 1995, 1997, 2000, 2002, and 2004. Two estimates were available for each of the years 1999, 2000, and 2002, the average of which was used.

Figure 1 shows the three exogenous estimates, the first-stage scores from models presented in Table 2, and the reference indicator. Table 3 shows correlations between the exogenous estimates, the first-stage scores, and the reference indicator. While the Eurostat and CZSO estimates are very close

Figure 1 Exogenous estimates, first-stage scores, and the reference indicator



Note: Sources of the exogenous estimates: Eurostat (2005), Eurostat (2023), Hanousek and Palda (2006), Czech Statistical Office (2023).
Source: Own construction

and range between 6 and 10%, the survey-based estimates are much higher (between 15 and 25%). The considerable differences in levels are due to different methodologies and likely due to different implied SE definitions.

Furthermore, Figure 1 shows that the survey-based estimate has a similar time path to the reference indicator, which means that they are probably based on comparable SE definitions. Moreover, Table 3 shows that they are highly correlated. In contrast, the Eurostat and CZSO estimates do not show the same trend as the reference indicator. The first-stage scores from both models have similar time paths and are highly correlated. However, the first-stage scores do not follow the same trend as any of the exogenous estimates and are negatively correlated with all of them.

Table 3 Correlations between the exogenous estimates, first-stage scores from the final model and the model used for robustness checking, and the reference indicator

	Exogenous estimates – Eurostat	Exogenous estimates – survey	Exogenous estimates – CZSO	First-stage scores – the best model	First-stage scores – robustness check	Reference indicator
Exogenous estimates – Eurostat	1.000					
Exogenous estimates – survey	NA	1.000				
Exogenous estimates – CZSO	NA	NA	1.000			
First-stage scores – the best model	–0.165	–0.299	–0.424	1.000		
First-stage scores – robustness check	–0.029	–0.517	–0.599	0.915	1.000	
Reference indicator	–0.166	0.909	–0.238	0.543	0.455	1.000

Notes: NA means that the correlation could not be calculated due to a lack of overlapping observations.
Source: Own construction

3.2 Calibration results with exogenous estimates by Eurostat

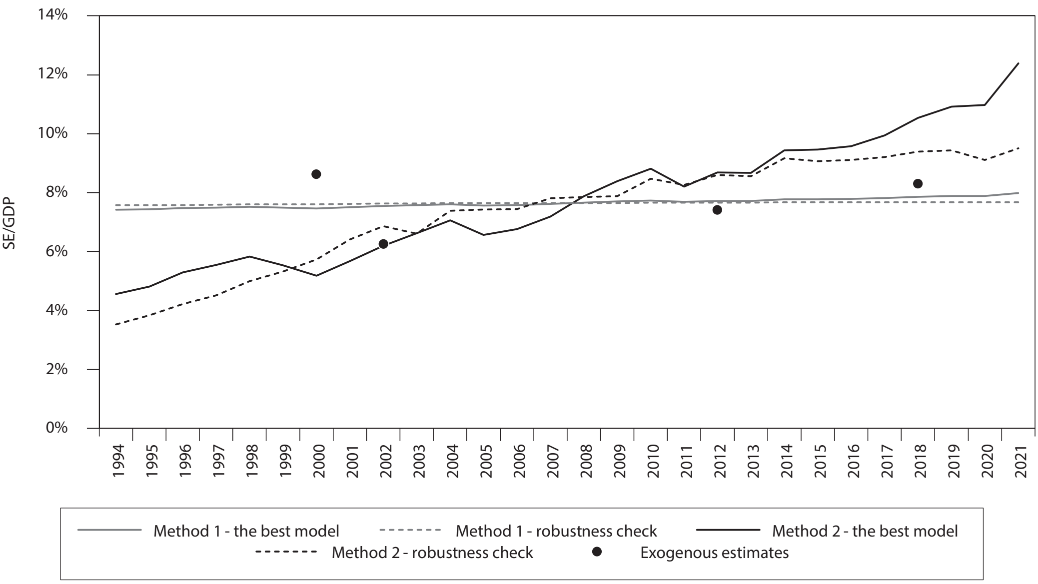
Figure 2 shows the calibration results using exogenous estimates by Eurostat. The observed types of implausible results are inverted trend, and implausibly low variability of the SE/GDP according to Method 1.

The inverted trend can be recognised in Figure 1, where both first-stage scores are declining, while in Figure 2, the resulting SE/GDPs of both methods are increasing (although very slightly for Method 1). In addition, a negative correlation between the exogenous estimates and the first-stage scores indicates the inverted trend in Method 1, and a negative correlation between the exogenous estimates and the reference indicator indicates the inverted trend in Method 2. The correlations are shown in Table 4. The implausible low variability of the SE/GDP according to Method 1 can be observed as a "too flat" time series of the resulting SE/GDP and by comparing the resulting standard deviation of the resulting SE/GDPs to the standard deviation of the first-stage scores in Table 4.

As seen from Table 4, none of the auxiliary regressions has a good fit (i.e., R^2 greater than 0.6 and coefficients statistically significant at a 10% level). The low variability of the resulting SE/GDP from Method 1 is caused mainly by the low variability of the exogenous estimates and the low correlation between the first-stage scores and the exogenous estimates.[5]

[5] Even if the correlation between the first-stage scores and the exogenous estimates were 1, the standard deviation of the resulting SE/GDP would be equal to the standard deviation of the exogenous estimates, which is 0.009.

Figure 2 Calibration results with national accounts-based exogenous estimates by Eurostat



Source: Own construction

Table 4 Decomposition of the standard deviation of the SE/GDP using exogenous estimates by Eurostat

	Method 1		Method 2	
	The best model	**Robustness check**	**The best model**	**Robustness check**
Standard deviation of the exogenous estimates	0.009	0.009	0.009	0.009
Standard deviation of the first-stage scores	0.046	0.040	0.046	0.040
Standard deviation of the first-stage scores for periods when the exogenous estimates are available	0.046	0.031	NA	NA
Correlation between the exogenous estimates and the first-stage scores	–0.165	–0.029	NA	NA
Standard deviation of the reference indicator for periods when the exogenous estimates are available	NA	NA	0.122	0.122
Correlation between the exogenous estimates and the reference indicator	NA	NA	–0.166	–0.166
Standard deviation of the SE/GDP	**0.002**	**< 0.001**	**0.021**	**0.018**
R^2 of the auxiliary regression	0.027	0.001	0.027	0.027
Significant coefficients in the auxiliary regression at a 10% level	NO	NO	NO	NO

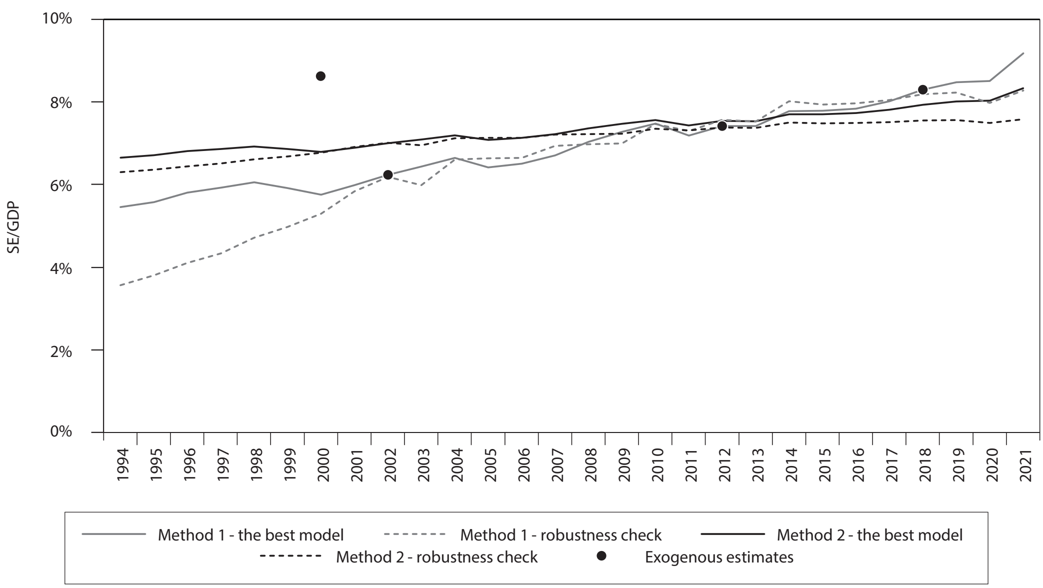
Notes: NA means that the parameter is not part of the standard deviation decomposition for the particular method.
Source: Own construction

Two things cause the much higher variability of the result of Method 2: the different methodology of the calculation and the standard deviation of the reference indicator being much higher than the standard deviation of the first-stage scores for the observations with exogenous SE estimates available. In Method 1, the low correlation between the exogenous estimates and first-stage scores deflates the resulting SE/GDP variability. In contrast, in Method 2, the low correlation between the exogenous estimates and the reference indicator inflates the resulting SE/GDP variability.

Regarding Method 1, the results for the best model and for the robustness check are similar. The results from the robustness check show that the lower variability is due to the very low correlation between the first-stage scores from the robustness check and the exogenous estimates. Regarding Method 2, slight divergences between the best model and the robustness check arise from different time paths of the first-stage scores.

Figure 2 shows that while the exogenous estimates from 2002, 2012, and 2018 have an increasing trend, the exogenous SE/GDP in 2000 is the highest and causes an overall trend of the exogenous estimates to be almost constant, which makes it an influential point in the regression. The inclusion or exclusion of the 2000 exogenous estimate notably impacts the correlation between the exogenous estimates and the first-stage scores (–0.165 with the 2000 observation and –1.000 without it for the best model), as well as the correlation between the exogenous estimates and the reference indicator (–0.166 with the 2000 observation and –0.715 without it for the best model), consequently affecting auxiliary regression coefficients and adjusted MIMIC coefficients. After excluding the 2000 observation, the standard deviation of the resulting SE/GDP from Method 1 increased for both models. In contrast, for Method 2, the SE/GDP of both models decreased.

Figure 3 Calibration results with national accounts-based exogenous estimates by Eurostat without the 2000 estimate



Source: Own construction

Figure 3 shows when the exogenous estimate from 2000 is excluded, the results of the two methods are much closer together for both the best model and the robustness check, and the SE/GDP according to Method 1 is much less flat. In addition, the R^2 of the auxiliary regressions is greater for both models and methods. The coefficients yielded by Method 1 are significant at a 10% level for both models.

Table 5 Decomposition of the standard deviation of the SE/GDP using exogenous estimates by Eurostat without the 2000 estimate

	Method 1		Method 2	
	The best model	Robustness check	The best model	Robustness check
Standard deviation of the exogenous estimates	0.008	0.008	0.008	0.008
Standard deviation of the first-stage scores	0.046	0.040	0.046	0.040
Standard deviation of the first-stage scores for periods when the exogenous estimates are available	0.039	0.023	NA	NA
Correlation between the exogenous estimates and the first-stage scores	–1.000	–0.990	NA	NA
Standard deviation of the reference indicator for periods when the exogenous estimates are available	NA	NA	0.120	0.120
Correlation between the exogenous estimates and the reference indicator	NA	NA	–0.715	–0.715
Standard deviation of the SE/GDP	**0.010**	**0.014**	**0.004**	**0.004**
R^2 of the auxiliary regression	0.999	0.981	0.511	0.511
Significant coefficients in the auxiliary regression at a 10% level	YES	YES	NO	NO

Notes: NA means that the parameter is not part of the standard deviation decomposition for the particular method.
Source: Own construction

Nevertheless, the inverted trend persists because the correlations are negative. Even though the exclusion of the 2000 exogenous estimate made the results of the two calibration methods much closer together, the authors do not advise excluding the influential points unless there are reasonable doubts about the reliability of the exogenous estimate in the particular period.

3.3 Calibration results with exogenous estimates from the survey

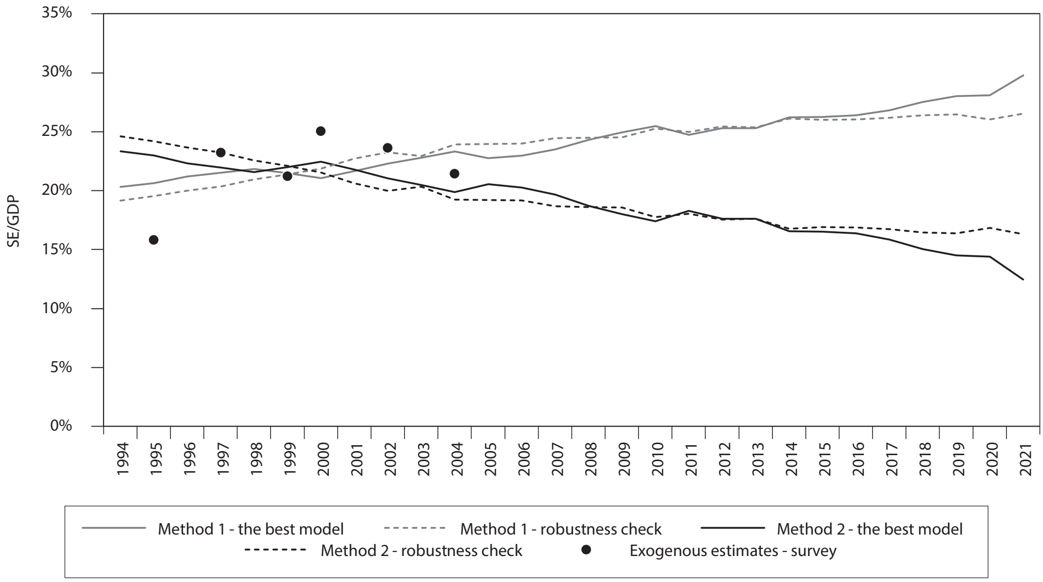
Figure 4 shows the results of calibration to survey-based estimates. The trends of the results of the two methods are opposite, which is caused by the inverted trend being present while applying Method 1 but not while applying Method 2. That is because the exogenous estimates positively correlate with the reference indicator but negatively with the first-stage scores. The exogenous estimates oscillate like a sine wave, which increases the risk of an inverted trend, especially in this case with few observations. Although the reference indicator and the first-stage scores are positively correlated (both having a global decreasing trend), the reference indicator has a strong local increasing trend between 1994 and 2002 (see Figure 1).

The auxiliary regressions have a bad fit for Method 1 for both models. The R^2 is 0.089 for the best model, and 0.267 for the robustness check, and the coefficients are not significant on a 10% level. In contrast, for Method 2, the auxiliary regression fit is much better for both models. The R^2 is 0.827, and the coefficients are statistically significant at a 10% level for both models. Generally, the results are very similar for the best model and for the robustness check.

The exogenous estimate from 1995 is an influential point that makes the trend of the exogenous estimates increasing, even though the other observations have a declining trend. The calibration results without the exogenous estimate from 1995 are shown in Figure 5.

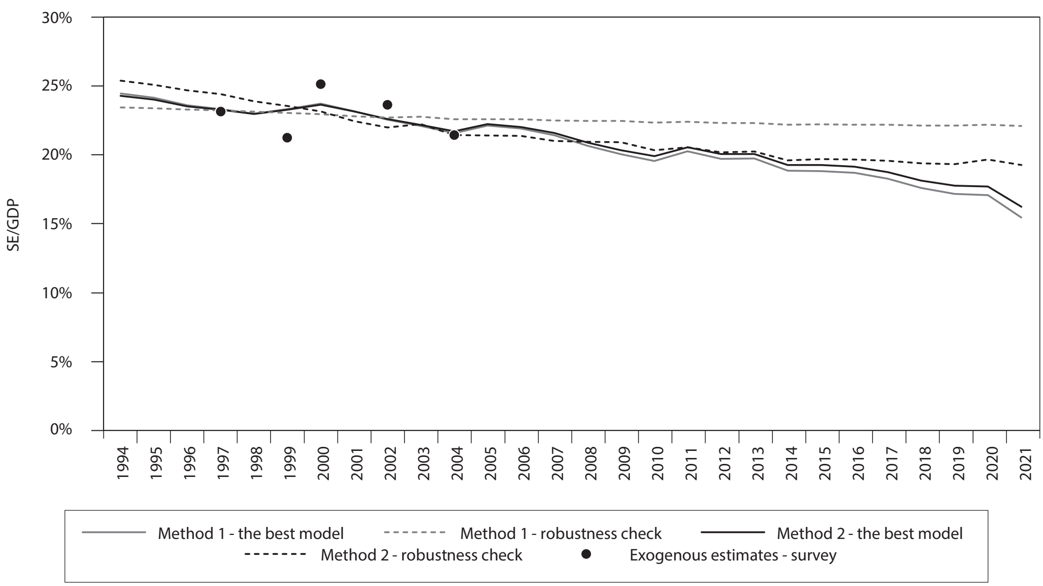
After the exclusion, the auxiliary regressions still have a bad fit for Method 1 and a slightly worse but still relatively good fit for Method 2. Nevertheless, the results of both methods are very similar and do not have an inverted trend. However, the reason for the similarity of the results is not the correlations being

Figure 4 Calibration results with survey-based exogenous estimates



Source: Own construction

Figure 5 Calibration results with survey-based exogenous estimates without the 1995 estimate



Source: Own construction

close to 1 or the standard deviations of the first-stage scores and the reference indicator being close (see Table 6). The reason is that $|corr(\hat{SE}_W^{FS\_est}, SE_W^{exog})|$ is approximately 2.5 times smaller than $\frac{1}{|corr(y_{1,W}, SE_W^{exog})|}$, and $\hat{\sigma}_{\hat{SE}_W^{FS\_est}}$ is approximately 2.5 times greater than $\sigma_{y_{1,W}}$.

However, the resulting SE/GDP decreased between 2008 and 2010 during the financial and economic crisis when it was expected to rise. The exogenous estimates do not cover this period. Therefore, the local trend between 2008 and 2010 is determined by the local trends of the exogenous estimates, the first-stage scores, and reference indicator for the observation with exogenous SE estimates available. Hypothetically, the declining trend of the estimated SE/GDP could result from a coincidence that the local trends of the exogenous estimates, the first-stage scores, and reference indicator are accordant even though the global trends differ.

The resulting SE/GDP from the robustness check using Method 1 has a significantly lower variability than the resulting SE/GDP from the best model. That is mainly because the first-stage scores used for the robustness check are much less correlated to the exogenous estimates than the first-stage scores from the best model.

Table 6 Decomposition of the standard deviation of the SE/GDP using survey-based exogenous estimates without the 1995 estimate

	Method 1		Method 2	
	The best model	**Robustness check**	**The best model**	**Robustness check**
Standard deviation of the exogenous estimates	0.014	0.014	0.014	0.014
Standard deviation of the first-stage scores	0.046	0.040	0.046	0.040
Standard deviation of the first-stage scores for periods when the exogenous estimates are available	0.015	0.023	NA	NA
Correlation between the exogenous estimates and the first-stage scores	0.529	0.162	NA	NA
Standard deviation of the reference indicator for periods when the exogenous estimates are available	NA	NA	0.037	0.037
Correlation between the exogenous estimates and the reference indicator	NA	NA	0.818	0.818
Standard deviation of the SE/GDP	**0.024**	**0.004**	**0.022**	**0.019**
R^2 of the auxiliary regression	0.280	0.026	0.670	0.670
Significant coefficients in the auxiliary regression at a 10% level	NO	NO	YES	YES

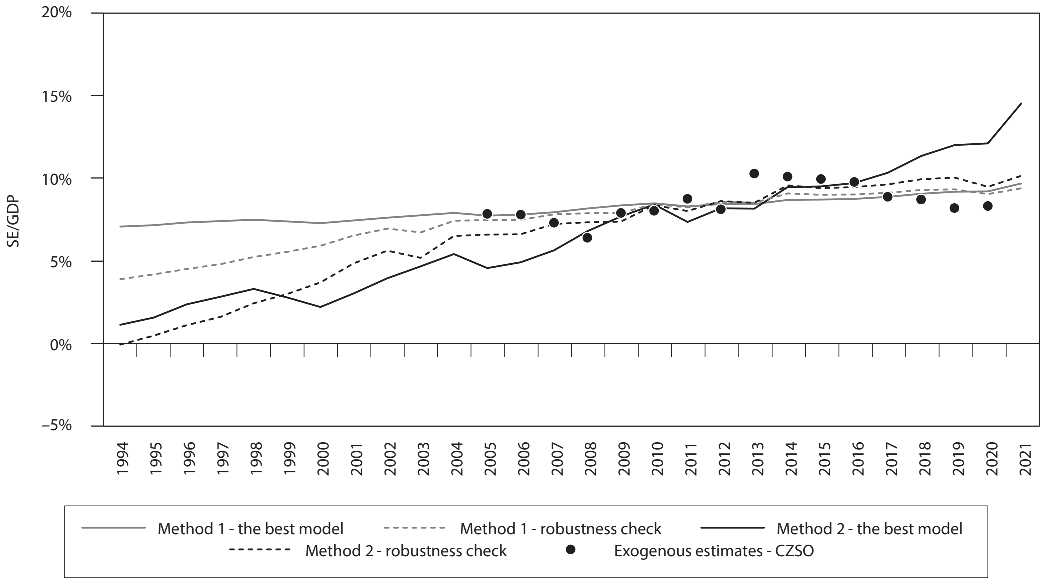
Notes: NA means that the parameter is not part of the standard deviation decomposition for the particular method.
Source: Own construction

3.4 Calibration results with exogenous estimates by CZSO

Despite having 16 years of available exogenous estimates by the CZSO, the calibration results displayed in Figure 6 are implausible. The observed types of implausibility include an inverted trend, implausibly low variability of the results for the robustness check with Method 1 for the robustness check, implausibly low values of the resulting SE/GDP from Method 2 at the beginning of the time series (and negative for the robustness check), and notable divergences in SE/GDP variabilities between Method 1 and Method 2.

The causes of the inverted trend, the implausibly low variability in Method 1, and the significant divergences in variability between resulting SE/GDP from Method 1 and Method 2 are the same as discussed in subsection 3.2. The implausibly low values from the results by Method 2 are caused by low values of

Figure 6 Calibration results with the national accounts-based estimate by CZSO



Source: Own construction

Table 7 Decomposition of the standard deviation of the SE/GDP using exogenous estimates by CZSO

	Method 1		Method 2	
	The best model	**Robustness check**	**The best model**	**Robustness check**
Standard deviation of the exogenous estimates	0.010	0.010	0.010	0.010
Standard deviation of the first-stage scores	0.046	0.040	0.046	0.040
Standard deviation of the first-stage scores for periods when the exogenous estimates are available	0.029	0.015	NA	NA
Correlation between the exogenous estimates and the first-stage scores	–0.424	–0.599	NA	NA
Standard deviation of the reference indicator for periods when the exogenous estimates are available	NA	NA	0.056	0.056
Correlation between the exogenous estimates and the reference indicator	NA	NA	–0.238	–0.238
Standard deviation of the SE/GDP	**0.007**	**0.017**	**0.036**	**0.031**
R^2 of the auxiliary regression	0.179	0.358	0.056	0.056
Significant coefficients in the auxiliary regression at a 10% level	NO	YES	NO	NO

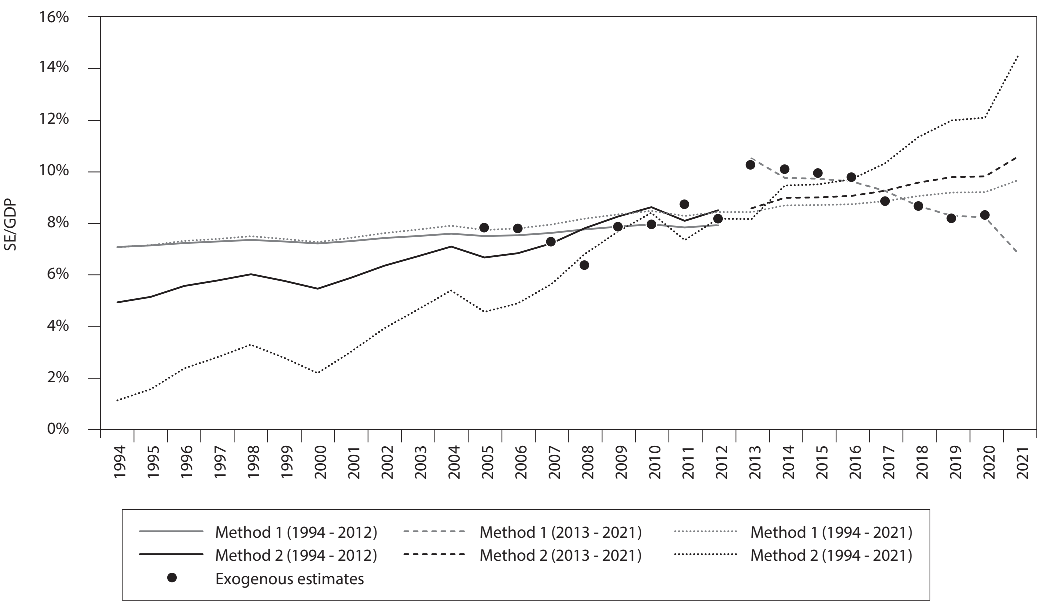
Notes: NA means that the parameter is not part of the standard deviation decomposition for the particular method.
Source: Own construction

the exogenous estimates placed at the end of the time series, and a steep upward slope of the estimated SE/GDP according to Method 2 (inverted due to the inverted trend). In addition, the low correlation between the exogenous estimates and the reference indicator inflated the resulting SE/GDP variability.

The trend of the exogenous estimates being opposite to the trends of the reference indicator and the first-stage scores for the observations with exogenous SE estimates available is likely caused by the implicit SE definition from the model not corresponding to the definition by CZSO. Moreover, Table 7 shows that the fits of auxiliary regressions for both models and methods are bad.

The results of both models show the same kinds of implausibility for both methods. Method 2 resulting SE/GDP from the robustness check is negative at the beginning of the time series. A significantly higher variability of the resulting SE/GDP from the robustness check in Method 1 is caused mainly by the lower variability of the first-stage scores for the observations with exogenous SE estimates available and the higher negative correlation between the first-stage scores and the exogenous estimates.

Figure 7 Robustness check results with exogenous estimates by CZSO



Source: Own construction

The calibration methods by Dell'Anno (2022) assume that the same economic principles apply for the entire period as for the period for which the exogenous estimates are available. The subsequent robustness check is employed to verify if this assumption holds by performing the calibrations for two periods separately. This test was performed only for the CZSO calibration since the other exogenous estimates have too few observations. The exogenous estimates were divided into two parts. The first part was 1994–2012 (the first eight observations of the exogenous estimates), and the second was 2013–2021.

Figure 7 shows that implausibly low values of the resulting SE/GDP at the beginning of the time series, according to Method 2, are no longer such an issue. However, the inverted trend is still present in all results except for Method 1 (2013–2021). Furthermore, there is still a significant divergence between the variabilities of Method 1 and Method 2 results in both periods, and an implausibly low variability for Method 1 (1994–2012). Table 8 shows that the fits of the auxiliary regressions are not good except for Method 1 (2013–2021).

As Table 8 shows, the resulting SE/GDP standard deviations are not stable in either method. The main driver is the instability of the correlations between the exogenous estimates and the first-stage scores for

Table 8 Robustness check with exogenous estimates by CZSO

	Method 1		Method 2	
	1994–2012	2013–2021	1994–2012	2013–2021
Standard deviation of the exogenous estimates	0.006	0.008	0.006	0.008
Standard deviation of the first-stage scores	0.028	0.023	0.028	0.023
Standard deviation of the first-stage scores for periods when the exogenous estimates are available	0.018	0.017	NA	NA
Correlation between the exogenous estimates and the first-stage scores	–0.258	0.958	NA	NA
Standard deviation of the reference indicator for periods when the exogenous estimates are available	NA	NA	0.065	0.044
Correlation between the exogenous estimates and the reference indicator	NA	NA	–0.247	–0.727
Standard deviation of the SE/GDP	**0.003**	**0.010**	**0.011**	**0.006**
R^2 of the auxiliary regression	0.066	0.918	0.061	0.529
Significant coefficients in the auxiliary regression at a 10% level	NO	YES	NO	YES

Notes: NA means that the parameter is not part of the standard deviation decomposition for the particular method.
Source: Own construction

Method 1 and between the exogenous estimates and the reference indicator for Method 2. The correlation between the exogenous estimates and the first-stage scores is relatively low and negative in the first period but highly positive in the second. The correlation between the exogenous estimates and the reference indicator is negative in both periods. However, it is much stronger during the second period.

The resulting SE/GDP for Method 1 (1994–2012) does not significantly differ from Method 1 (1994–2021). However, in 2013–2021, the resulting SE/GDP shows an opposite trend and is placed much higher than the resulting SE/GDP calibrated for the entire period. The SE/GDP resulting from Method 2 calibrated on separate periods has a much shallower slope than the resulting SE/GDP calibrated for the whole period. Overall, the resulting SE/GDPs calibrated for the separate periods significantly differ from the resulting SE/GDP calibrated for the whole period. Therefore, the assumption that the same economic principles apply for the entire period is likely unmet. The new methods are sensitive to the periods to which they are applied.

CONCLUSION

This paper explained and demonstrated implausible results in the SE estimation from MIMIC models. In order to convert the first-stage scores from the model to SE/GDP, they have to be calibrated using an exogenous estimate or series of exogenous estimates. However, calibration may lead to implausible results, such as an inverted trend, negative or implausibly high resulting SE/GDP, "too flat" time series of the resulting SE/GDP, or local trends not corresponding to reasonable expectations.

This paper has examined both the traditional calibration methods (i.e., methods by Giles and Tedds, 2002; Dell'Anno and Schneider, 2003; and Bajada and Schneider, 2005), and the methods newly developed by Dell'Anno (2022) and explained why these methods yielded implausible results.

The main focus was on the new methods by Dell'Anno (2022) that allow for using an extended time series of exogenous estimates (with at least two observations). Furthermore, apart from some of the traditional methods, neither the level nor the variability depends on the level of first-stage scores. However, even the new methods may lead to implausible results when specific data issues or economic phenomena are present. This paper described and demonstrated the data issues in detail, briefly mentioning the economic phenomena.

This paper concluded that one of the critical reasons for the implausible results was the misalignment of the SE definition implied by the MIMIC model and the SE definition used for the exogenous estimates. With the traditional methods, little attention is paid to the alignment, and testing is impossible if only one exogenous estimate is available. On the contrary, the new methods use auxiliary regressions whose poor fit may indicate misalignment between the SE definition implied by the model and the SE definition used for the exogenous estimates.

The alignment of the SE definitions mentioned above is essential not only for getting plausible results but also for the reliability of the resulting SE/GDP estimate. Suppose the first-stage scores based on one SE definition are calibrated to fit the exogenous estimates based on another SE definition. In that case, the resulting figure is unclear as to which definition of the SE is represented. Therefore, the reliability of the resulting SE/GDP is questionable, and it is not advisable to use it as a base for further research or policy-making implications.

When estimating the SE using the MIMIC model, the authors recommend testing the alignment of the SE definition implied by the model with the SE definition used for the exogenous estimates using either comparison of trends, correlation, or regression. If a misalignment is discovered, either another series of exogenous estimates should be used, or the MIMIC model should be redeveloped so that the latent variable reflects the exogenous estimates more clearly.

The authors do not believe that a more advanced calibration method can solve the issue of implausible results. No matter how good the method is, making reliable SE estimates from unreliable or incompatible data is impossible. Nevertheless, more advanced calibration methods may be able to make the resulting SE/GDP fit the exogenous estimates better or may not have such strict assumptions as the methods discussed in this paper.

Further research on this topic could concentrate on applying the new calibration methods to more countries to validate the methods' applicability and possibly discover more types of implausible results. Furthermore, alternative data sources or more advanced statistical techniques could be applied to improve the accuracy of SE estimates. Another suggestion for future research is to explore in more detail economic phenomena that may lead to implausible results and link implausible results to economic theories.

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References

BAJADA, C., SCHNEIDER, F. (2005). The Shadow Economies of the Asia-Pacific. *Pacific Economic Review*, 10(3): 379–401.
BOLLEN, K. A. (1989). *Structural Equations with Latent Variables.* Wiley. ISBN 978-1-118-61917-9
BREUSCH, T. (2005). *Estimating the Underground Economy using MIMIC Models.* EconWPA.

[6] Johannes Kepler University of Linz, Austria.
[7] University of Salerno, Italy.

BROWNE, M. W., CUDECK, R. (1993). *Testing Structural Equation Models.* SAGE. ISBN 978-0-8039-4507-4
BUEHN, A., SCHNEIDER, F. (2008). *MIMIC Models, Cointegration and Error Correction: An Application to the French Shadow Economy.* SSRN Scholarly Paper, Rochester, NY: Social Science Research Network.
CZECH STATISTICAL OFFICE. (2023). *Non-observed economy in the Czech Republic.* Prague: CZSO.
DELL'ANNO, R. (2007). The Shadow Economy in Portugal: An Analysis with the Mimic Approach online [online]. *Journal of Applied Economics*, 10(2): 253–277. <https://doi.org/10.1080/15140326.2007.12040490>.
DELL'ANNO, R. (2022). Measuring the unobservable: estimating informal economy by a structural equation modeling approach [online]. *International Tax and Public Finance.* <https://doi.org/10.1007/s10797-022-09742-0>.
DELL'ANNO, R., SCHNEIDER, F. (2003). The Shadow Economy of Italy and other OECD Countries: What Do We Know? [online]. *Journal of Public Finance and Public Choice*, 21(2–3): 97–120. <https://doi.org/10.1332/251569203X15668905422009>.
EUROSTAT. (2005). *Eurostat's Tabular Approach to Exhaustiveness* [online]. 5th Meeting of the GNI Committee. <https://www.dst.dk/Site/Dst/SingleFiles/GetArchiveFile.aspx?fi=739814884&fo=0&ext=intconsult>.
EUROSTAT. (2023). *Compilation of GNI – Czech Republic 2018.* CIRCABC database.
FEIGE, E. L. (2016). Professor Schneider's Shadow Economy (SSE): What Do We Really Know? A Rejoinder [online]. *Journal of Tax Administration*, 2(2). <http://jota.website/index.php/JoTA/article/view/109/78>.
GILES, D. E. A., TEDDS, L. M. (2002). *Taxes and the Canadian Underground Economy. Canadian Tax Foundation* [online]. <https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1002475>.
HANOUSEK, J., PALDA, F. (2006). Vývoj daňových úniků za pomoci markovských řetězců. *Czech Journal of Economics and Finance*, 56(3–4): 127–151.
HASSAN, M., SCHNEIDER, F. (2016). *Modelling the Egyptian Shadow Economy: A Currency Demand and A MIMIC Model Approach* [online]. <https://doi.org/10.13140/RG.2.1.2296.6804>.
KIRCHGÄSSNER, G. (2016). On Estimating the Size of the Shadow Economy [online]. *German Economic Review*, 18(1): 99–111. <https://doi.org/10.1111/geer.12094>.
MAENHOUT, M. A. (2016). *The Underground Economy: Evaluation and Replication of Schneider's Estimates.* Dissertation Summary, Brussels: University of Leuven.
MARMORA, P., MASON, B. J. (2021). Does the shadow economy mitigate the effect of cashless payment technology on currency demand? Dynamic panel evidence [online]. *Applied Economics*, 53(6): 703–718. <https://doi.org/10.1080/00036846.2020.1813246>.
SCHNEIDER, F., BUEHN, A., MONTENEGRO, C. E. (2010). *Shadow Economies All Over The World : New Estimates For 162 Countries From 1999 To 2007.* The World Bank, Policy Research Working Papers.
NCHOR, D. (2021). Shadow economies and tax evasion: The case of the Czech Republic, Poland and Hungary [online]. *Society and Economy*, 43(1): 21–37. <https://doi.org/10.1556/204.2020.00029>.
OYIBO, P. V., SCHNEIDER, F. (2022). How large is the size of Côte d'Ivoire's informal sector? A MIMIC approach. *Theoretical and Applied Economics*, Vol. XXIX, No. 4(633): 205–216.

ANNEX A – COMPLETE LIST OF CAUSES

Table A1 Complete list of causes for model building in Section 2

Cause	Group
NPL ratio of business loans	Economic situation of businesses
Real total Gross fixed capital formation	Economic situation of businesses
Real average gross wage in CZK	Economic situation of households
Real amount of loans to households	Economic situation of households
Real amount of loans to households per capita	Economic situation of households
Poverty headcount ratio at $6.85 a day (2017 PPP) (% of population)	Economic situation of households
Real amount of consumer loans to households	Economic situation of households
Real amount of consumer loans to households per capita	Economic situation of households
NPL ratio of loans to households	Economic situation of households
NPL ratio of consumer loans	Economic situation of households

Table A1 (continuation)

Cause	Group
Real amount of Non-bank consumer loans	Economic situation of households
Real amount of Non-bank consumer loans per capita	Economic situation of households
Number of housing allowance recipients	Economic situation of households
Real amount of housing allowances paid	Economic situation of households
Real amount of housing allowances paid per recipient	Economic situation of households
Percentage of people who receive housing allowances	Economic situation of households
Amount of loans to households per capita/average wage	Economic situation of households
Amount of consumer loans per capita/average wage	Economic situation of households
Amount of non-bank consumer loans per capita/average wage	Economic situation of households
Percentage of people with only primary education or no education (%)	Education
Percentage of people with high school education without maturita (%)	Education
Percentage of people who have high school education with maturita (%)	Education
Percentage of people who have university education (%)	Education
Personal income tax revenue/GDP (%)	Education
Percentage of people who have maturita exam	Education
Government consumption/GDP (%)	Freedom
Fraser Institute Freedom index	Freedom
Corruption index	Freedom
Economic freedom (part of the Heritage Index)	Freedom
Business freedom (part of the Heritage Index)	Freedom
Fiscal freedom (part of the Heritage Index)	Freedom
Regulation Quality Index from the World Bank	Freedom
Government Effectiveness Index from the World Bank	Freedom
Unemployment rate (%)	Job market
Self employment rate (%)	Job market
Inflation measured by CPI (%)	Others
Public social spending/GDP (%)	Others
Net migration	Others
Number of immigrants	Others
Indirect tax revenue/GDP (%)	Taxation
Tax revenue + social security revenue/GDP (%)	Taxation
Tax revenue without social security/GDP (%)	Taxation
Tax revenue from personal income and indirect taxes/GDP (%)	Taxation
Revenue from indirect taxes/total tax revenue	Taxation
Revenue from the personal income tax/total tax revenue	Taxation

Source: Own construction

ANNEX B – DECOMPOSING THE STANDARD DEVIATIONS OF THE RESULTING SE/GDP FROM THE NEW METHODS

For Method 1, the formula for calculating the resulting SE/GDP in Formula (15) can be rewritten as in Formula (23) which follows from Formulas (12) and (14):

$$ {}^{est\_1}\widehat{SE}_t = \hat{\rho}_0 + \hat{\rho}_1\, \widehat{SE}_t^{FS\_est}, \tag{23} $$

where ${}^{est\_1}\widehat{SE}_t$ is the SE/GDP estimated using Method 1, $\hat{\rho}_0$ and $\hat{\rho}_1$ are the OLS estimates of coefficients from Formula (13), and $\widehat{SE}_t^{FS\_est}$ are the first-stage scores. The standard deviation of the resulting SE/GDP is:

$$ \sigma_{est\_1\widehat{SE}} = |\hat{\rho}_1|\, \sigma_{\widehat{SE}^{FS\_est}}, \tag{24} $$

where $\sigma_{\widehat{SE}^{FS\_est}}$ is the standard deviation of the first-stage scores. $\hat{\rho}_1$ is a coefficient from a simple linear regression where the exogenous estimates SE_W^{exog} are the explained variable and $\widehat{SE}_W^{FS\_est}$ is the explanatory variable. Therefore, it can be calculated as:

$$ \hat{\rho}_1 = corr(\widehat{SE}_W^{FS\_est}, SE_W^{exog}) \frac{\sigma_{SE_W^{exog}}}{\sigma_{\widehat{SE}_W^{FS\_est}}}, \tag{25} $$

where $corr(\widehat{SE}_W^{FS\_est}, SE_W^{exog})$ is correlation between the first-stage scores and the exogenous estimates, $\sigma_{SE_W^{exog}}$ is the standard deviation of the exogenous estimates, and $\sigma_{\widehat{SE}_W^{FS\_est}}$ is the standard deviation of the first-stage scores for observations with available exogenous estimates. Therefore,

$$ \sigma_{est\_1\widehat{SE}_t} = \left| corr(\widehat{SE}_W^{FS\_est}, SE_W^{exog}) \frac{\sigma_{SE_W^{exog}}}{\sigma_{\widehat{SE}_W^{FS\_est}}} \right| \sigma_{\widehat{SE}^{FS\_est}} = |\sigma_{SE_W^{exog}}| \frac{\sigma_{\widehat{SE}^{FS\_est}}}{|\sigma_{\widehat{SE}_W^{FS\_est}}|} |corr(\widehat{SE}_W^{FS\_est}, SE_W^{exog})|, \tag{26} $$

where $\sigma_{est\_1\widehat{SE}_t}$ is the standard deviation of the resulting SE/GDP from Method 1. Since standard deviations cannot be negative, the $\sigma_{SE_W^{exog}}$ and $\sigma_{\widehat{SE}_W^{FS\_est}}$ can be written without absolute values.

For Method 2, the formula for calculating the resulting SE/GDP in Formula (19) can be rewritten as in Formula (27) which follows from Formulas (12) and (17):

$$ {}^{est\_2}\widehat{SE}_t = {}^{\Delta\mu\_est}\hat{\gamma}_0^{*} + \frac{\widehat{SE}_t^{FS\_est}}{{}^{REG}\hat{\lambda}_1}, \tag{27} $$

where ${}^{\Delta\mu\_est}\hat{\gamma}_0^{*}$ is the intercept calculated according to Formula (18), and ${}^{REG}\hat{\lambda}_1$ is the OLS estimate of the parameter ${}^{REG}\lambda_1$ from Formula (16). Therefore, its standard deviation is:

$$ \sigma_{est\_2\widehat{SE}_t} = \frac{\sigma_{\widehat{SE}^{FS\_est}}}{|{}^{REG}\hat{\lambda}_1|}, \tag{28} $$

where $\sigma_{est\_2_{\square}\hat{SE}_t}$ is the standard deviation of the resulting SE/GDP from Method 2. ${}^{REG}_{\square}\hat{\lambda}_1$ an OLS estimate of a coefficient from the regression described in Formula (16). Using the same rules as for $\hat{\rho}_1$ in Formula (25), ${}^{REG}_{\square}\hat{\lambda}_1$ can be rewritten as:

$$
{}^{REG}_{\square}\hat{\lambda}_1 = corr(Y_{1,W}, SE_W^{exog}) \frac{\sigma_{Y_{1,W}}}{\sigma_{SE_W^{exog}}} , \tag{29}
$$

where $corr(Y_{1,W}, SE_W^{exog})$ is the correlation between the reference indicator and the exogenous estimates, $\sigma_{Y_{1,W}}$ is the standard deviation of the reference indicator for observations with available exogenous estimates. The standard deviation of the resulting SE/GDP according to Method 2 is:

$$
\sigma_{est\_2_{\square}\hat{SE}_t} = \frac{\sigma_{\hat{SE}^{FS\_est}}}{\left| corr(Y_{1,W}, SE_W^{exog}) \frac{\sigma_{Y_{1,W}}}{\sigma_{SE_W^{exog}}} \right|} = \left|\sigma_{SE_W^{exog}}\right| \frac{\sigma_{\hat{SE}^{FS\_est}}}{\left|\sigma_{Y_{1,W}}\right|} \frac{1}{\left|corr(Y_{1,W}, SE_W^{exog})\right|} . \tag{30}
$$

Since standard deviations cannot be negative, the $\sigma_{SE_W^{exog}}$ and $\sigma_{Y_{1,W}}$ can be written without absolute values.

ANNEX C – DERIVING THE EQUALITY CONDITION OF RESULTS OF CALIBRATION METHOD 1 AND METHOD 2

For Method 1, the derivation is based on Formula (23). Formula (25) is used to derive $\hat{\rho}_1$. $\hat{\rho}_0$ is an OLS estimate of a constant from a simple linear regression where the exogenous estimates SE_W^{exog} are the explained variable and $\hat{SE}_W^{FS\_est}$ is the explanatory variable. Therefore, it can be calculated as:

$$
\hat{\rho}_0 = Mean(SE_W^{exog}) - \hat{\rho}_1 Mean(\hat{SE}_W^{FS_{est}}) = Mean(SE_W^{exog}) - corr(\hat{SE}_W^{FS_{est}}, SE_W^{exog}) \frac{\sigma_{SE_W^{exog}}}{\sigma_{\hat{SE}_W^{FS\_est}}} Mean(\hat{SE}_W^{FS_{est}}) , \tag{31}
$$

where $Mean(SE_W^{exog})$ is the mean of the exogenous estimates, and $Mean(\hat{SE}_W^{FS_{est}})$ is the mean of the first-stage scores for the observations with exogenous SE estimates available.

Therefore, the SE/GDP according to Method 1 can be rewritten as:

$$
\begin{aligned}
{}^{est\_1}_{\square}\hat{SE}_t &= Mean(SE_W^{exog}) - corr(\hat{SE}_W^{FS_{est}}, SE_W^{exog}) \frac{\sigma_{SE_W^{exog}}}{\sigma_{\hat{SE}_W^{FS_{est}}}} Mean(\hat{SE}_W^{FS_{est}}) + \\
&+ corr(\hat{SE}_W^{FS_{est}}, SE_W^{exog}) \frac{\sigma_{SE_W^{exog}}}{\sigma_{\hat{SE}_W^{FS_{est}}}} \hat{SE}_t^{FS_{est}} = \\
&Mean(SE_W^{exog}) + corr(\hat{SE}_W^{FS_{est}}, SE_W^{exog}) \frac{\sigma_{SE_W^{exog}}}{\sigma_{\hat{SE}_W^{FS_{est}}}} (\hat{SE}_W^{FS_{est}} - Mean(\hat{SE}_W^{FS_{est}})).
\end{aligned} \tag{32}
$$

For Method 2, the derivation is based on Formula (27). ${}^{REG}_{\square}\hat{\lambda}_1$ is derived in Formula (29), and ${}^{\Delta\mu\_est}_{\square}\hat{\gamma}_0^*$ is calculated in Formula (18). Therefore, the resulting SE/GDP according to Method 2 can be calculated as:

$$ {}^{est\_2}_{\square}\hat{SE}_{t} = Mean(SE_{W}^{exog}) - Mean\left(\frac{\hat{SE}_{\square}^{FS_{est}}}{{}^{REG}_{\square}\hat{\lambda}_{1}}\right) + \frac{\hat{SE}_{W}^{FS_{est}}}{corr(Y_{1,W}, SE_{W}^{exog})\,\frac{\sigma_{Y_{1,W}}}{\sigma_{SE_{W}^{exog}}}} \,. \tag{33} $$

After simplifications of the equations, the equality ${}^{est\_1}_{\square}\hat{SE}_{t} = {}^{est\_2}_{\square}\hat{SE}_{t}$ follows when:

$$ \frac{1}{\sigma_{\hat{SE}_{W}^{FS\_est}}}\left|corr(\hat{SE}_{W}^{FS\_est}, SE_{W}^{exog})\right| = \frac{1}{\sigma_{Y_{1,W}}}\,\frac{1}{\left|corr(Y_{1,W}, SE_{W}^{exog})\right|} \,. \tag{34} $$

The Real Equilibrium Exchange Rate of the Czech Koruna – the BEER Approach

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Abstract

The aim of this paper is to develop a model of the real equilibrium CZK/EUR exchange rate with all relevant explanatory variables. Emphasis will be placed on examining the longest possible time horizon, with a period of real convergence, but also economic stagnation. The model is based on the theoretical BEER approach using a cointegration or error correction model that distinguishes between short-term and long-term relationships. The results show that the Czech koruna strengthens when the productivity differential, terms of trade differential or gross fixed capital formation increase, while the koruna weakens when the VIX index representing global risk aversion increases. For the real interest rate differential, the hypothesis that the koruna strengthens in the short run during the improvement but weakens in the long run probably due to the country's risk premium, was confirmed. Moreover, it turned out that the start of the European Central Bank's quantitative easing led to strengthening of the Czech koruna, while the start of the CNB's foreign exchange interventions in 2013 led to weakening of the Czech koruna.

Keywords	***DOI***	***JEL code***
Equilibrium exchange rate, Czech Koruna, exchange rate misalignments, cointegration, error correction model	*https://doi.org/10.54694/stat.2023.50*	*C22, E44, F31, F41*

INTRODUCTION

The area of equilibrium exchange rates plays an important role in macroeconomics as well as economic policy, and the quest to explain medium- and long-term exchange rate developments has been a source of inquiry for over a century. It should be noted that the concept of equilibrium is purely theoretical, and the specific level of the equilibrium exchange rate depends on the model chosen (there is no consensus view). In general, the term equilibrium exchange rate is often used in public discourse, but it is non-trivial to precisely define and specify it using a model. One of the most common theories for assessing the long-term evolution of the exchange rate is purchasing power parity (PPP). For converging economies whose exchange rate shows appreciation symptoms (non-stationary time series), this theory is not complete (at least in the medium term) and the model needs to be supplemented with other determinants.

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Two theoretical approaches are currently distinguished in exchange rate equilibrium analysis. The first group of models introduces the conditions necessary to create an equilibrium into the model relationships and back-calculates the change in the current variables to achieve this goal. This is the so-called normative approach and includes, for example, the Fundamental Equilibrium Exchange Rate (FEER) (Williamson, 1985). The second group of models considers current variables and policies to determine future exchange rate equilibrium. This approach is called the positive concept and includes, for example, the Behavioural Equilibrium Exchange Rate (BEER) (Clark and MacDonald, 1998). This paper creates a real equilibrium exchange rate model based on the positive concept.

The analysis of the equilibrium exchange rate of the Czech koruna was dealt with by many authors (Čihák, 1999; Kreidl, 1997; Lazarová, 1997, etc.), mainly in the 1990s and 2000s during the robust transformation of the Czech economy. Disadvantages of the analyses were generally insufficiently short time series. The aim of this paper is to develop a model of the real equilibrium CZK/EUR exchange rate with all relevant explanatory variables. Emphasis will be placed on examining the longest possible time horizon (i.e. since the creation of the euro in 1999), because only then can the long-run relationships between the variables be observed. Over the past two decades, the Czech economy has undergone a period of significant real convergence but also economic stagnation, which has also been reflected in exchange rate movements.

In the first chapter, a literature review including a summary table of empirical models is developed. The methodology of the thesis, the variables included in the models and the results of the empirical analysis will be described. Chapter two describes the theoretical background of the included explanatory variables used for the long-run relationship and chapter three contains the empirical analysis itself, including a final evaluation of the evolution of the actual exchange rate against the model results.

1 LITERATURE REVIEW

This section summarizes the findings to date and describes the authors' selected models.[2] The emphasis will be on models of the Czech koruna, but also on other currency pairs. Finally, a summary table of existing models is created.

In the 1990s, the authors Frait and Komárek (1999) produced a thorough analysis of the Czech koruna. The model was built by combining the Natural Real Exchange Rate model (NATREX) and BEER and the main explanatory variables used are terms of trade, productivity, savings rate, and world real interest rates. Productivity was approximated by both real GDP per capita and labour productivity in industry.[3] The authors are aware of the limitations of the short time series, possible structural breaks, and insufficient data on fiscal policy. Cointegration analysis was used to estimate the model using both the Johansen approach based on the VAR model and the ARDL approach. In the first model using Johansen's approach, all signs came out as expected, i.e. an improvement in terms of trade, productivity and savings rate led to an appreciation of the domestic currency and an increase in the world interest rate led to a depreciation of the domestic currency. The productivity variable did not turn out to be statistically significant. In the second model using the ARDL approach, the model again came out with the expected signs. The ECM term came out negative and high indicating a rapid return of the model to equilibrium. All explanatory variables came out statistically significant. Finally, the authors compare their model with real data and conclude that the Czech koruna was overvalued just before 1997, which led to the currency crisis.

The author Rubaszek (2004) created an exchange rate model for the analysis of the Polish zloty. The model called the Balance of Payments Equilibrium Exchange Rate Model (BPEER) is based on the BEER

[2] Real equilibrium exchange rate models have also been discussed with a different methodology by the authors: Lazarová (1997), Šmídková (1999), Škop and Vejmělek (2009), Dudzich (2021), etc.

[3] The authors are aware of the limitations that both variables contain.

model but differs in its economic background. The BEER is described using the balance of payments identity as opposed to the BEER which is based on real uncovered interest parity (UIP). The main explanatory variables were domestic demand proxied by Polish real GDP, external demand proxied by euro area real GDP, net foreign assets, and the interest rate differential (three-month interbank rates). Cointegration analysis using Johansen's approach was used. The author was aware of the short time series, so he created an additional Fully Modified OLS model. In both models, all explanatory variables were statistically significant and consistent with economic theory. Changes in domestic and foreign demand (changes in demand on foreign trade) are offset by the exchange rate. Growth in net foreign assets and the interest rate differential led to appreciation of the domestic currency. Finally, the author mentions that according to this model, between 1995–99 the Polish zloty was close to its equilibrium value and in 2000–01 the Polish zloty was overvalued by about 10 to 15%. After a significant depreciation in 2002, the Polish currency returned to its equilibrium.

Komárek and Melecký (2005) used the BEER model to analyse the evolution of the Czech koruna. The model specification included an interest rate differential (real Czech and German lending rates), a productivity differential (real Czech and German GDP to employment), foreign direct investment, terms of trade, openness (a sum of exports and imports to GDP), net foreign assets and government consumption. For robustness, the authors developed three cointegration models namely Dynamic OLS, ARDL approach and Johansen approach. They divided the time period into short and medium periods. In the short period, they compare the exchange rate evolution using their model and the real evolution, and in the medium period, they apply cyclical adjustment to the model implying sustainable values, which they compare with the real values. The results indicated that the growth of productivity differential, interest differential and terms of trade led to the appreciation of the Czech koruna and these variables were statistically significant. Net foreign assets came out opposite in different models (different signs). The openness rate was significant only in one model with a positive relationship. Government consumption came out positive, although the authors expected a negative relationship because a rising government deficit should lead to a depreciation of the domestic currency. Finally, the authors compare the results with real developments and point out that the Czech koruna was undervalued by about 7% between 1994 and 2004.

Dufrenot and Égert (2005) analysed the equilibrium exchange rate using Central European emerging countries as a case study. Specifically, the authors analyzed the Czech Republic, Poland, Hungary, Slovakia and Slovenia. They used a combination of the BEER model and structural vector autoregression (SVAR) for the analysis. They analysed the BEER model using the Johansen cointegration approach. The structural model contains a two-equation approach. The exchange rate is explained by productivity differential, government deficit or surplus to GDP and current account balance of payments to GDP.[4] Further, the relative price level of non-tradable goods is explained by productivity differential and time trend. The authors are constrained not to build models for Slovakia and Slovenia because the cointegration relationship has not been econometrically confirmed. The results suggest that productivity growth leads to domestic currency appreciation for all three countries, suggesting the validity of the Balassa-Samuelson effect. The effect was weakest for the Czech Republic. The growth of the government deficit and the growth of the current account deficit of the balance of payments led to a depreciation of the domestic currency. Moreover, the so-called twin deficits were more strongly reflected in the depreciation of the domestic currency of Poland and Hungary. Finally, the authors conclude that productivity growth affects the exchange rate not only through prices in the services sector, but also through the non-tradable components of tradable goods prices and the growing ability of the tradable goods industry to produce higher quality goods.

[4] The authors deliberately omit the interest rate differential, whose impact on the exchange rate is not clear.

Pošta (2010) developed an equilibrium exchange rate analysis of the Czech koruna based on the BEER model. For the econometric analysis he used cointegration using the Johansen approach. The main explanatory variables he chose were the real interest rate differential, the productivity differential as the difference between the price level in the Czech Republic and the euro area, the real oil price in Czech koruna deflated by the PPI, net foreign assets in relation to GDP expressed as the accumulation of the current account of the balance of payments, government consumption to GDP and the ratio of total government debt to GDP. The author created three models where he included the explanatory variables in different ways. In the modelling, he included the interest differential variable outside the cointegrating vector. The largest positive effects on the appreciation of the Czech koruna were productivity growth, net foreign assets, and the interest rate differential. The government consumption, government debt and oil price variables did not turn out to be statistically significant. Finally, the author mentions that the Czech koruna was overvalued until 2008 according to the model, whereas in the financial crisis it tended to return to its equilibrium.

Pour and Illichmann (2022) attempted to develop an econometric model explaining the exchange rate using a modified BEER model. The authors rely on absolute purchasing power parity (PPP), but this needs to be extended to include other variables as this theory does not hold in the medium term for many countries. For the analysis, they used panel regression on a sample of 34 countries that had a floating currency regime during the time period. In addition to the mentioned exchange rate using PPP, the variables used were GDP per capita, interest rates (annual average of central banks), inflation, investment freedom, urbanization rate or exchange rate. The result shows that absolute PPP is appropriate to include in the exchange rate model (it comes out significantly with a parameter around one). Furthermore, the domestic currency tends to appreciate when GDP per capita, interest rates, investment freedom, urbanization rate and terms of trade rise. Conversely, when domestic inflation rises, the exchange rate tends to depreciate.[5] Finally, the authors conclude that the deviations of the exchange rate according to the constructed model from real developments are smaller than the deviations of the exchange rate according to purchasing power parity from real developments, and half of the deviation from the model tends to return to its equilibrium over a three-year period.

For clarity, Table 1 summarizes all the important parameters of the empirical work models.

Table 1 Summary review of empirical papers

Author	Estimation period	Background	Econometric method	REER deflator	Country	Expl. variables
Frait and Komárek	1992–1998/Q	BEER, NATREX	Cointegration (ARDL, JM)	CPI	CZ	*ToT, PROD, S, IRf*
Rubaszek	1995–2002/Q	BPEER (BEER)	Cointegration (JM), VECM	PPI	PL	*NFA, IRdif, FTD*
Komárek and Melecký	1994–2004/Q	BEER	Cointegration (DOLS, ARDL, JM), VECM	CPI	CZ	*PRODdif, IRdif, ToT,NFA, OR, GC*
Dufrenot and Égert	1993–2002/M	BEER	Cointegration (JM), VECM	CPI	CZ, PL, SK, SI, HU	*PRODdif, DEF, CA*
Pošta	2000–2009/Q	BEER	Cointegration (JM), VECM	CPI	CZ	*PRODdif, NFA, IRdif, GC, OIL*
Pour and Illichmann	2000–2020/A	BEER	Panel	CPI	34 countries	*PRODdif, IRdif, ToT,IF,UR, CPIdif*

Notes: M – monthly, Q – quarterly, A – annually, *ToT* – terms of trade, *PROD* – productivity, *NFA* – net foreign assets, *S* – savings rate, *IRf* – world interest rate, *IRdif* – interest differential, *PRODdif* – productivity differential, *FTD* – foreign trade demand, *OR* – openness rate, *GC* – government consumption, *DEF* – public deficit/surplus, *CA* – current account balance, *IF* – investment freedom, *UR* – urbanization rate, *OIL* – real price of oil, *CPIdif* – inflation differential.
Source: Authorial computation

[5] There is a possibility of endogeneity of inflation, i.e. an inverse relationship.

The table shows that the BEER model was the most frequently used to develop models based on the positive approach. Most of the authors used econometric cointegration method, either using Johansen approach or ARDL approach. Some of the most used explanatory variables were changes in productivity or changes in the interest rate in the home country relative to abroad, which in most models led to an appreciation of the domestic currency. However, in some models, a negative relationship with the real interest rate differential emerged, probably due to an increase in the country risk premium. The models also used the terms of trade variable, i.e., the ratio of export prices to import prices, which led to appreciation of the domestic currency during growth. In most models, growth in net foreign assets led to appreciation of the domestic currency. The authors tried to include the effect of fiscal policy on exchange rate movements, either through government consumption or government budget deficit, but these variables did not show a clear relationship (higher government consumption led to an appreciation of the domestic currency, but the resulting government deficit led to a subsequent depreciation of the domestic currency). Cost variables that the Czech economy must import, such as oil prices, had the expected negative relationship, but were often statistically insignificant.

2 FUNDAMENTAL DETERMINANTS

This paper is based on a positive approach to equilibrium real exchange rate modelling and is also based on the BEER model, which is adapted to the needs of a small export economy such as the Czech Republic. The BEER model in its basic form was defined by Clark and MacDonald (1998) and is based on uncovered interest parity (UIP). The basic form of the model is as follows:

$$SR_t = E_t(SR_{t+k}) \cdot \frac{1 + IR_{F,t}^{t+k}}{1 + IR_{D,t}^{t+k}}, \tag{1}$$

where SR_t is a spot rate, $E_t(SR_{t+k})$ is an expected spot rate, $t+k$ defines the maturity horizon of the bonds, $IR_{D,t}^{t+k}$ is a domestic nominal interest (yield) rate and $IR_{F,t}^{t+k}$ is a foreign nominal interest (yield) rate.
The second equation expresses the expected purchasing power parity in the absolute version:

$$E_t\left(\frac{P_{F,t+k}}{P_{D,t}}\right) = \frac{P_{F,t}}{P_{D,t}} \cdot \frac{1 + E_t(p_{F,t}^{t+k})}{1 + E_t(p_{D,t}^{t+k})}, \tag{2}$$

where $\frac{P_{F,t+k}}{P_{D,t}}$ is the purchasing power parity in the absolute version at the time t, and $\frac{1 + E_t(p_{F,t}^{t+k})}{1 + E_t(p_{D,t}^{t+k})}$ is the relationship for expected changes in foreign and domestic price levels.

Adding Formulas (1) and (2) we get:

$$RSR_t = E_t(RSR_{t+k}) \cdot \frac{1 + RIR_{F,t}^{t+k})}{1 + RIR_{D,t}^{t+k})}, \tag{3}$$

where $RIR_t^{t+k} = IR_t^{t+k} - E_t(p_t^{t+k})$ is the real interest rate (ex-ante, domestic or foreign currency), $RSR_t = SR_t \cdot \frac{p_{F,t}^{t+k}}{p_{D,t}^{t+k}}$ is the real exchange rate.

The real equilibrium exchange rate is therefore explained in Formula (3) by two variables, namely: the real interest differential with maturity $t+k$ and the expectation of the real exchange rate in period $t+k$. Under the market efficiency assumption, this variable can be approximated by a risk premium because the expected return from holding foreign currency is equal to the relative risk premium from holding this currency compared to holding domestic currency.

In this paper, the BEER model is supplemented with other variables that seem to be relevant for the Czech economy. The process of selecting the fundamental determinants entering the BEER model was influenced by known empirical experience and published recommendations. The consistency of the parameter results with the economic theory and their statistical significance were the decisive factors in the assessment of the different variants of the estimation equations. The model specification is based on the current account balance of payments equilibrium and the variables that restore the equilibrium.

2.1 Productivity differential

One of the main explanatory variables that the authors use in their real equilibrium exchange rate models is the labour productivity differential, in other words, the differential evolution of labour productivity in the domestic and foreign economies. There is strong evidence that faster labour productivity growth in the domestic economy relative to the foreign economy ultimately leads to an appreciation of the domestic currency in both nominal and real terms. This is because labour productivity growth tends to be associated with export growth, leading to a foreign trade surplus in the balance of payments, which ultimately leads to nominal and real appreciation of the domestic currency.

An alternative explanation for the appreciation of the real exchange rate (not the appreciation of the nominal exchange rate) through an increase in the domestic price level of internationally non-tradable goods is offered by the Balassa-Samuelson theorem. This argues, in simple terms, that all goods and services consumed in an economy can be divided into tradable (low international arbitrage costs) and non-tradable goods (high international arbitrage costs). The share of labour in value added in the two sectors of a given economy is roughly the same. The nominal exchange rate changes in line with changes in the relationship between domestic and foreign prices of tradable items. Initially, labour productivity is growing faster in the tradable goods sector due to FDI inflows. This allows nominal wages to rise faster. In the long run, however, wages must rise equally in all sectors, otherwise there would be a shift of labour supply to the sector with the higher wage growth rate. The law of supply and demand in the labour market therefore forces the same rate of wage growth in both sectors, regardless of the rate of growth of labour productivity. Maintaining the desired profitability forces firms in the non-tradable goods sector to raise final prices and there is upward pressure on the domestic price level and hence a real appreciation of the domestic currency. Thus, the expected hypothesis in our empirical model is the following: when the labour productivity differential (Czech economy versus euro area economy) increases, the real equilibrium CZK/EUR exchange rate appreciates.

2.2 Real interest rate differential

The inclusion of the real interest differential follows directly from the BEER approach described above. This approach is derived from uncovered interest parity (UIP), which was defined by I. Fisher (1896, 1930) and argues that a currency with a positive interest differential should depreciate to equalize the return in the domestic and foreign currency (speculator's equilibrium):

$$(1 + IR_{D,t}^{t+n}) = \frac{E_t(SR_{t+k})}{SR_t} \cdot (1 + IR_{F,t}^{t+n}), \tag{4}$$

where SR_t is a spot rate, $E_t\ (SR_{t+k})$ is an expected spot rate, $IR_{D,t}^{t+n}$ is a domestic interest (yield) rate and $IR_{F,t}^{t+n}$ is a foreign interest (yield) rate. The UIP was originally formulated under the assumption that speculators are risk neutral, i.e. without risk premium. The nominal interest rate can be decomposed using the following formula:

$$IR_t^{t+k} \approx RIR_t^{t+k} + E_t(p_t^{t+k}), \tag{5}$$

where RIR_t^{t+k} is a real interest rate and $E_t(p_t^{t+k})$ is an expected inflation rate. This equation can be supplemented by the effect of risk, which can be captured by the risk premium see Mandel and Vejmělek (2021):

$$RIR_t^{t+k} = rr + rp, \tag{6}$$

where rr is a real interest yield and rp is a risk premium. The real interest rate differential in relation to the exchange rate should be distinguished in the short and long run. In the short run, a rise in both nominal and real interest rates in the domestic economy attracts speculative capital, which creates demand for the domestic currency in the foreign exchange market, causing the domestic currency to appreciate. However, in the long run, the possibility that higher nominal and real interest rates are associated with a higher risk premium leading to a depreciation of the domestic currency cannot be ruled out. An example of this is the behaviour of speculators when interest rates rise in the face of a short-term government budget deficit, or the opposite behaviour of speculators when the government budget deficit in the long term translates into a risky increase in government debt. A wave of short-term speculative capital inflows and appreciation of the domestic currency is replaced by speculative capital outflows and a depreciation of the domestic currency. The expected hypothesis in our empirical model is the following: when the interest rate differential (Czech economy vs. euro area economy) increases, the real equilibrium CZK/EUR exchange rate appreciates in the short run but depreciates in the long run.

2.3 Terms of trade differential

Terms of trade is the ratio of export prices to import prices in the economy. The basis for realisation prices are invoice prices from major export and import transactions converted into domestic currency. The price indices therefore reflect, in addition to price developments, the effect of changes in foreign exchange rates. Terms of trade differential shows the evolution of this ratio between the domestic and the foreign economy. An improvement in terms of trade in the domestic economy, i.e. an increase in export prices, ultimately leads to a foreign trade surplus in the balance of payments, which leads to an appreciation of the domestic currency in nominal and real terms. Based on the literature review, a positive relationship is expected. i.e., an increase in the terms of trade differential (Czech economy versus euro area economy) leads to an appreciation of the real equilibrium CZK/EUR exchange rate.

2.4 VIX index

The global risk aversion of the financial market plays an important role for the exchange rate. If there is a financial crisis, a global pandemic or other global external shock, the financial market becomes averse to financial risks (risk-off phase) and starts to sell off emerging market currencies. Investors and speculators have observed that emerging economies handle crises worse than safe-haven economies. This situation may arise, among other things, because of the closing of carry trade positions by speculators who cancel deposits in high-yielding currencies (usually emerging markets) and promptly repay debts in low-yielding currencies that they have to buy in the foreign exchange market. (Mandel and Durčáková, 2016). One way to measure this global risk is the VIX index (the Chicago Board Options Exchange's Volatility Index) based on S&P 500 index options (Mandel and Vejmělek, 2021). As the Czech koruna belongs to the group of emerging economies based on historical data, the expected hypothesis is as follows: a rise in the VIX index leads ultimately to a depreciation of the real equilibrium CZK/EUR exchange rate.

2.5 Gross fixed capital formation

Gross fixed capital formation includes the value of the acquisition of tangible and intangible fixed assets (whether purchased, taken over free of charge or produced in-house), less the value of their sale and the value of assets transferred free of charge. In a small open economy, we assume that most of this investment goes into internationally tradable goods. Then, we can expect that the growth in gross fixed capital formation will increase exports of finished goods or reduce imports of finished goods, again leading to a foreign trade surplus in the balance of payments, which will lead to an appreciation of the domestic currency in nominal and real terms. Moreover, if gross fixed capital formation is created with the help of a foreign investor, this will ultimately increase the supply of foreign currency and the demand for domestic currency in the foreign exchange market, which should lead to an appreciation of the domestic currency (nominal and real terms) in the short run. The expected hypothesis in our empirical model is the following: when the gross fixed capital formation increases, the real equilibrium CZK/EUR exchange rate appreciates.

3 EMPIRICAL ANALYSIS

The empirical analysis used primary data available from public sources. These include the database of the Czech Statistical Office, the database of the Czech National Bank, the database of Eurostat, the database of the European Central Bank or the database of S&P Capital IQ. When selecting the data, an attempt was made to use the maximum time range of the data. The euro as the common currency of the euro area was established in 1999, and therefore estimates were made on quarterly data between Q1 1999 and Q2 2022.[6] Since the aim of the empirical analysis is to construct a real equilibrium of the koruna against the euro, bilateral variables of the Czech economy versus the euro area economy are included. All data are converted to a base index (2015 = 100) and are seasonally and calendar adjusted. All calculations will be performed in the statistical program R.

The real exchange rate (*RCZKEUR*) is calculated from the nominal exchange rate, which is deflated by the consumer price index based on the formula:

$$RCZKEUR_t = NCZKEUR_t \cdot \frac{HICPEA_t}{HICPCZ_t}, \tag{7}$$

where $NCZKEUR_t$ is the nominal exchange rate CZK/EUR and $HICP_t$ is the harmonised index of consumer prices according to the Eurostat methodology in the euro area, respectively in the Czechia.[7] The productivity differential (*PRODdif*) is measured as the ratio of real GDP to the number of employed persons (Czech Republic vs. euro area). The terms of trade differential (*ToTdif*) is measured as the ratio of export and import prices (Czechia vs. the euro area). The measurement of global risk was captured by the VIX index (*VIX*), see paragraph above. In calculating the real interest rate differential (*RIRdif*), an attempt was made to create an ex-ante variable that makes more sense from a theoretical point of view. One possibility was to use the inflation expectations published by individual central banks in the creation of the variable. For better statistical significance, the actual consumer price trend was used, see formula:

$$1 + RIRdif_t = \frac{1 + PRIBOR(1Y)_t}{HICPCZ_t} \Big/ \frac{1 + EURIBOR(1Y)_t}{HICPEA_t}, \tag{8}$$

where $PRIBOR(1Y)_t$ is the Prague Inter Bank Offered Rate with maturity of one year and $EURIBOR(1Y)_t$ is the Euro Inter Bank Offered Rate with maturity of one year. The variable real fixed capital formation to GDP (*RGFCF*) in the Czech Republic was also statistically significant and was therefore included in the model. During the model building process, the following variables were tested and eventually excluded due to inferior statistical significance or statistical insignificance. These variables include unit labour cost differential, government debt differential, real price of oil, export to import ratio, export to GDP ratio, foreign direct investment (FDI), current account to GDP, central bank foreign exchange intervention to GDP, etc. The list of time series, shortcuts and data sources used are shown in Table 2. Descriptive time series statistics are presented in Table 3.

6 More recent data are not available due to the way the real interest differential is constructed, see below.
7 A variable with the HICP one year ahead (ex-ante) was also created, but it gave worse statistical results in the model.

Table 2 List of time series used for econometric analysis

Shortcut	Meaning	Data source
RCZKEUR	Real exchange rate CZK/EUR	CNB, Eurostat
PRODdif	Productivity differential (CZ vs EA)	Eurostat
ToTdif	Terms of trade differential (CZ vs EA)	CZSO, ECB
RIRdif	Real interest rate differential (CZ vs EA)	CNB, ECB, Eurostat
VIX	VIX index	S&P Capital IQ
RGFCF	Real gross fixed capital formation to GDP (CZ)	CZSO

Notes: CZSO – Czech Statistical Office, CNB – Czech National Bank, ECB – European Central Bank, Eurostat – Statistical Office of the European Union.
Source: Authorial computation

Table 3 Descriptive time series statistics

Variable	Mean	Median	Maximum	Minimum	Standard deviation
RCZKEUR	104.16	98.75	145.34	79.77	15.48
PRODdif	93.07	99.48	128.03	53.05	19.83
ToTdif	100.28	100.71	103.96	94.94	2.15
RIRdif	100.19	99.69	105.38	90.99	2.58
VIX	116.13	107.73	294.61	57.64	42.39
RGFCF	100.87	100.73	110.97	93.35	3.90

Source: Authorial computation

The econometric model itself will be constructed by cointegration analysis using Johansen's approach based on the VAR model. If a cointegration vector is present, it is possible to build an error correction model (VECM) that puts together the short-run relationships – differentials and the long-run relationships – cointegration (Engle and Granger, 1987; Cipra, 2008; Hindls et al., 2018).

First, it is necessary to check that all time series are integrated in the same order. The stationarity of the time series was tested using the Augmented Dickey-Fuller test and the Phillips-Perron test (both with trend and constant). The results in Table 4 indicate that all time series except the VIX variable are integrated of order 1 and can be tested for cointegration relationship. The VIX index can only be included in the model as an exogenous variable.

Table 4 Unit root tests

Variable	ADF test (p-value)		PP test (p-value)		Order of integration
	Levels	1st diff.	Levels	1st diff.	
RCZKEUR	0.471	<=0.01	0.451	<=0.01	I(1)
PRODdif	0.686	<=0.01	0.638	<=0.01	I(1)
ToTdif	0.562	<=0.01	0.427	<=0.01	I(1)
RIRdif	0.142	<=0.01	0.258	<=0.01	I(1)
VIX	0.010	<=0.01	0.015	<=0.01	I(0)
RGFCF	0.377	<=0.01	0.303	<=0.01	I(1)

Source: Authorial computation

The length order of the leads and lags is determined based on some of the information criteria for the model selection. Based on Akaike's information criterion and Schwarz's criterion, the optimal lag length was chosen to be first order. Based on Johansen's cointegration test in Table 5, exactly one cointegration relationship was found (either with trace or with maximal eigenvalue statistics).

Table 5 Johansen's cointegration test

Trace statistics				**Maximal eigenvalue statistics**			
H_0	**H_1**	**Test stat.**	**Crit. values 5%**	**H_0**	**H_1**	**Test stat.**	**Crit. values 5%**
r = 0	r > 0	96.65	53.12	r = 0	r = 1	60.24	28.14
r ≤ 1	r > 1	26.41	34.91	r = 1	r = 2	12.93	22.00

Source: Authorial computation

In the final model, dummy variables were included because it improved the statistical results of the model. The following two dummy variables were created: the exchange rate commitment dummy variable (0,1) (*dummycommit*), where 2013Q4 = 1, and the quantitative easing by the European Central Bank dummy variable (0,1) (*dummyqe*), where 2009Q2 = 1, 2010Q3 = 1 and 2012Q1 = 1. During this period, the European Central Bank gradually began to use the Securities Markets Programme to purchase the total of €250 billion of covered bonds. In the first case, there should be a one-off depreciation

Table 6 Cointegration equation and error correction model for *RCZKEUR*

Cointegration equation		**Error correction model:**	**d RCZKEUR**
RCZKEUR(−1)	1	*error correction term*	−0.221
			[−7.752]
RGFCF(−1)	0.669	*d RCZKEUR(−1)*	−0.021
	[2.918]		[−0.231]
RIRdif(−1)	−1.464	*d RGFCF(−1)*	−0.205
	[−3.493]		[−2.026]
ToTdif(−1)	1.148	*d RIRdif(−1)*	−0.587
	[2.102]		[−3.562]
Constant	−172.963	*d ToTdif(−1)*	−0.225
	[−2.777]		[−0.759]
Coefficient of determination (R^2)	0.642	*PRODdif*	−0.113
			[−8.507]
		VIX	0.020
Adjusted R^2	0.598		[5.777]
		dummycommit	2.435
			[1.474]
F-statistic	14.69	*dummyqe*	−3.580
			[−3.504]

Note: The values of t-statistics are in square brackets.
Source: Authorial computation

of the Czech koruna against the euro. The exchange rate commitment between November 2013 and April 2017 was a situation where the Czech National Bank intervened against the appreciation of the Czech koruna below 27 CZK/EUR as part of unconventional monetary policy.[8] In the second case, quantitative easing by the European Central Bank should lead to a weakening of the euro and thus to a strengthening of the Czech koruna. The explanation can be described using the monetary approach to exchange rate, where an increase in the money supply in the domestic economy ultimately leads to a depreciation of the domestic currency (Vejmělek, 2000).

The final model included a constant in the cointegration vector. The variables *VIX* and *PRODdif* were included in the model as exogenous variables, *VIX* because it is cointegrated by a different order of integration and *PRODdif* because the overall quality of the model with respect to the coefficient of determination was enhanced in this combination. The results are summarized in Table 6.

All variables in the model were statistically significant at the 5% significance level except d *ToTdif* and *dummycommit* variables, which were statistically insignificant, which means significant at the 15% significance level. The adjustment parameter (*error correction term*) in the error correction model has a negative sign and is in the desired interval (–1, 0), which indicates a tendency for the exchange rate to return to equilibrium. Productivity growth in the Czech economy relative to the euro area leads to a real appreciation of CZK/EUR in the short run[9] and the Balassa-Samuelson effect can be confirmed. The Czech koruna also appreciated when terms of trade in the Czech economy improved relatively to the euro area. The real interest rate differential should be divided into short and long periods. In the short run, when the real interest rate differential rises, the koruna appreciates due to the attraction of short-term speculative capital; in the long run, on the other hand, the koruna depreciates because the real interest rate differential is mainly influenced by the country's risk premium. A model was also tested where the real interest differential is included as an exogenous variable (the hypothesis that the real interest rate is equal between countries in the long run and therefore has no effect on the exchange rate). However, this model showed much worse statistical results and since the variable is statistically significant even in the long run, it is concluded that this hypothesis does not hold for the Czech koruna. It has been shown that fixed capital formation does indeed lead to an appreciation of the Czech koruna in the short term.[10] The inclusion of the VIX index demonstrates that the Czech koruna is under pressure to depreciate in the face of rising global uncertainty, placing it in the category of riskier currencies. The dummy variable for quantitative easing by the European Central Bank shows a one-off appreciation of the Czech koruna confirming the effect of money supply on the exchange rate. The one-off depreciation of the koruna occurred after the announcement of foreign exchange interventions by the Czech National Bank.

Based on the model, the results expressed in basis index[11] were compared with the actual real CZK/EUR exchange rate deflated by the CPI. The results of the model, see Figure 1, show an appreciation trend of the Czech koruna, especially between 1999 and 2009, when the Czech economy grew faster relative to the euro area and thus real convergence occurred. Between 2010 and 2020, on the other hand, there is some appreciation stagnation of the Czech koruna. Since the onset of the financial crisis and the subsequent European debt crisis, the pace of real convergence has almost stopped for several years. Moreover, developments in this period are strongly influenced by the CNB's foreign exchange interventions between 2013 and 2017. The Czech koruna has been overvalued since 2010, but at the time of the exchange rate commitment the difference to the model has been minimized. Since 2019, the Czech koruna has been slightly undervalued and since 2021 the values have been rebalanced against the model.

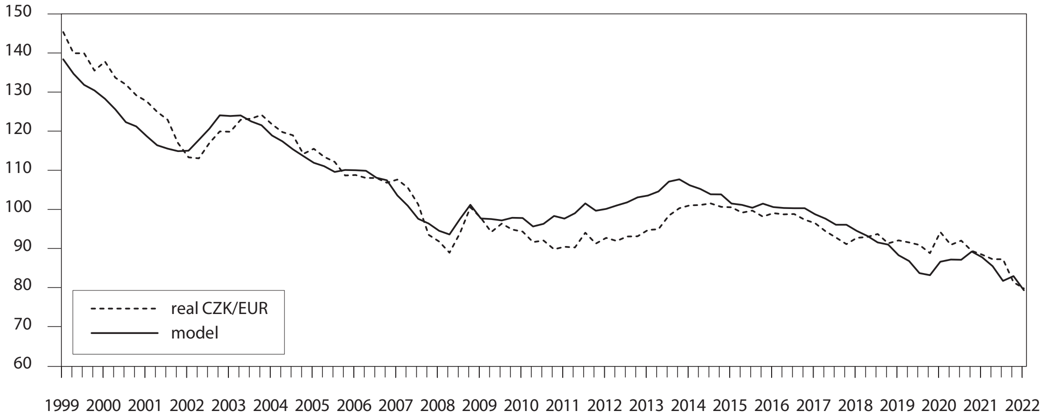
[8] The dummy variable with the end of foreign exchange interventions in 2017 was not statistically significant.

[9] In the case of a cointegrating vector, the signs of the variables must be perceived in reverse.

[10] In the long run, this relationship is no longer significant. It is likely that gross fixed capital formation will feed through into productivity growth.

[11] A drop in the index means a real appreciation of the Czech koruna.

Figure 1 Range of the equilibrium real CZK/EUR exchange rate

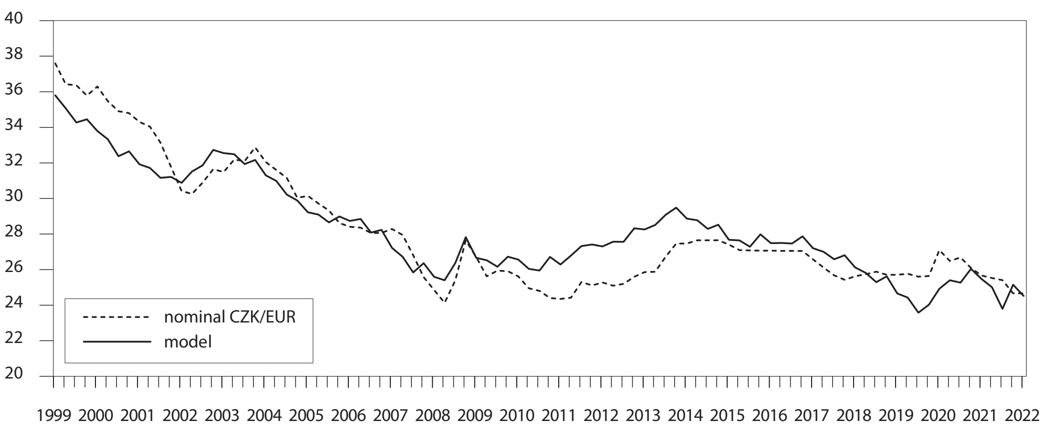


Source: Authorial computation

The resulting model can be converted into an equilibrium exchange rate in nominal terms based on knowledge of inflation in the Czech Republic and the euro area, see Figure 2. Subsequently, the percentage overvaluation or undervaluation of the actual CZK/EUR exchange rate relative to the model results was calculated, see Figure 3.

The results suggest that between 1999 and 2002 the Czech koruna was undervalued relative to the model by around 6%. From 2003 to 2009, the evolution of the real exchange rate was broadly in line with the model. In contrast, between 2010 and 2013, the Czech koruna was overvalued by around 9%. Between 2013 and 2017, the overvaluation of the Czech koruna has fallen to around 2%. Thus, if the CNB had not announced an exchange rate commitment, the Czech koruna would have been stronger. It is evident that the foreign exchange interventions had a long-term impact on the development of the Czech koruna (undervaluation) even after their termination. The deviation from the equilibrium of the model peaked in 2013, when the equilibrium exchange rate should have been around 29 CZK/EUR. After the outbreak

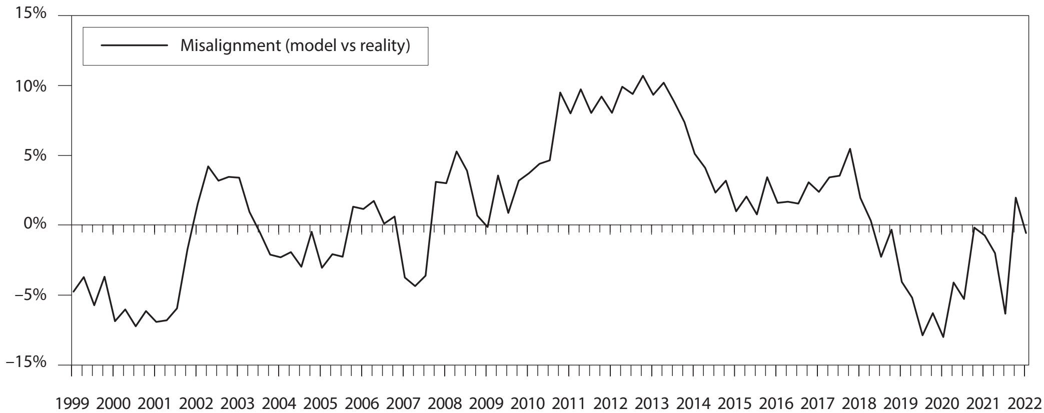
Figure 2 Range of the equilibrium CZK/EUR exchange rate in nominal terms



Source: Authorial computation

of the Covid-19 pandemic in 2020, the deviation decreased, and the exchange rate was in line with the model in 2022. Based on the actual CZK/EUR development after this date, it is known that the Czech koruna continued to appreciate in nominal terms until 2023, when it fell below the 24 CZK/EUR level.

Figure 3 Misalignment of the CZK/EUR exchange rate in nominal terms



Note: For misalignment, (+) indicates overvaluation and (-) undervaluation.
Source: Authorial computation

CONCLUSION

The aim of the academic paper was to analyse the determinants of the real equilibrium exchange rate with an emphasis on the long-run aspects. The empirical analysis was developed for the CZK/EUR currency pair over a long-time horizon between 1999 and 2022. The advantage of the long time series is the inclusion of the development of the Czech economy in the period of real convergence, but also in the period of economic stagnation, which is reflected, among other things, in the development of the Czech koruna. The model is based on the theoretical BEER approach, which is suitable for the study of a small open economy. The econometric method chosen was cointegration, or the error correction model, which has the advantage of distinguishing between short-run and long-run relationships. One of the most important variables for the analysis of the real equilibrium exchange rate appears to be the labour productivity differential between the Czech Republic and the euro area. When improving, the Czech koruna appreciated in both the short and long run, consistent with the Balassa-Samuelson effect. The terms of trade differential also proved to be an important variable, especially in the long run, when the Czech koruna appreciates during growth. The inclusion of the Czech Republic's real gross fixed capital formation also proved to be significant in the short and long run, with a positive effect on the appreciation of the Czech koruna.

For the real interest rate differential, the hypothesis that the Czech koruna appreciates in the short run when it rises because higher rates attract speculative capital was confirmed, while in the long run the Czech koruna depreciates because the country risk premium starts to be written into the real interest rate. Also included in the short-run relationship were the VIX index, which represents the global financial market's risk aversion. The growth of the VIX index leads to a weakening of the Czech koruna. It turned out that the Czech koruna is still one of the risky currencies and in case of an increase in global risk aversion the Czech koruna tends to depreciate. It was also a statistically significant variable when foreign exchange interventions started in 2013 and that led to a one-off depreciation of the Czech koruna. On the contrary, the dummy variable of the European Central Bank's quantitative easing led to a one-off depreciation of the euro or appreciation of the Czech koruna. Comparing the model results with the real data, it was possible to conclude that until 2002, the Czech koruna was significantly undervalued by around 6%, while between 2003 and 2009 it was roughly in line with the model. Since 2010, after the financial crisis, the Czech koruna has been overvalued by around 10%. Since the start of the CNB's foreign exchange interventions, the real exchange rate has moved closer to the model results and since 2019 the Czech koruna has been undervalued by around 8%. In 2022, the real exchange rate has returned to equilibrium in line with the model.

References

CLARK, P. B. MACDONALD, R. (1998). Exchange Rates and Economic Fundamentals: A Methodological Comparison of BEERs and FEERs. *IMF working papers* No. 67.

CIPRA, T. (2008). *Finanční ekonometrie.* Prague: Ekopress. ISBN 978-80-86929-93-4

CLARK, P. B. MACDONALD, R. (2000). Filtering the BEER: A Permanent and Transitory Decomposition. *IMF working papers* No. 144.

ČIHÁK, M. (1999). Real Exchange Rate: The Search for an Equilibrium. *Finance a úvěr*, 49(2).

DUDZICH, V. (2021). Real Exchange Rate Misalignments and Currency Crises in the Former Soviet Union Countries [online]. *Comparative Economic Studies*, 64: 384–416. <https://doi.org/10.1057/s41294-021-00178-9>.

DUFRENOT, G., ÉGERT, B. (2005). Real Exchange Rates in Central and Eastern Europe: What Scope for the Underlying Fundamentals? *Emerging Markets Finance & Trade*, 41(2).

ENGLE, R. F., GRANGER, C. W. J. (1987). Co-Integration and Error Correction: Representation, Estimation, and Testing [online]. *The Econometric Society*, 55(2). <https://doi.org/10.2307/1913236>.

FISHER, I. (1896). *Appreciation and Interest.* New York: Macmillan and Co.

FISHER, I. (1930). *The Theory of Interest.* New York: Macmillan and Co.

FRAIT, J., KOMÁREK, L. (1999). Dlouhodobý rovnovážný reálný kurz koruny a jeho determinanty. Institute of Economics of the Czech National Bank. *Prague Working Paper* No. 9.

HINDLS, R., ARLTOVÁ, M., HRONOVÁ, S., MALÁ, I., MAREK, L., PECÁKOVÁ, I., ŘEZANKOVÁ, H. (2018). *Statistika v ekonomii.* Prague: Professional publishing. ISBN 978-80-88260-09-7

KOMÁREK, L., KOPRNICKÁ, K., KRÁL, P. (2010). A Long-Term Real Appreciation as the Phenomenon of Economic Convergence [online]. *Politická ekonomie*, 58(1): 70–91. <https://doi.org/10.18267/j.polek.720>.

KOMÁREK, L., MELECKÝ, M. (2005). The Behavioural Equilibrium Exchange Rate of the Czech Koruna. Institute of Economics of the Czech National Bank, *Prague Working Paper* No. 5.

KOMÁREK, L., MOTL, M. (2012). Behavioural and Fundamental Equilibrium Exchange Rate of the Czech Koruna [online]. *Politická ekonomie*, 60(2). <https://doi.org/10.18267/j.polek.835>.

KREIDL. V. (1997). The Equilibrium Exchange Rate. *Finance a úvěr*, 47(10).

LAZAROVÁ, Š. (1997). Estimation of the Equilibrium Exchange Rate of the Koruna. *Finance a úvěr*, 47(10).

MANDEL, M., DURČÁKOVÁ, J. (2016). *Mezinárodní finance a devizový trh.* Prague: Management Press. ISBN 978-80-7261-287-1

MANDEL, M., VEJMĚLEK, J. (2021). Analysis of Relations in Uncovered Interest Rate Parity: Example of CZK/EUR Exchange Rate [online]. *Politická ekonomie*, 69(3): 340–359. <https://doi.org/10.18267/j.polek.1322>.

POŠTA, V. (2010). Estimation of the Behavioral Equilibrium Real Exchange Rate of the Czech Koruna. *Trends Economics and Management*, 7(4).

POUR, J., ILLICHMANN, V. (2022). Modelling Equilibrium Deviations of Exchange Rates from Purchasing Power Parity [online]. *Politická ekonomie*, 70(5): 531–551. <https://doi.org/10.18267/j.polek.1371>.

RUBASZEK, M. (2004). A Model of Balance of Payments Equilibrium Exchange Rate: Application to the Zloty. *Eastern European Economics*, 42(3).

ŠKOP, J., VEJMĚLEK, J. (2009). From PPP to Natrex – the Case of Czech Crown [online]. *Politická ekonomie*, 57(3): 323–343. <https://doi.org/10.18267/j.polek.687>.

ŠMÍDKOVÁ, K. (1999). Estimating the fundamental equilibrium exchange rate for the Czech economy [online]. *Prague Economic Papers*, 8(1). <https://doi.org/10.18267/j.pep.40>.

VEJMĚLEK, J. A monetary model of the exchange rate and its application to the Czech economy [online]. *Politická ekonomie*, 48(2). <https://doi.org/1010.18267/j.polek.127>.

WILLIAMSON, J. (1985). *The Exchange Rate System.* Institute for International Economics.

The Asymmetric Relation Between Money Supply and Inflation

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Abstract

In this study using yearly data, it is examined if the effect of money supply (broad money) on inflation is asymmetric or not. 38 countries which have 5% and above inflation rate in average during the period of 1989–2018 are investigated through the panel data analyses. The study differs from other researches, which use monetary shocks in explaining the asymmetric relation, in which that it uses broad money change intervals along with control variables to see the asymmetric impact. Using broad money change intervals, it is concluded that the relation between broad money and inflation is explained better in the asymmetric pooled and fixed effect panel data models, compared to the symmetric models. According to the results, the effects of negative and positive changes in money supply on inflation are not symmetric. Moreover, as broad money increases, inflation goes up further. In the light of this information, it is possible to mention an asymmetric relation between broad money and inflation.

Keywords	***DOI***	***JEL code***
Money supply, broad money, inflation, asymmetry	*https://doi.org/10.54694/stat.2023.24*	*C50, E30, E40*

INTRODUCTION

As known, in monetary economics, according to Fisher's transaction approach which is related to the quantity theory of money, there is a relationship between money supply (or the amount of money in circulation) and price level. The equation is $MV_T = PT$ and in the equation, (M) refers to the money supply, while (V_T) stands for the transactions velocity of circulation of money. The product of (M) and (V_T) should be equal to the product of the total amount of transactions (T) and the price level (P). From the perspective of income, quantity theory of money is later formulated as $MV_Y = PY$ as it is quite difficult to measure the volume and price level of transactions. In the income version of the quantity theory, (V_Y) is the income velocity of money, while (Y) is the real national income (or aggregate output). If it is assumed that (V_T) or (V_Y) and (T) or (Y) are stable, then the money supply and the price level will

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affect each other. Therefore, we can say that any increase in the amount of money in circulation will lead to an increase in price levels and, as said by Friedman (1963), "inflation is always and everywhere a monetary phenomenon".

Using monetary data might allow us to get information on the outlook for increase in price levels. Thus, monetary policy is important for the stock of money since it affects money supply. Decreasing policy rates make borrowing more attractive. However, monetary policy is not only a factor to affect broad money. Interest rates borrowers face or banks' lending criteria are also important (Berry et al., 2007). Investigating the monetary growth and inflation in Nigeria for the periods of 1982–1996 and 1996–2012, Moses et al. (2015) stated that the relationship between the two variables weakened in the second period and that this could be assigned to the developments in Nigerian financial system along with economy being more sophisticated.

While broad money is assumed a key driver of inflation, it might also determine the relationship between economic growth and inflation. Sare et al. (2019) stated that the ratio of broad money to GDP could be an indicator or a treshold in order for a country to keep its growth. According to the study, below the treshold, the effect of inflation can be even good for economic growth, as supported by Tobin (1965), Gregorio (1996), and Mundell (1965). Amassoma et al. (2018) examined the influence of money supply on inflation, however, they could not reach a relation in both long and short term. They stated the reason for this could be the recession in economy.

When investigating inflation, the expectation of inflation should also be taken into account. The formulation including the expectation of inflation can be shown as follows:

$$p_t = B\,E_t\,p_{t+1} + k\,X_t + shock_t,$$

where p is the inflation rate at time t, $E_t\,p_{t+1}$ is the expected rate of inflation at time $t+1$, X_t is the output gap (Coibion et al., 2018). If the government attempts to reduce unemployment by the use of monetary policy, then the rate of inflation will be on increase (El-Agraa, 2011). The expectation of inflation can also increase due to claims on central government. As mentioned by Ogunmuyiwa (2020), in addition to monetary policy, fiscal policy is another instrument to lower inflation. Adjusting the level of expenditures and taxes, especially governments in developing countries try to stabilize economy and curb price level increase. If the expenditures of central government are considerably more than taxes and there is a perception that the government will print money to pay its debts, then the expected rate of inflation will increase.

The effect of monetary growth on price levels might be indirect as well. As the amount of money in circulation increases, foreign exchange rates will increase, causing therefore costs to go up for countries with negative net exports. Odusola and Akinlo (2001) explained that the major causes of inflation were budget deficit (fiscal aspect), increase in money suppy (monetary aspect) and foreign exchange rate (balance of payments aspect). The balance of payment aspect of inflation is related to higher import prices due to foreign exchange rates rising.

In this study, the asymmetric relation between money supply and inflation is examined through broad money change intervals. The study differs from other researches, which use monetary shocks in explaining the asymmetric relation, in that it uses broad money change intervals along with control variables to see the asymmetric impact. The paper is organized as follows: Section 1 investigates the literature and provides theoretical background as well as empirical studies. Section 2 explains the methodolgy of study. As "monetary shocks" (error terms) may not reflect the real asymmetric impact of money supply on inflation, the nonlinear relation is analysed with growth rates in broad money divided into four groups. Besides, in order to get more reliable results, control variables are used in the models. In section 3, the asymmetric effect of broad money is assessed. First of all, unit root tests are performed and then panel data analyses are used. As per the chosen panel data models, otoregressive regression analyses are performed and the impact of broad money growth rates on inflation is examined. The last section is the conclusion section of the paper which summarizes and discusses the analysis results along with the limitations of study. The results are compared to the empirical studies in the litareture.

1 LITERATURE

In many researches in the literature, it is seen that the impact of money supply on inflation is investigated without taking different economic conditions, which potentially may influence the relation, into consideration. Conducting a research without using control variables will probably lead to inappropriate results. Even if different economic conditions are considered, adding these conditions into a regression analysis directly as independent variables would cause endogeneity concerns. Therefore, the relation between broad money and inflation might be established as follows, using control variables:

$$p_t = a + \sum_{i=1}^{m} A_i P_{t-i} + \sum_{i=0}^{m} B_i M_{t-i} + \sum_{i=0}^{m} C_i CV_{t-i} + \varepsilon_t , \tag{1}$$

where: p is the inflation rate, M is the broad money growth, CV is the control variables.

In line with the literature, the control variables may be chosen as real GDP growth rate, change in official exchange rate, claims on central government and domestic bank credits.

Let's consider real GDP growth rate. As Kennedy (2000) stated, real GDP growth requires increase in money supply to meet extra money demand. However, if increase in money supply is bigger than the real GDP growth rate, the value of money will decrease, therefore leading to inflation.

Similarly, bank credit is an important factor for broad money to grow. As bank credits increase, so will broad money. On the other hand, as Marshal et al. (2015) concluded, bank credit and economic growth had a short term relation as well. For this reason, it is likely that there could be a endogeneity problem. Although money is created on the basis of credit, some researches showed that credit was not the main determinant of money supply or deposits, and that deposits were the main factor of credits, and that deposits could exceed the effect of credits on money supply. Therefore credits can be taken into account as a control variable (Tiryaki and Hasanov, 2022).

The amount of claims on central government might be a signal for a government to print money. Because, as the claims on central government increase, it will be difficult for the government to pay its debts. Therefore, due to potential default risk, expected inflation rate would rise and eventually result in price level escalating. As supported by Klein and Ichimura (2000), claims on central government and financial deficit are closely related to each other and financial deficit will lead enterprises and residents to hold more currency and deposit. As a result of this, increase in reserve money and broad money would arise.

As for exchange rate, it is assumed that monetary authority would like to adjust money supply in order to stabilize exchange rates, which means the monetary authority either decreases or increases interest rates depending on the changes in foreign exchange rates. For an illustration, for BRICS countries, Si et al. (2018) concluded that there were co-movement and causality relation between exchange rates and interest rates. In addition, as said by Bianchi and Deschamps (2018), Singapore's monetary policy is based on the exchange rate. For this reason, it is deemed fit to consider exchange rate as a control variable.

As theoretical background, another approach to the quantity theory of money is the Cambridge cash-balance approach that is $M^d = kPY$. In this formula, (M^d) is the demand for money and (P) is the price level. (Y) reflects the real national income, while (k) is the proportion of nominal income which people want to hold in money. As per the formula, considering money-market equilibrium, demand for money and supply of money need to be equal. Thus, in equilibrium $M = kPY$ (Cesarano, 2008; Runde, 1994; Buthelezi, 2023). The literature offers also non-monetary theories. These are demand pull inflation (demand is not met by supply), cost push inflation (cost increases faster than productivity), profit push inflation (profit is the main cause of inflation), imported inflation (the effect of external inflation) and politically caused inflation (O'Neill, 2017; Alpago, 2021).

In accordance with the monetarist theories, Şahin (2019) stated that inflation in Turkey rises as money supply increases, and that money supply with budget deficit leads to inflation (Şahin, 2019; Kaya and Öz, 2016). Similarly, the studies by Chaundhary and Parai (1991), Altıntaş et al. (2008), Lozano (2008), Bakare et al. (2014), Koyuncu (2014), and Dekkiche (2022) are the empirical studies implying that money supply and inflation show a significant relation. On the other hand, Şahin and Karanfil (2015) did not reach a causality relation between the variables (Şahin, 2019). Koti and Bixho (2016) found that money supply did not cause inflation in Albania, while Ditimi et al. (2018) indicated that money supply was not an important factor in Nigeria's inflation, using the ARDL-ECM method (Dekkiche, 2022).

In the literature, it is seen that the asymmetric effect of monetary policy on inflation is investigated through positive or negative monetary shocks. Examining the relation between money growth and inflation, the study by Cooray and Kheraief (2018) revealed that the response of inflation to positive and negative monetary shocks was asymmetric. Just as the study by Cooray and Kheraief (2018), the researches by Olayiwola and Ogun (2019), and Khundrakpam (2013) focused on positive and negative monetary shocks, concluding that monetary policy or money supply had asymmetric impact on prices. This can raise the question whether different amounts of change in broad money could also cause asymmetric effect on inflation. Thus, instead of asymmetric impact of positive or negative monetary shocks, the asymmetric relation between money supply and inflation might be examined via the change in broad money. In addition, in the literature, it seems that these monetary shocks are the error terms of autoregressive models. However, since the error terms may arise from variables which are not used in the model, it may not be meaningful to call these residuals as monetary shocks. For an illustration, if real economic growth in a country is considerably high, then it can be necessary to further increase money supply compared to previous year and if the economic growth is not taken into consideration in the model, this will result in error term in the model increasing and indicate a positive monetary shock which is actually not.

2 METHODOLOGY

In this study using the Worldbank indicators and yearly data, the countries which had 5% or greater inflation rate in average during the period of 1989–2018 are examined as De Grauwe and Polan (2005) suggest that the relation between monetary growth and inflation is weak for countries that have low inflation. The period used is from 1989 to 2018 and the countries are shown in the Appendix 1. Although there are other countries which had minimum 5% or greater inflation rate in average during the period of 1989–2018, those countries could not be analyzed due to missing data of some variables used in this study. In addition, when the period is extended, the number of missing data increases. Hence, the period is defined as 1989–2018 to analyze as much data (many country) as possible.

The variables used in the study are illustrated through the Appendix 2. In line with the literature, the control variables are chosen as real GDP growth rate, change in official exchange rate, claims on central government and domestic bank credits. Therefore, RGDP, EXCH, COCG and DBC are included in the models as control variables. M is used for the symmetric relation, while MG1, MG2, MG3 and MG4 are the groups defined as per the changes in broad money and incorporated into the models investigating the asymmetric relation. The broad money is defined in the Worldbank indicator note as "the sum of currency outside banks; demand deposits other than those of the central government; the time, savings, and foreign currency deposits of resident sectors other than the central government; bank and traveler's checks; and other securities such as certificates of deposit and commercial paper".

As stated before, the study differs from other researches, which use monetary shocks in explaining the asymmetric relation, in that it uses the broad money change intervals along with control variables to see the asymmetric impact. In the light of aforementioned literature and as a new alternative approach to the asymmetric relation, based on the distribution of observations, the percentage changes in broad money (the observations) are defined and divided into four groups as follows:

Group 1: the changes in broad money less than 0% (reduction in broad money),
Group 2: the changes in broad money between 0% and 25%,
Group 3: the changes in broad money between 25% and 50%,
Group 4: the changes in broad money greater than 50%.

The groups and their intervals are defined according to the explanations stated below:

- In the literature, to detect assymetric relation, it seems that negative monetary shocks are also included in the studies. For this reason, in order to see the negative impacts of reduction in broad money on inflation, the first group is composed of negative changes in broad money.
- For the positive changes in broad money, the second and third group intervals are defined as 0%–25% and 25%–50%, respectively. The reason for this is that some observations in the real GDP growth variable have quite high values. For instance, the growth rate of Eswatini in 1990 reached 21 percent. Therefore, in order to better understand the assymetric positive impact of broad money on inflation, it is deemed fit to define the intervals greater than 21%. This would also allow us to see the impact of excess broad money growth on inflation (all the broad money growths will be higher than all the real GDP growths in the third and fourth groups). As known, if the money supply rises faster than real output, prices will usually increase. The second reason is that as the intervals get smaller, the number of groups increase, decreasing the number of observations in the groups. This could statistically cause bias and distorted results. Therefore, the fourth group includes all the broad money changes greater than 50% in order to limit the number of groups.

After that, as per the groups, the relation between broad money and inflation is re-established as stated below:

$$p_t = a + \sum_{i=1}^{m} A_i p_{t-i} + \sum_{i=0}^{m} D_1 B_{1i} MG1_{t-i} + \sum_{i=0}^{m} D_2 B_{2i} MG2_{t-i} + \sum_{i=0}^{m} D_3 B_{3i} MG3_{t-i} + \sum_{i=0}^{m} D_4 B_{4i} MG4_{t-i} + \sum_{i=0}^{m} C_i CV_{t-i} + \varepsilon_t , \qquad (2)$$

where:

$D_1=1, D_2=0, D_3=0$ *and* $D_4=0$; if the change in broad money belongs to Group 1,
$D_1=0, D_2=1, D_3=0$ *and* $D_4=0$; if the change in broad money belongs to Group 2,
$D_1=0, D_2=0, D_3=1$ *and* $D_4=0$; if the change in broad money belongs to Group 3,
$D_1=0, D_2=0, D_3=0$ *and* $D_4=1$; if the change in broad money belongs to Group 4,
CV is the control variables.

As we do not have specific data on expected rates of inflation to cover all the countries in this study, the control variables used might also be considered as proxy variables for expectations about inflation. Therefore, expected rates of inflation are indirectly incorporated into the analyses as control variables. For instance, claims on central government and domestic bank credits could be the indirect factors that people consider could decrease or increase inflation. As mentioned in the literature section, as claims on central government or domestic bank credits increase, the expected rate of inflation would increase.

In order to state that there is an asymmetric relation between broad money and inflation, within the scope of this study, it is investigated whether the following two requirements are met:

i) the adjusted R squared of Formula (2) should be greater than that of Formula (1),

ii) instead of $H_0 = [B_i^{Group1} = B_i^{Group2} = B_i^{Group3} = B_i^{Group4}]$ implying a symmetric relation, $H_A = [B_i^{Group1} \neq B_i^{Group2} \neq B_i^{Group3} \neq B_i^{Group4}]$ showing an asymmetric relation should be accepted.

The first condition implies that Formula (2), which is an asymmetric relation, has a better explanation on the relation between broad money and inflation, while the second condition indicates that different changes in broad money have different effects on inflation.

3 EMPIRICAL STUDY

Within the scope of the study, first of all, unit root tests are applied in order to decide whether the variables can be used at I(0) or not. Im-Pesaran-Shin (2003) and Levin-Lin-Chu (2002) unit root test results shown in Table 1 and Table 2 respectively imply that:

- according to both tests, p, M and RGDP can be used at I(0),
- although COCG (Intercept and Trend) is not stationary at I(0) as per Im-Pesaran-Shin Unit Root Analysis, it is deemed stationary at I(0) as both COCG (Intercept) and COCG (Intercept & Intercept and Trend) are stationary at I(0) as per Im-Pesaran-Shin and Levin-Lin-Chu unit root analyses,
- both tests indicate that DBC is not stationary at I(0). Thus, the log difference of DBC, ld_DBC, is used in the analyses investigating the symmetric and asymmetric relations.

In addition, as M is stationary at (0); MG1, MG2, MG3 and MG4 are deemed stationary at I(0) as well.

Table 1 Im-Pesaran-Shin unit root analysis

Variable	I(0) / I(1)	Intercept		Intercept and trend	
		Im-Pesaran-Shin t-bar	Significance level	Im-Pesaran-Shin t-bar	Significance evel
p	I(0)	–2.76771	***	–3.26773	***
	I(1)	–	–	–	–
M	I(0)	–4.1183	***	–4.54576	***
	I(1)	–	–	–	–
RGDP	I(0)	–4.17596	***	–4.54576	***
	I(1)	–	–	–	–
EXCH	I(0)	–4.2592	***	–4.59112	***
	I(1)	–	–	–	–
COCG	I(0)	–1.82384	**	–2.12721	–
	I(1)	–	–	–	–
DBC	I(0)	–0.977661	–	–1.82642	–
	I(1)	–4.5421	***	–4.64426	***

Note: For the model with intercept, the critical values are –1.72, –1.77 and –1.88 for 10%, 5% and 1% respectively. For the model with intercept and trend, the critical values are –2.35, –2.41 and –2.51 for 10%, 5% and 1%. ***, **,* reflect significance level of 1%, 5% and 10% respectively.
Source: Own construction

Table 2 Levin-Lin-Chu unit root analysis

Variable	I(0) / I(1)	Intercept			Intercept and trend		
		Coefficient	t ratio	z-score [p value]	Coefficient	t ratio	z-score [p value]
p	I(0)	−0.57891	−23.279	−16.2876 [0.0000]	−0.66247	−24.423	−15.3744 [0.0000]
	I(1)	–	–	–	–	–	–
M	I(0)	−0.72906	−26.061	−20.1039 [0.0000]	−0.83735	−28.450	−19.7083 [0.0000]
	I(1)	–	–	–	–	–	–
RGDP	I(0)	−0.71486	−25.473	−18.6035 [0.0000]	−0.80596	−28.067	−18.0388 [0.0000]
	I(1)	–	–	–	–	–	–
EXCH	I(0)	−0.98298	−32.793	−27.3207 [0.0000]	−0.99555	−33.140	−23.9638 [0.0000]
	I(1)	–	–	–	–	–	–
COCG	I(0)	−0.14557	−10.775	−3.74581 [0.0001]	−0.22131	−12.671	−2.67754 [0.0037]
	I(1)	–	–	–	–	–	–
DBC	I(0)	−0.063047	−5.871	0.880028 [0.8106]	−0.153	−9.873	0.733616 [0.7684]
	I(1)	−0.8077	−27.621	−20.782 [0.0000]	−0.84611	−29.214	−19.1121 [0.0000]

Source: Own construction

Since 38 countries are examined and panel data is used in the study, it is needed to decide what panel data analysis model to be used. Therefore; F test, Breusch-Pagan Test and Hausman test are applied for comparison. The null and alternative hypotheses for these tests are shown below:

F test	Breusch-Pagan test	Hausman test
H_0 = Pooled Panel Data Analysis H_A = Fixed Effect Panel Data Analysis	H_0 = Pooled Panel Data Analysis H_A = Random Effect Panel Data Analysis	H_0 = Random Effect Panel Data Analysis H_A = Fixed Effect Panel Data Analysis

As per Table 3 which reflects model selection, it is seen that both pooled and fixed effects prevail against random effect panel data and that it will be more appropriate to apply pooled panel data. However, for the avoidance of any doubt, both pooled and fixed effect panel data analyses are applied to see the relations.

The pooled and fixed effect panel data analyses results are shown in Table 4 and Table 5 respectively. As planned, the analyses are performed on both symmetric and asymmetric relation. As per the results, both P values and adjusted R squared values of the asymmetric relations are better than those of symmetric relations.

After concluding that adjusted R squared values of asymmetric relation are greater, as the second condition, the hypotheses which are $H_0 = [B_i^{Group1} = B_i^{Group2} = B_i^{Group3} = B_i^{Group4}]$ and $H_A = [B_i^{Group1} \neq B_i^{Group2} \neq B_i^{Group3} \neq B_i^{Group4}]$ are tested and the results thereof are shared in Table 6.

As per Table 6, for both pooled and fixed effect models, the F statistics have an implication that the betas of different groups are not statistically equal to each others, thus indicating that the relation is asymmetric.

Table 3 Panel Data Analysis model selection

Tests used			Selected model
F test	**Breusch-Pagan test**	**Hausman test**	
Symmetric relation			
0.876151 (0.681787)	–	–	Pooled
–	2.08892 (0.148371)	–	Pooled
–	–	33.3836 (0.006572)***	Fixed effect
Asymmetric relation			
1.16221 (0.235279)	–	–	Pooled
–	0.304118 (0.581313)	–	Pooled
–	–	45.4549 (0.007418)***	Fixed effect

Note: ***, **, * reflect significance level of 1%, 5% and 10% respectively.
Source: Own construction

Table 4 Pooled Panel Data Analysis results

Independent variable	Dependent variable: inflation	
	Panel data analysis: pooled	
	Symmetric relation	**Asymmetric relation**
Constant	0.882625 (0.0859)*	1.72424 (0.0152)**
p_1	0.611151 (1.07e-122)***	0.540793 (1.36e-096)***
M	0.200412 (3.79e-029)***	–
M_1	0.0851506 (3.98e-06)***	–
M_2	–0.0331218 (1.58e-069)***	–
MG1	–	–0.184506 (0.0338)**
MG1_1	–	–0.587201 (4.50e-011)***
MG1_2	–	0.154766 (0.0778)*
MG2	–	0.102975 (0.0105)**
MG2_1	–	0.100769 (0.0122)**

Table 4 (continuation)

Independent variable	Dependent variable: inflation Panel data analysis: pooled Symmetric relation	Asymmetric relation
MG2_2	–	–0.050833 (0.1631)
MG3	–	0.134193 (3.16e-07)***
MG3_1	–	0.123031 (1.70e-06)***
MG3_2	–	–0.0339215 (0.1266)
MG4	–	0.242289 (5.97e-036)***
MG4_1	–	0.141102 (9.71e-012)***
MG4_2	–	–0.0314155 (4.89e-066)***
RGDP	–0.679792 (9.33e-019)***	–0.587916 (2.04e-015)***
RGDP_1	0.165466 (0.0359)**	0.166504 (0.0272)**
RGDP_2	0.118097 (0.1047)	0.138205 (0.0473)**
EXCH	–0.000319 (0.5992)	–0.000436 (0.4496)
EXCH_1	0.000509 (0.4023)	0.0003094 (0.5911)
EXCH_2	2.32036e-05 (0.9693)	9.35856e-06 (0.9869)
COCG	–0.150150 (0.0045)***	–0.167587 (0.0009)***
COCG_1	0.347071 (5.84e-06)***	0.330745 (5.40e-06)***
COCG_2	–0.177569 (0.0007)***	–0.127888 (0.0100)**
Id_DBC	–6.79550 (1.82e-05)***	–6.97568 (4.26e-06)***
Id_DBC_1	3.90248 (0.0121)**	3.04975 (0.0395)**
Id_DBC_2	–2.20387 (0.1256)	–2.15244 (0.1179)
P-value (F)	4.8e-290***	1.4e-306***
Adjusted R squared	0.745976	0.773218
Durbin-Watson	2.078846	2.075212

Source: Own construction

Table 5 Fixed Effect Panel Data Analysis results

	Dependent variable: inflation Panel data analysis: fixed effect	
Independent variable	**Symmetric relation**	**Asymmetric relation**
Constant	1.85907 (0.0085)***	3.23739 (0.0003)***
p_1	0.566570 (2.66e-096)***	0.494842 (1.44e-075)***
M	0.191828 (2.72e-025)***	–
M_1	0.087383 (4.36e-06)***	–
M_2	–0.029899 (6.52e-052)***	–
MG1	–	–0.182592 (0.0402)**
MG1_1	–	–0.596345 (4.87e-011)***
MG1_2	–	0.111542 (0.2111)
MG2	–	0.083138 (0.0444)**
MG2_1	–	0.088592 (0.0312)**
MG2_2	–	–0.06602 (0.0769)*
MG3	–	0.109276 (4.90e-05)***
MG3_1	–	0.109711 (2.80e-05)***
MG3_2	–	–0.05218 (0.0227)**
MG4	–	0.236651 (8.00e-033)***
MG4_1	–	0.151358 (9.65e-013)***
MG4_2	–	–0.028307 (4.08e-050)***
RGDP	–0.695415 (1.13e-017)***	–0.590117 (3.27e-014)***
RGDP_1	0.118634 (0.1477)	0.131189 (0.0918)*
RGDP_2	0.079877 (0.2962)	0.113066 (0.1207)
EXCH	–0.00053 (0.3924)	–0.00069 (0.2359)

Table 5		(continuation)
	Dependent variable: inflation Panel data analysis: fixed effect	
Independent variable	**Symmetric relation**	**Asymmetric relation**
EXCH_1	0.000275 (0.6581)	6.9860e-06 (0.9905)
EXCH_2	–0.000175 (0.7767)	–0.00028 (0.6273)
COCG	–0.15153 (0.0061)***	–0.174298 (0.0009)***
COCG_1	0.332102 (1.54e-05)***	0.317535 (1.22e-05)***
COCG_2	–0.155858 (0.0041)***	–0.10632 (0.0382)**
ld_DBC	–7.17756 (8.39e-06)***	–6.85155 (7.65e-06)***
ld_DBC_1	3.43576 (0.0291)**	2.90239 (0.0514)*
ld_DBC_2	–2.56010 (0.0799)*	–2.09729 (0.1311)
P-value (F)	2.4e-260	6.3e-280
Adjusted R squared	0.744817	0.77457
Durbin-Watson	2.050529	2.06198

Source: Own construction

Table 6 Fixed Effect Panel Data Analysis results

	F Statistic	
Null & alternative hypotheses	**Pooled**	**Fixed effect**
$H_0 = [B_i^{Group1} = B_i^{Group2} = B_i^{Group3} = B_i^{Group4}]$ $H_A = [B_i^{Group1} \neq B_i^{Group2} \neq B_i^{Group3} \neq B_i^{Group4}]$	14.4672 (3.52964e-022)***	15.2546 (2.08619e-023)***

Source: Own construction

DISCUSSION AND CONCLUSION

In this study, it is investigated whether there is an asymmetric relation between money supply (broad money) and inflation. Unlike previous studies in the literature, the asymmetric relation is not examined through monetary shocks which are positive or negative error terms of autoregressive models of money supply or interest rates as monetary shocks may not reflect a real shock. Instead, broad money change intervals are used in order to see the nonlinear relation. In the study, real GDP growth rate, change in official exchange rate, claims on central government and domestic bank credits are added into the models as control variables and growth rates in broad money are divided into four groups. The first group reflects negative rates, in other words, reduction in broad money, while the fourth group is the growth rates which are above 50%. Using broad money change intervals, it is concluded that the relation between broad money and inflation is explained better in the asymmetric pooled and fixed effect panel data models, compared to the symmetric models, as the betas of groups are not statistically equal. According to the results, the effects of negative and positive changes in money supply on inflation are not symmetric. Moreover, as broad money increases, inflation goes up further. In the light of this information, it is possible to state that the effect of money supply on inflation is not symmetric but asymmetric. Since all the broad money growths are higher than all the real GDP growths in the third and fourth groups, another economic interpretation is that as the excess broad money growth rises, its impact on inflation increases. In this regard, considering the control variables are included in the model, the results of this study are also in line with the monetarist theory of inflation and even imply that there is a non-linear relation between the variables. The results obtained are similar to the results of analyses by Cooray and Kheraief (2018), Olayiwola and Ogun (2019), and Khundrakpam (2013), indicating an asymmetric relation. Therefore, as mentioned by Alpago (2021), taking into consideration the statement that inflation is a result of poor monetary and fiscal policy might be more effective in making decisions about how to fight inflation. Furthermore, inflation is discussed differently in developing and developed countries (Shaikh et al., 2022). In developed countries, inflation might be more related to monetary approach but in developing countries, it may not be purely monetary. Since even political and structural factors could play a role in inflation, it would be quite challanging to decompose inflation into its demand-pull, monetary, cost-push and structural components as the process is dynamic and the shocks to prices are mixed (Totonchi, 2011; Esumanba et al., 2019, Shaikh et al., 2022). For this reason, in case that more variables defined by non-monetary theories are included in studies to be carried out, it is likely to obtain interesting analysis results.

References

ALPAGO, H. (2021). Inflation as an Instrument of Monetary Regulation [online]. *İmgelem,* 5(9): 281–293. <https://doi.org/10.53791/imgelem.994473>.

ALTINTAŞ, H., ÇETINKAYA, H., TABAN, S. (2008). Türkiye'de Bütçe Açığı, Parasal Büyüme ve Enflasyon Arasındaki İlişkinin Ekonometrik Analizi: 1992–2006. *Anadolu Üniversitesi Sosyal Bilimler Dergisi*, 8(2): 185–208.

AMASSOMA, D., SUNDAY, K., ONYEDIKACHI, E. E. (2018). The influence of Money Supply on Inflation in Nigeria [online]. *Journal of Economics and Management,* 31(1): 5–23. <https://doi.org/10.22367/jem.2018.31.01>.

BAKARE, I. A. O., ADESANYA, O. A., BOLARINWA, S. A. (2014). Empirical Investigation Between Budget Deficit, Inflation and Money Supply in Nigeria. *European Journal of Business and Social Sciences*, 2(12): 120–134.

BERRY, S. et al. (2007). Interpreting Movements in Broad Money. *Bank of England Quarterly Bulletin*, Q3: 376–388.

BIANCHI, P., DESCHAMPS, B. (2018). The Effectiveness of Exchange-Rate-Based Monetary Policy: Evidence from Survey Data [online]. *Applied Economics Letters,* 25(18): 1261–1265. <https://doi.org/10.1080/13504851.2017.1414930>.

BUTHELEZI, E. M. (2023). Impact of Money Supply in Different States of Inflation and Economic Growth in South Africa [online]. *Economies,* 11(64). <https://doi.org/10.3390/economies11020064>.

CESARANO, F. (2008). *Monetary Systems and Monetary Theory*. Money and Monetary Systems, Cheltenham: Edward Elgar Publishing, 79–95.

CHAUDHARY, M. A. S., PARAI, A. K. (1991). Budget Deficit and Inflation: The Peruvian Experience [online]. *Applied Economics*, 23(6): 1117–1121. <https://doi.org/10.1080/00036849100000015>.

COIBION, O., GORODNICHENKO, Y., KAMDAR, R. (2018). The Formation of Expectations, Inflation and The Phillips Curve [online]. *Journal of Economic Literature,* 56(4): 1447–1491. <https://doi.org/10.1257/jel.20171300>.

COORAY, A., KHERAIEF, N. (2019). Money Growth and Inflation: New Evidence from a Nonlinear and Asymmetric Analysis [online]. *The Manchester School,* 87(4): 1–35. <https://doi.org/10.1111/manc.12258>.

DE GRAUWE, P., POLAN, M. (2005). Is Inflation Always and Everywhere a Monetary Phenomenon? [online]. *Scandinavian Journal of Economics,* 107: 239–259. <https://doi.org/10.1111/j.1467-9442.2005.00406.x>.

DEKKICHE, D. (2022). Impact of Money Supply on Inflation Rate in Egypt: A VECM Approach [online]. *Economics and Business,* 36: 134–148. <https://doi.org/10.2478/eb-2022-0009>.

DITIMI, A., SUNDAY, K., ONYEDIKACHI, E. E. (2018). The Upshot between Money Supply and Inflation in Nigeria [online]. *Journal of Economic Cooperation and Development*, 39: 83–108. <https://doi.org/10.1515/vjes-2017-0021>.

EL-AGRAA, A. M. (2011). *The European Union: Economics and Policies*. Cambridge University Press.

ESUMANBA, S. V., DANAA, N., KONADU, N. E. K. (2019). The Impact of Money Supply on Inflation Rate in Ghana [online]. *Research Journal of Finance and Accounting,* 10(16): 156–170. <https://doi.org/10.7176/RJFA>.

FRIEDMAN, M. (1963). *Inflation: Causes and Consequences*. New York: Asia Publishing House.

GREGORIO, J. (1996). Inflation, Growth, and Central Banks: Theory and Evidence [online]. *World Bank Policy Research Working Paper 1575*, Policy Research Department, Macroeconomics and Growth Division. <https://doi.org/10.1596/1813-9450-1575>.

IM, K. S., PESARAN, M. H., SHIN, Y. (2003). Testing for Unit Roots in Heterogeneous Panels [online]. *Journal of Econometrics,* 115: 53–74. <https://doi.org/10.1016/S0304-4076(03)00092-7>.

KAYA, M. G., ÖZ, E. (2016). Enflasyon, Bütçe Açığı ve Para Arzı İlişkisinin Türkiye Ekonomisi Açısından Değerlendirilmesi: 1980–2014 Dönemi [online]. *Manisa Celal Bayar Üniversitesi İ.İ.B.F Yönetim ve Ekonomi*, 23(3): 639–651. <https://doi.org/10.18657/yonveek.281800>.

KENNEDY, P. E. (2000). *Macroeconomic Essentials: Understanding Economics in the News*. Cambridge MA: The MIT Press.

KHUNDRAKPAM, J. K. (2013), *A Note on Differential Asymmetric Effects of Money Supply and Policy Rate Shocks in India* [online]. <https://mpra.ub.uni-muenchen.de/53058/1/MPRA_paper_53058.pdf>.

KLEIN, L. R., ICHIMURA, S. (2000). *Econometric Modeling of China*. World Scientific Publishing Company.

KOTI, S., BIXHO, T. (2016). Theories of Money Supply: The Relationship of Money Supply in a Period of Time T-1 and Inflation in Period T – Empirical Evidence from Albania [online]. *European Journal of Multidisciplinary Studies*, 1(1): 294–302. <https://doi.org/10.26417/ejms.v1i1.p294-302>.

KOYUNCU, F. T. (2014). Causality Network Between Budget Deficit, Money Supply and Inflation: An Application to Turkey. *International Journal of Business and Social Science*, 5–10(1): 225–235.

LEVIN, A., LIN, C. F., CHU, C. S. J. (2002). Unit Root Tests in Panel Data: Asymptotic and Finite-sample Properties [online]. *Journal of Econometrics,* 108: 1–24. <https://doi.org/10.1016/S0304-4076(01)00098-7>.

LOZANO, I. (2008). Budget Deficit, Money Growth and Inflation: Evidence From Colombia Case, Banco De La Republica [online]. *Borradores de Economia*, 5127: 1–25. <https://doi.org/10.32468/be.537>.

MARSHAL, I., SOLOMON, I. D., ONYEKACHI, O. (2015). Bank Domestic Credits and Economic Growth Nexus in Nigeria (1980–2013) [online]. *International Journal of Finance and Accounting,* 4(5): 236–244. <https://doi.org/10.5923/j.ijfa.20150405.02>.

MOSES, K. T. et al. (2015). Monetary Growth and Inflation Dynamics in Nigeria. *CBN Working Paper Series CBN/WPS/01/2015/02*, Central Bank of Nigeria, 2–21.

MUNDELL, R. (1965). Growth, Stability and Inflationary Finance [online]. *Journal of Political Economy,* 73: 97–109. <https://doi.org/10.1086/258999>.

ODUSOLA, A. F., AKINLO, A. E. (2001). Output, Inflation and Exchange Rate in Developing Countries: An Application to Nigeria [online]. *The Developing Economies,* 39(2): 199–222. <https://doi.org/10.1111/j.1746-1049.2001.tb00900.x>.

OLAYIWOLA, A. S., OGUN, T. P. (2019). Asymmetric Effect of Monetary Policy Shocks on Output and Prices in Nigeria. *African Journal of Economic Review,* 7(1): 238–248.

OGUNMUYIWA, M. S. (2020). Managing the Inflation Problem in Nigeria using the Fisco-Monetary Approach [online]. *Athens Journal of Business & Economics,* 6(3): 239–252. <https://doi.org/10.30958/ajbe.6-3-4>.

O'NEILL, R. (2017). What Is Inflation? In: *Inflation*, Springer International Publishing.

RUNDE, J. (1994). Keynesian uncertainty and liquidity preference. *Cambridge Journal of Economics,* 18: 129–144.

SARE, Y. A., IBRAHIM, M., MUSAH, A. (2019). Examining the Threshold Effects of Inflation on Economic Growth in Ghana. *Ghanaian Journal of Economics,* 7: 5–23.

SHAIKH, P. A., MUHAMMAD, F., KHAN, S. K. (2022). The Dynamic Theories of Inflation [online]. *Pak. Journal of Int'L Affairs*, 5(2): 956–971. <https://doi.org/10.52337/pjia.v5i2.488>.

SI, D. K. et al. (2018). Co-Movement and Causality Between Nominal Exchange Rates and Interest Rate Differentials in BRICS Countries: A Wavelet Analysis. *Romanian Journal of Economic Forecasting,* 21(1): 5–19.

ŞAHİN, B. E. (2019). Analysis of the Relationship between Inflation, Budget Defıcıt and Money Supply in Turkey by ARDL Approach: 1980–2017 [online]. *Journal of Life Economics*, 6(3): 297–306. <https://doi.org/10.15637/jlecon.6.018>.

ŞAHİN, İ., KARANFİL, M. (2015). Türkiye Ekonomisinde 1980–2013 Dönemi Para Arzının Enflasyon Üzerindeki Etkisi. *Business and Economics Research Journal*, 6(4): 97–113.

TIRYAKI, G., HASANOV, M. (2022). Do Credits Affect Money Supply and Deposits, or Vice Versa, or Interconnected? [online]. *Journal of BRSA Banking and Financial Markets*, 16(2): 217–245. <http://doi.org/10.46520/bddkdergisi.1178353>.

TOBIN, J. (1965). Money and Economic Growth [online]. *Econometrica,* 33: 671–684. <https://doi.org/10.2307/1910352>.

TOTONCHI, J. (2011). Macroeconomic Theories of Inflation. *International Proceedings of Economics, Development & Research*, 4: 36

WORLD BANK. (2020). *World Bank Indicators* [online]. [cit. 6.8.2020]. <https://www.worldbank.org>.

APPENDICES

Appendix 1 The countries investigated

Algeria	Guatemala	Nigeria
Bangladesh	Haiti	Pakistan
Bhutan	Iceland	Papua New Guinea
Bolivia	India	Paraguay
Botswana	Indonesia	Peru
Chile	Israel	Philippines
Costa Rica	Jamaica	Sri Lanka
Dominican Republic	Kenya	Sudan
Ecuador	Madagascar	Trinidad and Tobago
Egypt, Arab Rep.	Mauritius	Tunisia
El Salvador	Mexico	Turkey
Eswatini	Myanmar	Uruguay
Ghana	Nepal	

Appendix 2 The variables used

Variable	Explanation
p	Inflation
M	Change in broad money
MG1	Group 1 – Change in broad money (less than 0%)
MG2	Group 2 – Change in broad money (between 0% and 25%)
MG3	Group 3 – Change in broad money (between 25% and 50%)
MG4	Group 4 – Change in broad money (greater than 50%)
RGDP	Real GDP growth rate
EXCH	Change in official exchange rate
COCG	Claims on central government (% of GDP)
DBC	Domestic bank credits (% of GDP)

Source: The data is derived from Worldbank indicators

Unconventional Monetary Policy Response to Covid-19 and Its Impact on Inflation in Morocco

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Abstract

This study explores the impact of unconventional monetary policy on Morocco's economy during the Covid-19 pandemic. We used a hybrid model combining a financial dynamic stochastic general equilibrium (DSGE) model with a standard epidemiology model, enabling us to consider both economic and epidemiological factors. Our results indicate that unconventional monetary policy cannot fully mitigate the adverse effects of a pandemic, except for an exogenous increase in Central Bank claims. We also found that Morocco's high inflation is partly due to Bank Al-Maghrib's unconventional monetary measures in response to the pandemic. Our research underscores the importance of monetary authorities balancing the benefits and risks of unconventional monetary policy. While it can stimulate the economy during crises, it should be used judiciously to avoid long-term negative effects. Incorporating epidemiological factors into macroeconomic models is crucial for understanding the intricate interplay between the economy and public health crises.

Keywords	***DOI***	***JEL code***
Monetary policy, financial DSGE, epidemiology model	*https://doi.org/10.54694/stat.2024.8*	*E1, E5, E6, H5, I1*

INTRODUCTION

The Covid-19 pandemic has wrought significant disruption across global economies, prompting fiscal authorities worldwide to devise and implement stabilization packages to support households and businesses. In response, major central banks, including the Federal Reserve and Bank Al-Maghrib (BAM), have

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rapidly deployed their Financial Crisis Toolkits. BAM, for instance, undertook several measures to bolster the economy: it reduced the key rate from 2.25% to 2% on March 17, followed by a further reduction to 1.5% on June 16, thereby easing the financing conditions for households and companies. Additionally, BAM enhanced its refinancing program for small and medium-sized enterprises (SMEs) to encompass both investment and operating credits, and banks were granted access to BAM's refinancing instruments in dirham and foreign currency. Tailored measures were also developed specifically for banks.

Prior to the pandemic, the integration of epidemiology into macroeconomic theory was sparse, with notable exceptions in microeconomics, as evidenced by the works of Horan and Wolf (2005), Horan and Fenichel (2007), Fenichel et al. (2011), Lenhart and Workman (2007), and Morin et al. (2015). More recently, researchers have employed Susceptible-Infected-Recovered (SIR) epidemiological models to assess the potential economic impacts of pandemics on a macroeconomic scale, aligning with the macro model proposed by Eichenbaum et al. (2020b). Yet, the role of financial intermediaries in epidemic-economic systems has been largely uncharted territory, and the focus on how monetary policies can alleviate the economic toll of pandemics has been limited.

Our research methodology draws inspiration from the approach of Verónica Acurio Vásconez et al. (2021), utilizing a hybrid macroeconomic and epidemiological model. We meld a dynamic stochastic general equilibrium (DSGE) model akin to Smets and Wouters (2007), which includes a financial sector as conceptualized by Gertler and Karadi (2011), with a Susceptible-Exposed-Infected-Recovered (SIR) epidemiological model. This fusion allows us to probe the efficacy of monetary policy interventions in the current crisis, culminating in a financial DSGE-SIR model.

This innovative model encompasses six distinct entities: households, financial intermediaries, non-financial goods producers, capital producers, retailers, and the government. It reflects the actions of a central bank that wields both conventional and unconventional monetary policy tools. Our study goes beyond the frameworks of Smets and Wouters (2007) and Gertler and Karadi (2011) in terms of methodology. It does this by combining the financial DSGE model with an SIR epidemiological model, rather than just assuming that the number of infected people affects the economy on both the supply and demand sides. On the demand side, we agree with Faria-e Castro's (2020) statement that the number of infected people shows how bad the disease is. This means that the marginal utility of household consumption goes down. Because people may be less willing to buy things in crowded places because they are more likely to get sick. On the supply side, the pandemic's economic ramifications could be reflected in households' labor supply decisions. We hypothesize that labor supply is contingent upon the number of healthy individuals, as determined by the SIR model, and that these dynamics result in consumption and output contractions. Furthermore, households that consume less may exhibit reduced work incentives, resulting in a curtailed labor supply. The government's role in providing unemployment benefits to those incapacitated by illness is also considered. Using the latest research from Moustabchir et al. (2023) and El Ouazzani et al. (2023), our study recognizes that "pandemic loans" from central banks and other non-traditional monetary policy measures might help lessen some of the bad effects of a pandemic crisis. Also, Moustabchir et al.'s (2024) study of the war between Russia and Ukraine and its effects on the Moroccan economy shows how important it is to take risk premium shocks into account when making economic policy, since they can cause inflation and real currency depreciation. These insights are instrumental in understanding the broader economic landscape within which our model operates, as well as the complex interplay of global events and policy responses.

We evaluate the effects of unconventional monetary policy through modeling this policy by quantitative easing, particularly in form of quantitative easing (QE) or loan policy. We model successive cuts in the policy rate as a Central Bank liquidity injection into the real sector in the form of claims that do not pass through private banks. This measure can be understood as a light form of "helicopter money" (Friedman, 1969), in the sense that the injected liquidity goes directly to the real sector without the direct involvement of fiscal authorities or private banks. According to our model's simulation results, a reduction in the policy rate can encourage household consumption, but its effect on output may be ambiguous and influenced by a variety of factors. However, if the policy is implemented for an extended period or excessively, it may result in increased inflation and excessive credit growth, which could have adverse impacts on investment and financial stability.

The next sections of the paper are organized as follows: Section 1 provides an empirical analysis of the literature that emphasizes the role of monetary policy in resolving the issues presented by the Covid-19 situation. In Section 2, we go into the basic ideas underpinning the SIR (Susceptible-Infectious-Recovered) model while also illustrating the possibility of integrating it with the financial DSGE (Dynamic Stochastic General Equilibrium) model. Additionally, Section 3 details the calibration approach applied to calculate parameters relevant to the Moroccan economy as well as provides the findings gained from numerical summations. Lastly, the study finishes with a final section summarizing the important results and implications.

1 LITERATURE REVIEW

The Covid-19 pandemic has presented unprecedented challenges to emerging market economies (EMEs), as highlighted by Carvalho et al. (2021). These authors have illuminated the difficulties faced by EMEs, including reduced capital inflows and a sharp decline in commodity prices. They observed that EMEs with more flexible exchange rate regimes and lower levels of external debt were able to respond more effectively to the crisis through a combination of monetary and fiscal policies. This study serves as a starting point for our analysis, establishing a framework for understanding the importance of macroeconomic stability and financial resilience.

Complementing this, Hasanov and Bulut (2021) analyzed the impact of the pandemic on macroeconomic variables in Turkey, concluding that the government's policy response helped stabilize the economy. Similarly, Escaith and MacGregor (2021) examined the repercussions of the pandemic on Latin American economies, revealing that countries with more flexible exchange rates and stronger policy frameworks fared better during the crisis. These studies reinforce the argument that effective monetary policy is crucial for supporting economic stability in EMEs during a pandemic.

Belke et al. (2021) explored the impact of Covid-19 on inflation dynamics and monetary policy in the euro area, finding a significant decline in inflation expectations and an increase in uncertainty about the inflation outlook. They advocate for the European Central Bank (ECB) to adopt a more flexible approach to inflation targeting, including a greater emphasis on forward guidance and more active use of unconventional policy tools such as asset purchases. This analysis is essential to our discussion as it underscores the importance of international policy coordination in facing the challenges posed by the pandemic.

The use of Dynamic Stochastic General Equilibrium (DSGE) models to analyze the impact of monetary policy during the Covid-19 pandemic has been highlighted by Bauer and Rudebusch (2021), who found that the Federal Reserve's policy response successfully stabilized inflation and output, although there might be long-term costs associated with the use of unconventional policy tools such as asset purchases. Galesi and Sgherri (2021) noted that the pandemic led to a significant decline in economic activity and inflation in the euro area and that the ECB's policy actions helped mitigate the negative effects of the crisis. Similarly, Lemoine and Lindé (2021) discovered that the pandemic led to a significant decline in economic activity and inflation in France and that the ECB's policy actions were effective in stabilizing financial markets and supporting the economy.

However, these authors also noted that the use of unconventional policy tools had an increased the risk of financial instability in the long run. Coibion et al. (2021) used a DSGE model to analyze the impact of the pandemic on the U.S. economy and found that the Federal Reserve's policy response had been effective in stabilizing inflation and output. They also noted that the use of unconventional policy tools had been necessary in a crisis of this magnitude, but that their long-term use had risks.

Our research connects two streams of literature by incorporating two popular models. Firstly, we build upon the widely used method of modeling epidemics, based on the seminal contribution of Kermack and McKendrick (1927) and its extension to include asymptomatic infected individuals (Prem et al. 2020). Secondly, we incorporate this modified SEIR model into a financial New Keynesian business cycle framework, similar to the one developed by Gertler and Karadi (2011). Our complete framework is most similar to the approach taken by Eichenbaum, Rebelo, and Trabandt (2020a), who demonstrate that a DSGE model with an SIR component can effectively capture macroeconomic processes during an epidemic.

This integration offers a unique perspective, allowing for a more precise analysis of the economic repercussions of the pandemic. It also emphasizes the need for a careful assessment of the short- and long-term costs and benefits of different policy actions. By linking these models, we provide a deeper understanding of the complex challenges faced by central banks and policymakers in responding to a crisis of such magnitude.

2 THE MODEL

2.1 Equations of SEIR model

The global health crisis of the coronavirus has led to the use of epidemiological mathematical models to make decisions on health and politics. The susceptible-infected-removed (SIR) model, which originated in the early 20th century, is a popular model in epidemiology for depicting infectious disease transmission. In this research, we use the susceptible-exposed-infected-removed-deceased (SEIR) model, an extension of the SIR model. As its name suggests, the model consists of five components:

- the number of susceptible individuals S: individuals who are healthy but can contract the disease;
- the number of exposed individuals E: persons who are infected but not yet infectious;
- The number of infected individuals I: who suffer from the disease and can spread it to susceptible individuals;
- the number of recovered individuals R: who have contracted the disease but have recovered and are immune to future infections;
- the number of deceased individuals D: who have contracted the disease but have died.

The S, E, I, and R compose the model's name (SEIR). For simplicity, we normalize the total population N to 1. Then S, E, I and R can be interpreted as shares or proportions of individuals of each class in the general population.

Furthermore, we focus only on transmitting the disease in a location or a closed economy. The standard SEIRD model assumes that no births occur and no people enter or exit the location. Hence, the population N is constant over time. For any time t, we have: $S_t + E_t + I_t + R_t + D_t = 1$ Susceptible individuals might get infected in three ways: buying consumer products, working, and having random interactions unrelated to economic activity. The transmission function reveals the frequency of newly infected individuals:

$$T_t = \pi_1(S_tC_t^s)(I_tC_t^i) + \pi_2(S_tL_t^s)(I_tL_t^i) + \pi_3S_tI_t\,, \tag{1}$$

Let N be the total population, and let S, E, I, R, and D be the number of susceptible, exposed, infected, recovered, and deceased individuals, respectively. The rates of change of these variables over time can be described by the following system of differential equations:

$$\frac{dS}{dt} = -\beta SI\,, \tag{2}$$

$$\frac{dE}{dt} = \beta SI - \sigma E \,, \tag{3}$$

$$\frac{dI}{dt} = \sigma E - (\gamma + \mu)I \,, \tag{4}$$

$$\frac{dR}{dt} = \gamma I \,, \tag{5}$$

$$\frac{dD}{dt} = \mu I \,, \tag{6}$$

where β is the effective contact rate, σ is the rate at which exposed individuals become infectious, γ is the recovery rate, and μ is the mortality rate. This system of equations describes how the number of individuals in each compartment changes over time as the disease spreads through the population.

2.2 Households

To link the DSGE model with the SIR modeling of the Covid-19, we assume that the disease can affect the economy through both the demand and supply sides Faria-e-Castro (2020). Assume that in the economy, there are infinitely many identical households. The representative household has the following expected lifetime utility U:

$$U = E_t \sum_{t=0}^{\infty} \beta_t \left\{ Health\left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\varphi}}{1+\varphi} \right) \right\}, \tag{7}$$

where β is a discount factor. C_t and L_t are consumption and labor supply at time t, respectively. The key difference between our model and the literature lies in the introduction of term Health is related variable that affects consumption and labor supply. With the health variable including the variables of SIR model.

$$Health_t = \{S_t, I_t, R_t\}.$$

Therefore, the utility function become:

$$U = E_t \sum_{t=0}^{\infty} \beta_t \left\{ s_t\left(\frac{C_{t,s}^{1-\sigma}}{1-\sigma} - \frac{L_{t,s}^{1+\varphi}}{1+\varphi} \right) + e_t\left(\frac{C_{t,e}^{1-\sigma}}{1-\sigma} - \frac{L_{t,e}^{1+\varphi}}{1+\varphi} \right) + i_t\left(\frac{C_{t,i}^{1-\sigma}}{1-\sigma} - \frac{L_{t,i}^{1+\varphi}}{1+\varphi} \right) + r_t\left(\frac{C_{t,r}^{1-\sigma}}{1-\sigma} - \frac{L_{t,r}^{1+\varphi}}{1+\varphi} \right) \right\}. \tag{8}$$

The representative household is subject to a budget constraint that correlates its expenditures with its resources:

$$P_t C_{H,t} + B_{t+1} \leq b_t i_t L_{t,i} + W_t(s_t L_{t,s} + r_t L_{t,r}) + R_t B_t + T_t + P_t D_t \,. \tag{9}$$

The variables $C_{H,t}$ and $L_{t,s}$, $L_{t,i}$, $L_{t,r}$ means the consumption and hours worked of susceptible, infected and recovered hoseholds, respectively.

The variables B_t, D_t, W_t, b_t and P_t mean government bond held by the household, dividends earned by the household, wages paid by firms, unemployment compensation received by infected peoples and the aggregate price level respectively.

The law of motion for the stock of capital is:

$$K_{t+1} = Inv_t + (1 - \delta)K_t \,, \tag{10}$$

The first-order conditions for $C_{t,s}$, $C_{t,e}$, $C_{t,i}$, et $C_{t,r}$ are:

$$C_{t,s}^{-\sigma} = \lambda_t^b P_t - \lambda_t^\tau \pi_1(I_t C_t^I) \,, \tag{11}$$

$$C_{t,e}^{-\sigma} = \lambda_t^b P_t \,, \tag{12}$$

$$C_{t,i}^{-\sigma} = \lambda_t^b P_t \,, \tag{13}$$

$$C_{t,r}^{-\sigma} = \lambda_t^b P_t \,, \tag{14}$$

here, λ_t^b is the Lagrange multiplier on the household budget constraint. The first-order conditions for $L_{t,s}$, $L_{t,i}$ and $L_{t,r}$ are:

$$L_{t,s}^{\varphi} = \lambda_t^b W_t + \lambda_t^\tau \pi_2(I_t N_t^i) \,, \tag{15}$$

$$L_{t,e}^{\varphi} = \lambda_t^b b_t \,, \tag{16}$$

$$L_{t,i}^{\varphi} = \lambda_t^b b_t \,, \tag{17}$$

$$L_{t,r}^{\varphi} = \lambda_t^b W_t \,.$$

The first-order condition for K_{t+1} is:

$$\lambda_t^b P_t = \beta \lambda_{t+1}^b [R_{t+1}^k + P_{t+1}(1 - \delta)] \,, \tag{18}$$

2.3 Firms and Calvo pricing

The model distinguishes between two types of firms: Intermediate Non-Financial Firms and Retailers. The first type of firm produces a range of differentiated goods, operating in an imperfectly competitive market. The second type of firm produces final goods and is characterized by a representative firm that aggregates the production of a range of intermediate firms j ∈ [0,1]. The aggregation function, based on the Dixit-Stiglitz model, is defined as:

$$Y_t = \left(\int_0^1 Y_{i,t}^{\frac{1}{\theta}} di \right)^{\theta}, \theta > 1 \,, \tag{19}$$

where θ is the elasticity of substitution between differentiated goods. This representative firm maximizes its profit according to the following equation:

$$Prof_t = P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di = P_t \left(\int_0^1 Y_{i,t}^{\frac{1}{\theta}} di \right)^{\theta} - \int_0^1 P_{i,t} Y_{i,t} di \,. \tag{20}$$

Profit maximization implies the following demand schedule for intermediate products:

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\frac{\theta}{\theta-1}} Y_t , \tag{21}$$

here, $P_{i,t}$ denotes the price of intermediate input i in units of the final good. The price of output is given by:

$$P_t = \left(\int_0^1 P_{i,t}^{-\frac{1}{\theta-1}} di\right)^{-(\theta-1)} . \tag{22}$$

For *Intermediate Non-Financial Firms:* The production function of intermediate good firm i is a Cobb-Douglas function given by:

$$Y_{i,t} = AK_{i,t}^{\eta} L_{i,t}^{1-\eta} , \tag{23}$$

where $L_{i,t}$ and $K_{i,t}$ are the labor and capital employed by firm i, respectively. A is the total factor productivity (TFP) level of the economy, which is shared by all intermediate good firms and is assumed to be constant. Intermediate good firms maximize profits:

$$\pi_{i,t} = P_{i,t} Y_{i,t} - mc_t Y_{i,t} . \tag{24}$$

The firm's maximization problem gives the following first-order conditions for $K_{i,t}$ and $L_{i,t}$:

$$\eta \frac{Y_t(i)}{K_t(i)} mc_t = r_{K,t} , \tag{25}$$

$$(1 - \eta) \frac{Y_t(i)}{L_t(i)} mc_t = w_t . \tag{26}$$

Calvo pricing: Nominal price rigidity is imposed using Calvo (1983) style price-setting frictions. With probability $(1 - \theta)$, the firm reoptimizes $\tilde{P}_t$. With probability θ, $P_{i,t} = P_{i,t+1}$. The firm chooses its optimal price at time t, $\tilde{P}_t$, to maximize:

$$\max_{P_t} \sum_{j=0}^{\infty} (\theta\beta)^j \lambda_{t+j}^b (\tilde{P}_t Y_{i,t+j} - P_{t+j} mc_{t+j} Y_{i,t+j}) , \tag{27}$$

subject to the demand function.

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\frac{\gamma}{\gamma-1}} Y_t , \tag{28}$$

here, mc_t denotes the real marginal cost at time t:

$$mc_t = \frac{W_t^{\alpha} (R_t^k)^{1-\alpha}}{P_t A \alpha^{\alpha} (1 - \alpha)^{1-\alpha}} . \tag{29}$$

The optimal price is given by:

$$\check{p}_t = \left[(1-\theta)\left(\frac{1-\theta\pi_t^{\frac{1}{\gamma-1}}}{1-\theta}\right)^{\gamma} + \theta\,\frac{\frac{\gamma}{\gamma-1}}{\check{p}_t - 1}\right]^{-1} . \tag{30}$$

2.4 Government and monetary policy

For the government sector, we assume that government consumption of final goods is constant and that transfers are automatically adjusted at each date. The government's expenditures include consumption of final goods, payments to non-financial intermediaries, and unemployment benefits, while its revenue includes lump-sum taxes and interest on debt. The government faces the following budget constraint:

$$G_t + \tau\phi_t Q_t K_{t+1} + b_t(1 - L_t) + \phi_t Q_t Z_t = T_t + (R_{k,t} - R_t)\, B_{g,t} + B_{g,t+1} . \tag{31}$$

Unconventional monetary policy:

$$\phi_t = \bar{\phi}_t + \omega E_t[(logR_{k,t+1} - logR_{t+1}) - (logR_k - logR)] . \tag{32}$$

where $\bar{\phi}_t$ represents pandemic loans modeled as quantitative easing, but not exactly helicopter money as proposed by Friedman (1969), $\omega > 0$ is the Central Bank credit feedback parameter, and $(logR_k - logR)$ is the steady-state risk premium.

The Central Bank also conducts conventional monetary policy based on a Taylor rule, which relates the nominal interest rate R_t^* to its past value R_{t-1}^*, to the nominal inflation π_t^*, and to the output gap $(Y_t^* - Y_{t-1}^*)$:

$$R_t^* = \alpha R_{t-1}^* + (1-\alpha)\,\alpha_\pi \pi_t^* + (1-\alpha)\,\alpha_y(Y_t^* - Y_{t-1}^*) , \tag{33}$$

where α is the degree of interest rate smoothing, α_π is the weight on inflation, and α_y is the weight on the output gap.

We also have the Fisher relation that links nominal interest rates set by the Central Bank to the gross real interest rate set by the market:

$$1 + i_t = R_{t+1}\mathbb{E}_t\Pi_{t+1} . \tag{34}$$

Finally, market clearing conditions establish that production is divided between consumption, net investment, government expenditures on goods, and government financial intervention:

$$Y_t = C_t + I_{n,t} + f\left(\frac{I_{n,t}}{I_{n,t-1}}\right) I_{n,t} + G + \tau\psi_t Q_t K_{t+1} . \tag{35}$$

3 CALIBRATION AND MONETARY POLICY TO REDUCE THE EFFECTS OF COVID-19

3.1 Calibration

In this section, we outline the approach used to calibrate parameters for Morocco. Similar to Eichenbaum et al. (2020a, 2020b), we assume that each period in the model equates to one week, and it takes an average of 14 days for a patient to recover or die. As our model is weekly, we set $\pi_r + \pi_d = 7 \div 14$.

The computation of mortality rates is based on direct standardization, which eliminates disparities caused by factors affecting the mortality rate. This method is commonly used in demography for global comparisons when investigating demographic phenomena such as mortality and fertility. The aim is to assimilate the structure of the researched population to that of a reference group for which accurate information on Covid-19 is available (South Korea conducted the most tests for Covid-19 at the start of the pandemic); Borelli and Góes, 2020; Eichenbaum et al., 2020a, 2020b). We calculate a death rate in Morocco of 0.017%, which implies $\pi_d = 7 \times 0{,}017\backslash 14$.

As in Eichenbaum et al. (2020a, 2020b), we set $\mu_1 = 3.1949 \times 10^{-7}$, $\mu_2 = 1.5936 \times 10^{-4}$ and $\mu_3 = 0{,}4997$. These values must also satisfy the following system.

Let $D = \mu_1 C^2 + \mu_2 L^2 + \mu_3$ then we have:

$$\begin{aligned} \gamma_1 &= \frac{\mu_1 C^2}{D}, \\ \gamma_2 &= \frac{\mu_2 L^2}{D}, \\ \gamma_3 &= \frac{\mu_3}{D}, \end{aligned} \tag{36}$$

where C and L represent consumption and labor supply, measured by the number of hours worked. Note that the values γ_1, γ_2 and γ_3 represent the percentage of transmission that occurs in the market, at work, and in other activities, respectively. In the literature, these proportions are approximated to be 1/6, 1/6, and 2/3, respectively (Borelli and Góes, 2020; Eichenbaum et al., 2020a, 2020b).

The macroeconomic parameters in our model are in line with the literature on dynamic stochastic general equilibrium models and have been determined for the Moroccan case. We set the discount factor to its standard value of 0.98, a common choice in quarterly models. The utility function assigns a weight of 35 percent to leisure time, while the elasticity of substitution between intermediate inputs is calibrated to achieve a product markup of 25 percent. The degree of price stickiness is determined by expressing the standard quarterly Calvo probability of 0.70 in weekly units. The monetary policy reaction function includes interest rate feedback parameters for inflation and the output gap, which are set to 1.4 and 0.6 (converted to weekly), respectively, as per the standard Taylor rule. The elasticity of capital

Table 1 Parameter values

Parameter	Value	Description
π_d	0.001	Probability of death
π_r	0.499	Probability of recovery
ε_0	0.001	Initial infection
β	0.99	Discount factor
δ	0.025	Capital depreciation
α	0.3	Marginal productivity of labor
A	1	PTF SS
π_1	3.1949×10^{7}	Probability of a susceptible person becoming infected by consumption
π_2	1.5936×10^{-4}	Probability of a susceptible person becoming infected by hours worked
π_3	0,4997	Probability of a susceptible person becoming infected by other activities
π_r	1.5	Inflation rate Taylor Rule
π_x	0.5/52	Output gap

Source: Authors

β is calibrated assuming a labor share of approximately 3/4, while the bankers' survival rate is fixed at 0.970, representing an average tenure of 9 years. The share of unemployment compensation ξ is set at 0.6. The parameters for private banks, μ and , are determined to achieve a steady-state risk premium of 120 basis points and a leverage ratio of 5, consistent with Gertler and Karadi (2011). Table 1 provide a summary of the calibrated model parameter values. The multiple processes of solving and simulating the model were implemented using Matlab and Dynare software version 5.1 Documented in Adjemian et al. (2022).

3.2 Macroeconomic effects of infection shocks

Before registering of the first case of Covid-19 in Morocco in March 2020, the country's economy was growing steadily. In 2019, the Gross Domestic Product (GDP) increased by 2.3%, and the unemployment rate dropped to 9.2%. The government also launched several initiatives to attract foreign investment and promote economic growth, such as the Industrial Acceleration Plan and the Green Morocco Plan. However, the Covid-19 pandemic had a significant impact on the Moroccan economy. The government had to implement strict measures to control the spread of the virus, such as imposing a nationwide lockdown and closing borders, which led to a sharp contraction of economic activity. The International Monetary Fund (IMF) estimated that Morocco's GDP contracted by 6.3\% in 2020, while the unemployment rate increased to 12.5\%.

Similar to the methodology used by Eichenbaum et al. (2020), to consider the effect of the pandemic on labor supply and consumer demand simultaneously, both parameters π_1 and π_2 must be positive.

In this part, the transmission mechanism for a rise in infected persons in the economy is described. The dynamic responses of endogenous variables to the pandemic illness are shown in Figure 2 in the appendix and summarized in Table 2. The major consequence of the shock is a rise in the proportion of infected I_t, which is progressively mirrored a week later by an increase in the proportion of recovered R_t. A greater proportion of diseased persons translates to fewer susceptibles that are exposed E_t and quarantined Q_t. The figure shows that the model captures the main features of the recession. Output, consumption, investment, and hours worked fell sharply by 8%, 9%, 11%, and 12%, respectively. The decline in consumption reflects the likely decline in consumer demand. The large decline in investment reflects the size of the labor supply shock. As the data shows, the pandemic recession is accompanied by slight deflation. Moreover, we observe that households significantly reduce their consumption and work hours to reduce the probability of being infected.

The decline in output is a direct result of the increase in the proportion of infected individuals. Note that the reduction in production is caused by both supply and demand factors. On the one hand,

Table 2 Summary of effects of infection shocks

Economic and epidemiological indicators	Before the pandemic	During the pandemic
GDP growth	+2.3\%	–6.3\%
Unemployment rate	9.2\%	12.5\%
Economic production	-	–8.0\%
Consumption	-	–9.0\%
Investment	-	–11.0\%
Hours worked	-	–12.0\%
Inflation rate	1.2\%	–0.5\%
Proportion of infected individuals	-	High
Mortality rate	-	High

Source: Authors

the decrease in the marginal utility of consumption reduces the demand for output. On the other hand, the supply of output decreases due to the fall in TFP. Finally, Figure 1 also illustrates the impact of a pandemic in a model where prices are both flexible and sticky. We can observe that the recession is slightly more severe when prices are rigid. The main difference between the two models is related to inflation. The flexible price model predicts a greater decline in prices than the sticky price model.

Figure 1 Epidemic as a shock to consumption demand and labor supply

Note: See the online version of *Statistika: Statistics and Economy Journal* No. 3/2024: <*https://doi.org/10.54694/stat.2024.8*>.
Source: Authors

3.3 Monetary policy shock during a pandemic

In this part, we evaluate the use of Unconventional monetary policy by Bank al Maghrib to mitigate the effects of the Covid-19 pandemic and how this policy can create today's inflation. As well as the Vásconez et al. (2021) we suppose of Unconventional monetary policy by Bank al Maghrib. Our definition of this policy is an extreme version of Quantitaive easing policy, but not quite "helicopter money" as described by Friedman (1969). Instead of distributing money directly to people with no prospect of being returned, the Central Bank raises its percentage of total claims issued, and firms subsequently acquire capital without having to transit through private banks. Thus our unconventional policy directly influence demand by motivating investment, and should be conceived of as growing Central Bank intermediation rather than expanding the money supply.

Figure 2 IRFs to unconventional monetary policy shocks

Note: See the online version of *Statistika: Statistics and Economy Journal* No. 3/2024: <*https://doi.org/10.54694/stat.2024.8*>.
Source: Authors

Lowering the policy rate is a commonly used monetary policy tool to stimulate economic growth. The theoretical basis for this policy is that by lowering the interest rate, households and firms are encouraged to borrow more money and spend it on consumption and investment, respectively. This increased spending leads to a boost in aggregate demand, which can help increase output and employment. Empirical studies have shown that lowering the policy rate can indeed stimulate household consumption. For example, research has found that when interest rates are lowered, households are more likely to take out mortgages and consumer loans, which can lead to increased spending on housing and durable goods. However, the impact of lowering the policy rate on output is not significant. While some studies have found a positive relationship between interest rates and output, others have found no significant impact. One reason for this is that the transmission mechanism from interest rates to output is complex and may be influenced by factors such as credit availability, exchange rates, and government policies.

It is possible that the non-significant impact on output in this scenario could be due to other factors that are dampening the effectiveness of the policy. For example, if households are highly indebted or if there is a lack of investment opportunities, then even with lower interest rates, they may not increase spending as much as expected. Additionally, if there are supply-side constraints, such as a shortage of skilled workers or limited access to raw materials, then even if demand increases, output may not be able to keep up without compromising the effects of Covid-19. Overall, while lowering the policy rate can encourage households to consume more, its impact on output may be less clear and influenced by various other factors. Lowering the policy rate can also lead to an increase in inflation if it stimulates too much demand in the economy. As households and firms borrow more and spend more, the increased demand can push up prices, especially if there is limited capacity to produce more goods and services.

Inflation erodes the purchasing power of money, which can lead to a decline in real investment and a decrease in the net wealth of banks. Additionally, higher inflation can lead to a rise in interest rates, which can further reduce investment and hurt the banking sector.

The sharp increase in credit offered by banks in response to lower interest rates is a common phenomenon, as it becomes more profitable for banks to lend money when the interest rates are low. However, excessive lending can lead to an increase in credit risk and potentially to financial instability if borrowers are unable to repay their loans. It is important for monetary authorities to monitor inflation and credit growth closely and adjust interest rates accordingly to maintain stability in the economy. If inflation becomes too high, central banks may need to raise interest rates to cool down the economy and prevent inflation from spiraling out of control. Similarly, if credit growth becomes excessive, regulatory authorities may need to impose limits on bank lending to prevent a buildup of financial risks.

Overall, while lowering the policy rate can stimulate economic activity during Covid-19, it can also lead to higher inflation and excessive credit growth, which can have negative effects on investment and financial stability. Monetary authorities need to carefully balance the benefits and risks of this policy tool and use it judiciously to achieve their policy objectives. Our results are in line with those proposed by Sharma et al. (2020), Céspedes et al. (2020), and Kiley (2020).

CONCLUSION

The Covid-19 pandemic has had a significant impact on the global economy, including the Moroccan economy. The pandemic has led to a sharp decline in economic activity, disruptions in supply chains, and a reduction in international trade. In response to the economic challenges posed by the pandemic, Bank Al-Maghrib, the central bank of Morocco, has taken a number of measures to support the economy and ensure financial stability. In March 2020, Bank Al-Maghrib lowered its policy rate by 25 basis points to 2%. This decision was aimed at supporting economic activity and ensuring the availability of credit to households and businesses. Additionally, the central bank implemented a number of liquidity support measures, such as reducing reserve requirements and providing loans to banks at a lower interest rate.

Studies of other nations have shown that these measures have had a positive impact on household consumption in Morocco. For example, a study by the African Development Bank found that the reduction in interest rates led to an increase in bank lending to households and businesses. This, in turn, contributed to an increase in consumption and investment, which helped to support economic growth.

However, the impact of these measures on output is less clear. While some studies have found a positive relationship between interest rates and output, others have found no significant impact. One reason for this is that the transmission mechanism from interest rates to output is complex and may be influenced by various factors, such as the availability of credit and supply-side constraints. Overall, Bank Al-Maghrib's policy response to the Covid-19 pandemic has been aimed at supporting economic activity and ensuring financial stability. While the measures taken have had a positive impact on household consumption, their impact on output is less clear and may be influenced by various factors. In addition, some studies have also highlighted the potential risks of these measures. For example, the reduction in interest rates could lead to higher inflation and a decrease in the net wealth of banks, which could affect their ability to provide credit in the future. Therefore, it is important for Bank Al-Maghrib to carefully balance the potential benefits and risks of its monetary policy decisions in response to the Covid-19 pandemic.

In light of these challenges, our study aims to provide a deeper understanding of the potential impact of unconventional monetary policy on the Moroccan economy in the context of the Covid-19 pandemic. By using a DSGE model that incorporates both epidemiological and economic dynamics, furthermore, our study contributes to the broader theoretical and empirical literature on the use of unconventional monetary policy in the context of the Covid-19 pandemic. By building on previous research in this area, we aim to further our understanding of the mechanisms through which these policies can impact economic outcomes and the conditions under which they are most effective. Ultimately, our study may supply information for policy debates around the use of unconventional monetary policy in the context of future economic shocks, both in Morocco and other countries around the world.

References

ABHOFF, S., BELKE, A., OSOWSKI, T. (2021). Unconventional monetary policy and inflation expectations in the euro area [online]. *Economic Modelling*, 102: 105564. <https://doi.org/10.1016/j.econmod.2021.105564>.

BDFR, S., LEMOINE, M., LINDÉ, J. (2021). *Fiscal Stimulus in Liquidity Traps: Conventional or Unconventional Policies?* [online]. <https://doi.org/10.1016/j.euroecorev.2022.104324>.

BULÍŘ, A., VLČEK, J. (2021). Monetary transmission: Are emerging market and low-income countries different? [online]. *Journal of Policy Modeling*, 43(1): 95–108. <https://doi.org/10.1016/j.jpolmod.2020.06.006>.

CANDILA, V., MAXIMOV, D., MIKHAYLOV, A., MOISEEV, N., SENJYU, T., TRYNDINA, N. (2021). On the relationship between oil and exchange rates of oil-exporting and oil-importing countries: From the great recession period to the Covid-19 era [online]. *Energies*, 14(23): 8046 <https://doi.org/10.3390/en14238046>.

CARVALHO, E. D., SILVA, R. R., ARAÚJO, F. H., DE AL RABELO, R., DE CARVALHO FILHO, A. O. (2021). An approach to the classification of Covid-19 based on CT scans using convolutional features and genetic algorithms [online]. *Computers in Biology and Medicine*, 136: 104744. <https://doi.org/10.1016/j.compbiomed.2021.104744>.

CÉSPEDES, L. F., CHANG, R., VELASCO, A. (2020). Macroeconomic policy responses to a pandemic. *Covid-19 in Developing Economies*, 1: 175–186.

CLARK, A., JIT, M., WARREN-GASH, C., GUTHRIE, B., WANG, H. H., MERCER, S. W., JARVIS, C. I. et al. (2020). Global, regional, and national estimates of the population at increased risk of severe Covid-19 due to underlying health conditions in 2020: a modelling study [online]. *The Lancet Global Health*, 8(8): e1003-e1017. <https://doi.org/10.1016/S2214-109X(20)30264-3>.

COIBION, O., GEORGARAKOS, D., GORODNICHENKO, Y., KENNY, G., WEBER, M. (2024). The effect of macroeconomic uncertainty on household spending [online]. *American Economic Review*, 114(3): 645–677. <https://doi.org/10.1257/aer.20221167>.

COLABELLA, A. (2021). Do ECB's Monetary Policies Benefit EMEs? A GVAR Analysis on the Global Financial and Sovereign Debt Crises and Postcrises Period [online]. *Oxford Bulletin of Economics and Statistics*, 83(2): 472–494. <https://doi.org/10.1111/obes.12403>.

EICHENBAUM, M. S., REBELO, S., TRABANDT, M. (2022). Epidemics in the New Keynesian model [online]. *Journal of Economic Dynamics and Control*, 140: 104334. <https://doi.org/10.1016/j.jedc.2022.104334>.

EICHENBAUM, M., DE MATOS, M. G., LIMA, F., REBELO, S., TRABANDT, M. (2020). *Expectations, Infections, and Economic Activity* [online]. <https://doi.org/10.3386/w27988>.

EL OUAZZANI, H., OUAKIL, H., MOUSTABCHIR, A. (2023). A simulation of the macroeconomic effects of the Russia–Ukraine War on the Moroccan economy using the DSGE model [online]. *African Development Review.* <https://doi.org/10.1111/1467-8268.12726>.

ESCAITH, H., KHORANA, S. (2021). *The impact of the Covid-19 pandemic on merchandise trade in Commonwealth countries.* Commonwealth.

FARIA-E-CASTRO, M. (2021). Fiscal policy during a pandemic [online]. *Journal of Economic Dynamics and Control*, 125: 104088. <https://doi.org/10.1016/j.jedc.2021.104088>.

FRIEDMAN, M. (1970). A theoretical framework for monetary analysis. *Journal of Political Economy*, 78(2): 193–238.

GERTLER, M., KARADI, P. (2011). A model of unconventional monetary policy [online]. *Journal of Monetary Economics*, 58(1): 17–34. <https://doi.org/10.1016/j.jmoneco.2010.10.004>.

HORAN, R. D., FENICHEL, E. P. (2007). Economics and ecology of managing emerging infectious animal diseases [online]. *American Journal of Agricultural Economics*, 89(5): 1232–1238. <https://www.jstor.org/stable/30139466>.

HORAN, R. D., WOLF, C. A. (2005). The economics of managing infectious wildlife disease [online]. *American Journal of Agricultural Economics*, 87(3): 537–551. <https://doi.org/10.1111/j.1467-8276.2005.00746.x>.

KERMACK, W. O., MCKENDRICK, A. G. (1927). A contribution to the mathematical theory of epidemics [online]. *Proceedings of the Royal Society of London*, Series A, 115(772): 700–721.< https://doi.org/10.1098/rspa.1927.0118>.

KILEY, M. T. (2020). *Pandemic recession dynamics: The role of monetary policy in shifting a U-shaped recession to a V-shaped rebound* [online]. Finance and Economics Discussion Series, Washington. <https://doi.org/10.17016/FEDS.2020.083>.

LENHART, S., WORKMAN, J. T. (2007). *Optimal control applied to biological models* [online]. Chapman and Hall. <https://doi.org/10.1201/9781420011418>.

MOUSTABCHIR, A., OUAKIL, H. (2023). Impacts économiques de la Covid-19 sur l'économie marocaine: simulations à travers un modèle hybride Sais-infectés-rétablis et le modèle économique keynésien (SIR-NKM) [online]. *Repères et Perspectives Economiques*, 7(1). <https://doi.org/10.34874/IMIST.PRSM/RPE/39432>.

MOUSTABCHIR, A., OUAKIL, H. (2023). Vaccination, treatment and containment policy to reduce the effects of the Covid-19 pandemic in Morocco [online]. *African Development Review*, 35(4): 351–364. <https://doi.org/10.1111/1467-8268.12714>.

MOUSTABCHIR, A., OUAKIL, H., EL OUAZZANI, H. (2023). Monetary policy through the pandemic loans instrument to address the effects of Covid-19: hybrid financial DSGE-SIR [online]. *Repères et Perspectives Economiques*, 7(2). <https://doi.org/10.34874/IMIST.PRSM/RPE/43218>.

MOUSTABCHIR, A., OUAKIL, H., EL OUAZZANI, H., LECHHEB, H. (2024). A DSGE model approach to examining the impact of the Russian–Ukrainian War and oil shocks on the Moroccan economy [online]. *African Development Review.* <https://doi.org/10.1111/1467-8268.12741>.

SHARMA, S., FRATTO, C., VANNIER, B. H., MIRCHEVA, M., DE PADUA, D., WARD, M. H. P. (2021). *Unconventional monetary policies in emerging markets and frontier countries.* International Monetary Fund.

SMETS, F., WOUTERS, R. (2007). Shocks and frictions in US business cycles: A Bayesian DSGE approach [online]. *American Economic Review*, 97(3): 586–606. <https://doi.org/10.1257/aer.97.3.586>.

VÁSCONEZ, V. A., DAMETTE, O., SHANAFELT, D. W. (2021). Macroeepidemics and unconventional monetary policy: coupling macroeconomics and epidemiology in a financial dsge-sir framework 1. *Covid Economics*, 199.

Modelling Risk Dependencies in Insurance Using Survival Clayton Copula

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Abstract

Our aim in this paper is to show the use of survival Clayton copula as a suitable tool for modelling risk dependencies in insurance. A purpose-built simulation of an adequate upper tail dependence can be an important part of the aggregation of risks in an insurer's internal models. The occurrence of extreme values of the aggregate random variable might have a very negative impact on the insurer when securing coverage of unexpected losses. The upper conditional quantile exceedance probability of the copula is a suitable indicator. In addition an analysis of its effect on the level of modelling of the risk scenario is available. This effect is measured using the Tail Value at Risk of the aggregate random variable. To simplify our description of the given principle for aggregating risks we will in this paper only consider the two-dimensional case. The programming language R was used to simulate the values of the joint distribution of the marginal random variables.

Keywords	***DOI***	***JEL code***
Dependence modelling, survival Clayton copula, conditional quantile exceedance probability, joint distribution, Tail Value at Risk, risk aggregation	*https://doi.org/10.54694/stat.2024.15*	*C63, G22*

INTRODUCTION

The aggregation of risk is at the present time very topical in the insurance sector and much time is devoted to it in the context of risk management in insurers' internal models. The preferred solution for risk aggregation is the use of multi-dimensional copula functions. Many authors have covered this area. In the context of risk aggregation they describe the use of various copula functions and some have carried out comparisons with the approach based on the standard formula in the Solvency II directive

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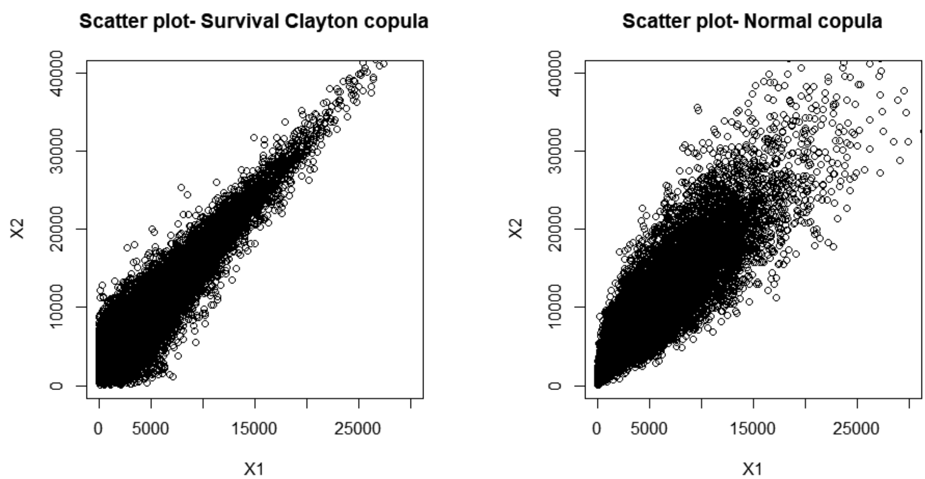
(Nguyen and Molinari, 2011; Eling and Jung, 2020; Marri and Moutanabbir, 2022; Ghosh, Chakraborty and Watts, 2022; Pfeifer, Strassburger and Philipps, 2020; Allen, Mcaleer and Singh, 2017). We should emphasise that insurers will develop their own copula functions to manage risk the most effectively (Milek, 2020). The classical approach attempts to choose a suitable copula function, reflecting the risk dependencies, using scatter plots of their values. Copulas can be constructed either non-parametrically or parametrically using the maximum likelihood method, good fit tests, or information criteria (Remillard, Genest and Beaudoin, 2009; Chen and Huang, 2007; Joo, Shin and Heo, 2020; Yang, 2022; Cuevas, Yela and Achcar, 2019). In many cases analyses of various dependence coefficients or association measures are used to construct suitable copulas (Adès, Provost and Zang, 2024; Gijbels, Veraverbeke and Omelka, 2011; Nicolas and Garcin, 2021).

Rank correlation measures have a more important application, as for example in Spearman´s Rho, Kendall´s tau (Embrechts, Lindskog and McNeil, 2001). These measures can be used to characterize the trend of changes in the values of the marginal distributions in the analyzed ordered pairs. These association measures may not however capture tail dependencies which in the case of insurance risks are very important – see Figure 1.

Figure 1 Scatter plots of the values of a joint distribution with Kendall tau values 0.7, generated using the chosen copula functions



Source: Own construction, customized in R

Many authors therefore prefer various estimation approaches for determining the upper-tail dependency coefficients for a given copula function (Hua and Joe, 2011; Gijbels, Kika and Omelka, 2020; Charpentier and Segers, 2009; De Luca and Rivieccio, 2012).

An important indicator of the tail-dependency of the copula, or the joint distribution, is the upper conditional quantile exceedance probability $cqep_U(u)$ which we will go into in more detail later (Milek, 2020; Mucha and Škrovánková, 2022). It relates to the answer to the question: What is the probability that the value of the second marginal distribution exceeds the 95% quantile given that the value of the first marginal distribution has already exceeded that quantile (Cuypers, 2020)? In the context of modelling risk dependencies with the help of copulas the value $cqep_U(u)$ of the copula is "handed over" to the generated joint distribution. This distribution has this indicator at the same level as the copula used. We could therefore say that it "hands over" certain risk information which is encoded in that copula function.

An innovative approach in this area could indeed be to use the copula function as a tool to generate a given risk level scenario for the tail dependency, even though it is not evident in the scatter plot of the data. Based on this approach a given capital requirement represents a guarantee of covering unexpected losses even in the case of the occurrence of extreme values (Pinda, Mucha and Smažáková, 2022). The mentioned conditional quantile exceedance probability could be the parameter of the risk scenario. The survival Clayton copula is the proposed tool for the modelling (Mucha, 2023; Hofert, Kojadinovic, Maechler and Yan, 2018). It is suitable for describing the upper tail dependency between the risks. It is also referred to as the HRT (heavy right tail) copula (Di Bernardino and Prieur, 2018). Depending on the parameter of the survival Clayton copula it is therefore possible to model various levels of the mentioned conditional quantile exceedance probability of the generated joint distribution. As the parameter $\theta \to 0$ the survival Clayton copula tends towards the position of the independent copula and as $\theta \to \infty$ it tends towards the position of the comonotonic copula. In this paper we will concentrate on analysing the calculation of the Tail Value at Risk of the aggregate random variable depending on the value of the parameter of the survival Clayton copula for selected marginal risk distributions. Our aim is to show the use of survival Clayton copula as a suitable tool for modelling risk dependencies in insurance.

1 METHODS AND METHODOLOGY OF ANALYSIS

A copula is a mathematical object which describes the dependency structure between risks and it is the base for determining the joint distribution of their marginal distributions.

1.1 Copula functions

A two-dimensional copula is the joint distribution function of two equally distributed random variables $U_1 \sim U(0; 1)$ and $U_2 \sim U(0; 1)$. We can express the copula $C(u_1; u_2)$ as follows:

$$C(u_1; u_2) = P(U_1 \leq u_1; U_2 \leq u_2). \tag{1}$$

Sklar's theorem provides the theoretical foundation for copula function theory, which makes clear the role of copulas in determining two-dimensional distributions in the context of their dependency structure. Let C be a two-dimensional copula and F_1, F_2 one-dimensional distribution functions. The function $F(x_1; x_2)$ defined by form:

$$F(x_1; x_2) = C(F(x_1); F(x_2)), \tag{2}$$

is the joint distribution function with marginal distribution functions F_1, F_2.

Given that we will be considering the Clayton copula, we will introduce a definition of Archimedean copulas. The key concept is a copula generator. Mathematically speaking a copula generator is a continuous decreasing function $\phi(\cdot) : \langle 0; 1\rangle \to \langle 0; \infty\rangle$, such that $\phi(1) = 0$ (if $\phi(0) = \infty$, called a strict copula generator). The notation $\phi^{-1}(\cdot) : \langle 0; \infty\rangle \to \langle 0; 1\rangle$, represents the inverse function to ϕ (if it is not a strict generator we need to use the pseudo-inverse function which takes the value zero everywhere in the interval $\langle \phi(0); \infty\rangle$). Then

$${}_AC(u_1; u_2) = \phi^{-1}(\phi(u_1) + \phi(u_2)), \tag{3}$$

is a two-dimensional Archimedean copula (Cipra, 2015; McNeil, Frey and Embrechts, 2015). The generator for Clayton copula takes the form ${}^{CLC}\phi(u) = \frac{1}{\theta} \cdot (u^{-\theta} - 1)$, with parameter $\theta > 0$. In the two-dimensional case we can write this in the form:

$$^{CLC}_{\theta}C(u_1; u_2) = (u_1^{-\theta} + u_2^{-\theta} - 1)^{-\frac{1}{\theta}} . \tag{4}$$

1.2 The survival copula

Let C be a two-dimensional copula for which we have $\mathbf{U} \sim C$. Then $\mathbf{V} = (\mathbf{1} - \mathbf{U}) \sim \bar{C}$, i.e. $\mathbf{V} = \mathbf{1} - \mathbf{U} = (1 - u_1; 1 - u_2)$, is a random vector whose distribution is given by the survival copula $\bar{C}$ corresponding to the copula C (Hofert, Kojadinovic, Maechler and Yan, 2018). We can also express the survival copula as follows:

$$\bar{C}(\mathbf{u}) = \sum_{J \subseteq \{1,2\}} (-1)^{|J|} \cdot C((1 - u_1)^{I(1 \in J)}, (1 - u_2)^{I(2 \in J)}), \mathbf{u} \in (0; 1)^2 , \tag{5}$$

where the sum is over all the subsets of the set $\{1, 2\}$, $|J|$ is the number of elements of a given subset, $I(j \in J)$ is the indicator of $j \in \{1, 2\}$. For the two-dimensional survival copula $\bar{C}$ we then have:

$$J = \{\{1; 2\}; \{\emptyset\}; \{1\}; \{2\}\}, j \in \{1; 2\} , \tag{6}$$

$$\begin{aligned}\bar{C}(\mathbf{u}) = (-1)^2 \cdot C((1 - u_1)^1; (1 - u_2)^1) + (-1)^0 \cdot C((1 - u_1)^0; (1 - u_2)^0) + \\ + (-1)^1 \cdot C((1 - u_1)^1; (1 - u_2)^0) + (-1)^1 \cdot C((1 - u_1)^0; (1 - u_2)^1) ,\end{aligned} \tag{7}$$

from which we obtain:

$$\bar{C}(\mathbf{u}) = C(1 - u_1; 1 - u_2) + 1 - (1 - u_1) - (1 - u_2) = -1 + u_1 + u_2 + C(1 - u_1;1 - u_2) . \tag{8}$$

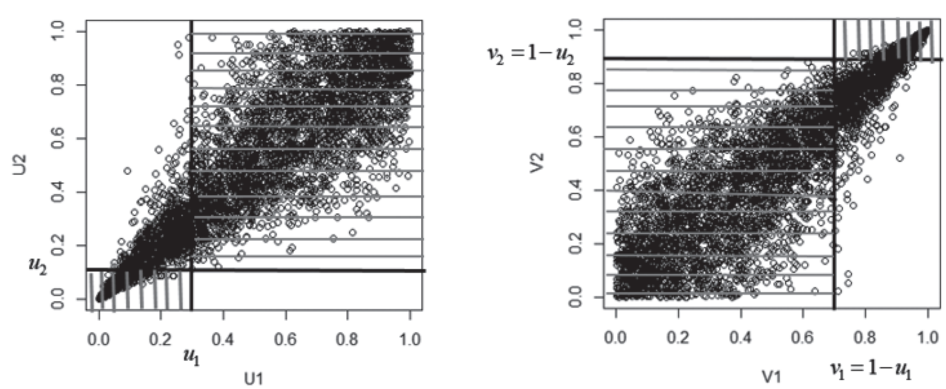
If we denote the probability density function of the copula C as c, then for the probability density function of the survival copula corresponding to C we have:

$$\bar{c}(\mathbf{u}) = c(1 - u_1; 1 - u_2), \mathbf{u} \in (0; 1)^2 . \tag{9}$$

For the copula and survival copula the following relationships also apply:

$$\begin{aligned}P(U_1 > u_1; U_2 > u_2) = P(V_1 \le v_1; V_2 \le v_2) = \bar{C}(v_1; v_2) , \\ P(V_1 > v_1; V_2 > v_2) = \mathrm{P}(U_1 \le u_1; U_2 \le u_2) = C(u_1; u_2) .\end{aligned} \tag{10}$$

Figure 2 Scatter plot of the generated copula values (on the left) and the corresponding survival copula values (on the right)



Source: Own construction, customized in R

These probabilities can be determined by statistical processing of the generated values (Figure 2).

1.3 Using copulas to simulate the values of the joint distribution

To generate the values of the marginal distributions and determine ordered pairs of the joint distribution what is important is the size of the values that are in a given pair. In the aggregation process these values are added together which then affects the value of the aggregate random variable. In aggregation by addition the order of the values in the marginal distributions is completely random. When using copula functions however the order is in a certain sense coordinated and hence this approach is considered to be more sophisticated.

1.3.1 Simulation of the values of the Clayton copula and the survival Clayton copula

Before we present the algorithm for generating the values of the two-dimensional copula function using conditional probabilities, we will introduce some notation:

$$P(U_2 \leq u_2 \mid U_1 = u_1) = C_{2|1}(u_2 \mid u_1)\,, \tag{11}$$

whereby we can also express this as:

$$C_{2|1}(u_2 \mid u_1) = \frac{\partial C(u_1, u_2)}{\partial u_1}\,. \tag{12}$$

The algorithm for generating values of the two-dimensional copula function C, i.e. the vector $\mathbf{U} = (U_1, U_2) \sim C$, respectively its values (u_1, u_2), is based on the Rosenblatt transformation.

1. We transform the vector $\mathbf{u} = (u_1, u_2) \in (0; 1)^2$ into the vector $\mathbf{u}' = (u_1', u_2') \in (0; 1)^2$, which we can write as $\mathbf{U}' = R_C(\mathbf{U})$, so that:

$$u_1' = u_1,\ u_2' = C_{2|1}(u_2 \mid u_1) = P(U_2 \leq u_2 \mid U_1 = u_1). \tag{13}$$

2. We transform the vector $\mathbf{u}' = (u_1', u_2') \in (0; 1)^2$ into the vector $\mathbf{u} = (u_1, u_2) \in (0; 1)^2$ using the quantile function as follows:

$$u_1 = u_1',\ u_2 = C_{2|1}^{-1}(u_2' \mid u_1), \tag{14}$$

which we can write as $\mathbf{U} = R_C^{-1}(\mathbf{U}')$ (Hofert, Kojadinovic, Maechler and Yan, 2018).

For a random sample of values of the vector $\mathbf{u} = (u_1, u_2) \in (0; 1)$ for the Clayton copula we then have:

$$u_1 = u_1',\ u_2 = C_{2|1}^{-1}(u_2'|u_1) = \left[1 + \left(u_2'^{\frac{-\theta}{\theta+1}} - 1\right) \cdot u_1^{-\theta}\right]^{-\frac{1}{\theta}}. \tag{15}$$

The presented approach can also be applied to generate values of the survival Clayton copula using Formula (9).

For this two-dimensional copula function, namely for:

$\bar{C} \sim \mathbf{V} = \mathbf{1} - \mathbf{U} = (1 - u_1, 1 - u_2) = (v_1, v_2)$, we get:

$$v_1 = 1 - u_1 = 1 - u_1', v_2 = 1 - u_2 = 1 - \left[1 + \left(u_2'^{\frac{-\theta}{\theta+1}} - 1\right) \cdot u_1^{-\theta}\right]^{-\frac{1}{\theta}}. \tag{16}$$

1.3.2 Generated values of the joint distribution using copula functions

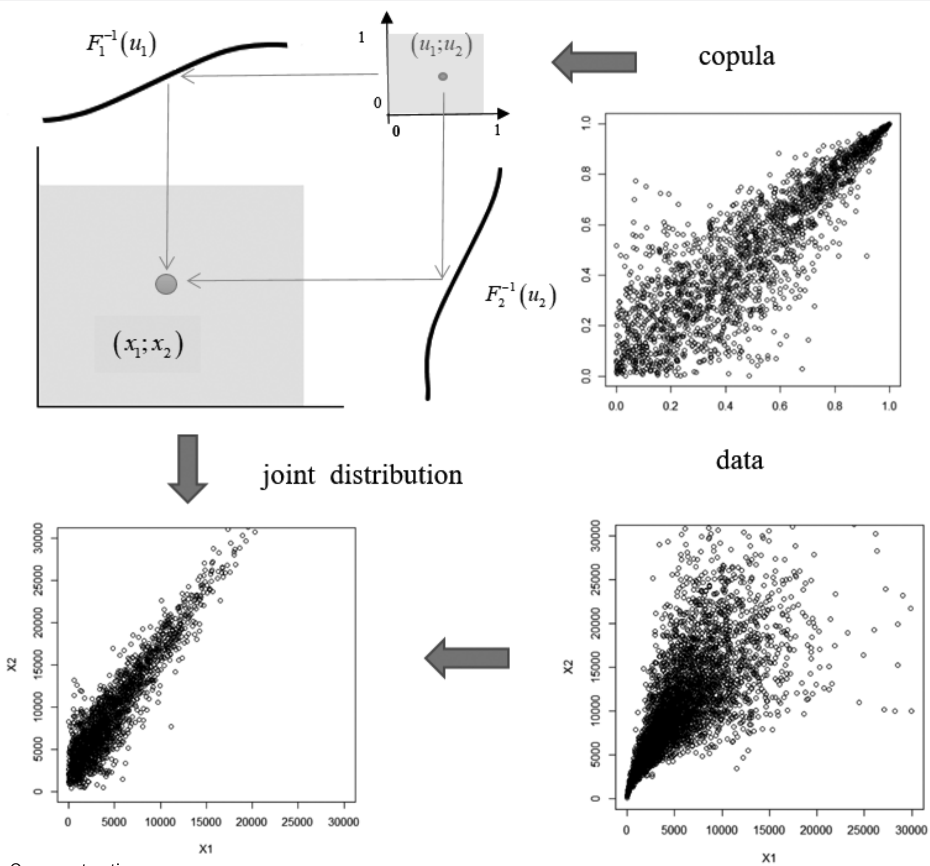
Using Sklar's theorem and the inverse transformation method we obtain values of the joint two-dimensional distribution F as per the equations:

$$W_i = F_i(X_i) \Leftrightarrow X_i = F_i^{-1}(W_i), i = 1, 2, \tag{17}$$

where:

$$\mathbf{W} = (W_1, W_2) \sim C; W_i \sim U(0; 1), i = 1, 2. \tag{18}$$

Figure 3 Generated values of the joint distribution using copula functions



Source: Own construction

1.4 Conditional quantile exceedance probability (cqep)

The conditional quantile exceedance probability (*cqep*) is an important indicator regarding modelling tail dependencies between insurance risks (Milek, 2020; Mucha and Škrovánková, 2022). We can distinguish between the upper and lower conditional probability of exceeding the quantile, whereby for the upper version $cqep_U(u)$ we consider the values $u \in (0.5; 1)$, respectively $u \to 1^+$, and for the lower version $cqep_L(u)$ the values $u \in (0; 0.5)$, respectively $u \to 0^-$. Just to recap, given the assumption $U_1 \sim U(0; 1)$; respectively $U_2 \sim U(0; 1)$, we have for their quantiles $F_{U_2}^{-1}(u) = u$; respectively $F_{U_1}^{-1}(u) = u$. For $cqep_U(u)$ copulas we get the result:

$$
\begin{aligned}
{}^{C}cqep_U(u) &= P(U_2 > F_{U_2}^{-1}(u) \mid U_1 > F_{U_1}^{-1}(u)) = \frac{P(U_2 > F_{U_2}^{-1}(u) \wedge U_1 > F_{U_1}^{-1}(u))}{P(U_1 > F_{U_1}^{-1}(u))} = \\
&\frac{1 - P(U_2 \le F_{U_2}^{-1}(u)) - P(U_1 \le F_{U_1}^{-1}(u)) + P(U_1 \le F_{U_1}^{-1}(u) \wedge U_2 \le F_{U_2}^{-1}(u))}{1 - P(U_1 \le F_{U_1}^{-1}(u))},
\end{aligned}
\tag{19}
$$

$$
{}^{C}cqep_U(u) = \frac{1 - 2u + C(u; u)}{1 - u} = \frac{\bar{C}(u; u)}{1 - u}\,. \tag{20}
$$

The copula indicator ${}^{C}cqep_U(u)$ therefore expresses the probability of exceeding the quantile $F_{U_2}^{-1}(u) = u$ of the random variable $U_2 \sim U(0; 1)$ on the assumption that the quantile $F_{U_1}^{-1}(u) = u$ of the random variable $U_1 \sim U(0; 1)$ was exceeded.

By analogy we can derive an expression for the lower conditional quantile exceedance probability $cqep_L(u)$ of the copula:

$$
{}^{C}cqep_L(u) = P(U_2 \le F_{U_2}^{-1}(u) \mid U_1 \le F_{U_1}^{-1}(u)) = \frac{P(U_2 \le F_{U_2}^{-1}(u) \wedge U_1 \le F_{U_1}^{-1}(u))}{P(U_1 \le F_{U_1}^{-1}(u))}, \tag{21}
$$

$$
{}^{C}cqep_L(u) = \frac{C(u; u)}{u}\,. \tag{22}
$$

For the joint distribution with marginal distributions of the random variables $X_1; X_2$, which we generate based on the given copula function, we get for the conditional quantile exceedance probability $cqep_U(u)$ using Sklar's theorem the following:

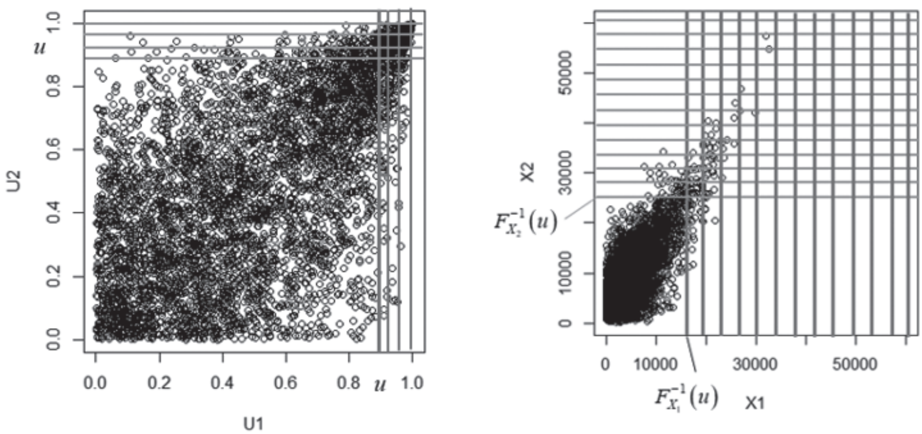
$$
\begin{aligned}
{}^{JD}cqep_U(u) &= P(X_2 > F_{X_2}^{-1}(u) \mid X_1 > F_{X_1}^{-1}(u)) = \frac{P(X_2 > F_{X_2}^{-1}(u) \wedge X_1 > F_{X_1}^{-1}(u))}{P(X_1 > F_{X_1}^{-1}(u))} = \\
&\frac{1 - P(X_2 \le F_{X_2}^{-1}(u)) - P(X_1 \le F_{X_1}^{-1}(u)) + P(X_1 \le F_{X_1}^{-1}(u) \wedge X_2 \le F_{X_2}^{-1}(u))}{1 - P(X_1 \le F_{X_1}^{-1}(u))} = \\
&\frac{1 - 2u + C(u; u)}{1 - u} = \frac{\bar{C}(u; u)}{1 - u}\,.
\end{aligned}
\tag{23}
$$

The joint distribution indicator ${}^{JD}cqep_U(u)$ expresses the probability of exceeding the quantile $F_{X_2}^{-1}(u) = {}^{2}x_u$ of the random variable X_2 on the assumption that the quantile $F_{X_1}^{-1}(u) = {}^{1}x_u$ of the random variable X_1 was exceeded.

By analogy we can also derive a formula for the lower conditional quantile exceedance probability ${}^{JD}cqep_L(u)$ of the joint distribution:

$$^{JD}cqep_L(u) = P(X_2 \leq F_{X_2}^{-1}(u) \mid X_1 \leq F_{X_1}^{-1}(u)) = \frac{P(X_2 \leq F_{X_2}^{-1}(u) \wedge X_1 \leq F_{X_1}^{-1}(u))}{P(X_1 \leq F_{X_1}^{-1}(u))} = \frac{C(u;u)}{u}. \tag{24}$$

Figure 4 Graphical interpretation of ${}^{C}cqep_U(u)$ (on the left) and ${}^{JD}cqep_U(u)$ (on the right) as scatter plots of the generated values of the copula and the joint distribution



Source: Own construction

The upper conditional quantile exceedance probability ${}^{C}cqep_U(u)$ and ${}^{JD}cqep_U(u)$ can be determined statistically from their generated values, Figure 4. It is clear from this that the joint distribution retains the same value for the conditional quantile exceedance probability as the copula used to generate the values. It can be asserted that the copula "passes to the joint distribution particular genetic information" concerning the tail dependencies, respectively a particular risk scenario, and we have:

$$^{C}cqep_U(u) = {}^{JD}cqep_U(u), \text{ respectively } {}^{C}cqep_L(u) = {}^{JD}cqep_L(u). \tag{25}$$

1.4.1 Conditional quantile exceedance probability for the survival Clayton copula

To derive a formula for calculating ${}^{C}cqep_U(u)$ for the survival Clayton copula we will use the fact that it is a rotated copula with respect to the Clayton copula. Let C be a two-dimensional copula and let $\mathbf{U} \sim C$. For $\mathbf{r} \in \{0;1\}^2$, we call $rot_r(C)$ the rotated copula with regard to C, if $\mathbf{U} \sim C$ this is equivalent to:

$$((1-r_1) \cdot U_1 + r_1 \cdot (1-U_1); (1-r_2) \cdot U_2 + r_2 \cdot (1-U_2)) \sim rot_r(C). \tag{26}$$

If the copula C has probability density function c, the probability density function of the rotated copula $rot_r(C)$ is as follows (Hofert, Kojadinovic, Maechler and Yan, 2018):

$$rot_r(c)(\mathbf{u}) = c((1-r_1) \cdot u_1 + r_1 \cdot (1-u_1); (1-r_2) \cdot u_2 + r_2 \cdot (1-u_2)); \mathbf{u} \in (0;1)^2. \tag{27}$$

It is obvious that the survival Clayton copula is a 180° rotated copula with respect to the Clayton copula. Hence we can obtain the following expression for the values of the vector $\mathbf{r} = (1; 1)$:

$$(1 - U_1; 1 - U_2) = \mathbf{1} - \mathbf{U} = \mathbf{V} \sim rot_{(1;1)}(C) = \bar{C}\,, \tag{28}$$

$$\bar{c}(\mathbf{v}) = rot_{\mathbf{r}}\,(c)(\mathbf{v}) = c(1 - u_1; 1 - u_2);\, \mathbf{v} \in (0; 1)^2\,. \tag{29}$$

Given the above we have for the value of the conditional quantile exceedance probability for the survival copula ${}^{\bar{C}}cqep_U(v)$:

$${}^{\bar{C}}cqep_U(v) = {}^{C}cqep_L(1 - v);\, v \in (0.5; 1),\, v = 1 - u\,. \tag{30}$$

We can also derive this as follows, where we treat the survival copula $\bar{C}$ as a copula using Formula (8):

$$\begin{aligned}
{}^{\bar{C}}cqep_U(v) &= P(V_2 > F_{V_2}^{-1}(v) \mid V_1 > F_{V_1}^{-1}(v)) = \frac{P(V_2 > F_{V_2}^{-1}(v) \wedge V_1 > F_{V_1}^{-1}(v))}{P(V_1 > F_{V_1}^{-1}(v))} = \\
&\frac{1 - P(V_2 \le F_{V_2}^{-1}(v)) - P(V_1 \le F_{V_1}^{-1}(v)) + P(V_1 \le F_{V_1}^{-1}(v) \wedge V_2 \le F_{V_2}^{-1}(v))}{1 - P(V_1 \le F_{V_1}^{-1}(v))} = \\
&\frac{1 - 2 \cdot v + \bar{C}(v; v)}{1 - v} = \frac{1 - 2 \cdot (1 - u) + 1 - 2 \cdot u + C(u; u)}{1 - (1 - u)} = \frac{C(u; u)}{u} = {}^{C}cqep_L(u)\,.
\end{aligned} \tag{31}$$

For the lower conditional quantile exceedance probability of the Clayton copula ${}^{CLC}cqep_L(u)$, given Formula (22), we get:

$${}^{CLC}cqep_L(u) = \frac{(2 \cdot u^{-\theta} - 1)^{-\frac{1}{\theta}}}{u}\,, \tag{32}$$

and for the upper conditional quantile exceedance probability of the survival Clayton copula ${}^{SCLC}cqep_U(1 - u)$ we get using Formula (31):

$${}^{SCLC}cqep_U(1 - u) = {}^{SCLC}cqep_U(v) = \frac{(2 \cdot (1 - v)^{-\theta} - 1)^{-\frac{1}{\theta}}}{(1 - v)};\, v \in (0.5; 1),\, v = 1 - u\,. \tag{33}$$

When modelling a risk scenario represented by the value ${}^{SCLC}cqep_U(v)$ it can be identified with regard to the parameter of the survival Clayton Copula using Formula (33).

2 RESULTS AND DISCUSSION

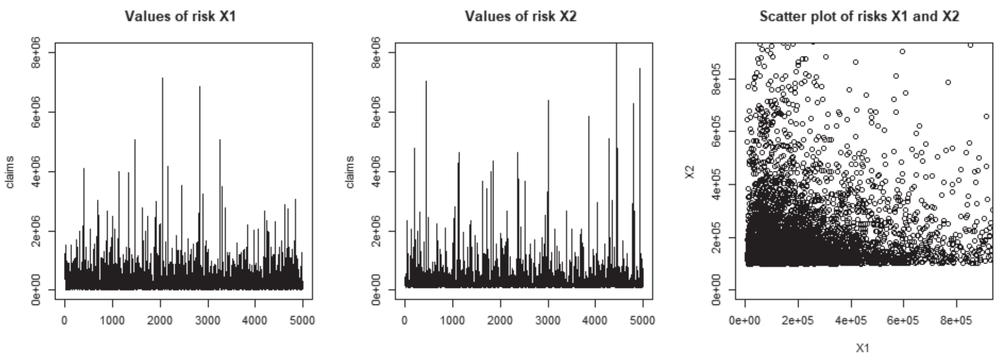
In this section we will deal with a practical example of the aggregation of two non-life insurance risks X_1, X_2 using the survival Clayton copula. From the statistical data used we have assumed their marginal distributions, for parameter estimation for the truncated Pareto distribution see Aban, Meerschaert and Panorska (2006):

$$X_1 \sim LN(12; 1),\, X_2 \sim Pa^{truncated}(100\ 000; 1\ 000\ 000; 1.5), \tag{34}$$

with characteristics:

$$E(X_1) = 268\ 337.3,\ E(X_2) = 270\ 270.3,\ {}^1x_{0.95} = 843\ 060.6,\ {}^2x_{0.95} = 727\ 546.7\ . \tag{35}$$

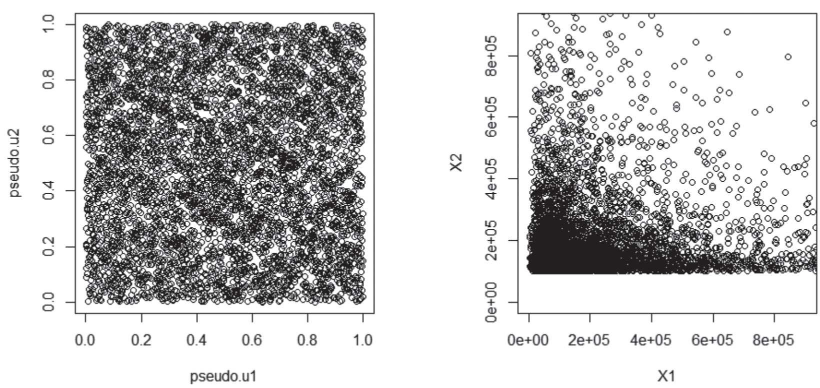
Figure 5 Graphical representation values of the risks X_1, X_2



Source: Own construction, customized in R

To start with we will present the classical approach to aggregation of these risks based on fitting a suitable copula given the values of their marginal distributions, Figure 5. Using the package `VineCopula` and the function `BiCopEstList`, we chose, based on the information criteria *AIC* and *BIC*, the best model for the copula function given the presented values (Nagler et al., 2023). When using pseudo-observations obtained in accordance with the principle for constructing empirical distribution functions using Formula (36) we need to consider their independence, Figure 6 (on the left).

Figure 6 Graphical representation of the pseudo-observations and scatter plot of their joint distribution simulated using a chosen Normal copula



Source: Own construction, customized in R

Remark 1: We will consider a random vector $\mathbf{X} = (x_{i1}; x_{i2})$, $i = 1, ..., n$, where n is the number of points in the scatter plot. We obtain the pseudo-observations using the following:

$$u_{ij} = \frac{r_{ij}}{n+1},\ i = 1, ..., n,\ j = 1, 2\ , \tag{36}$$

where r_{ij} is the order of precedence of x_{ij} amongst all the x_{kj}, $k = 1, ..,n$ (Hofert, Kojadinovic, Maechler and Yan, 2023).

We chose as the most suitable a Normal copula with parameter $\rho = 0.009$, where, as $\rho \to 0$, it converges to the independent copula confirming our earlier assumption. We obtain the aggregation of the risks using the joint distribution simulated using this copula and by addition the points shown in the scatter plot on the right of Figure (6). We determine the values of the aggregate variable using the following:

$$s_i = x_{i1} + x_{i2},\ i = 1, ..., n\ , \tag{37}$$

where: x_{i1}, x_{i2} are elements of the random vector of the generated joint distribution.

In order to measure the effect of the aggregation achieved using the chosen copula function we use the risk measure *TVaR* (Bargés, Cossete and Marceau, 2009). This represents the expected value of the aggregate variable on the assumption that its quantile s_p was exceeded. We write this as:

$$TVaR_p(S) = E(S \mid S > s_p)\ . \tag{38}$$

In the classical approach to risk aggregation using a Normal copula function used by us so far, we calculated the value of this measure by simulating the joint distribution using the library `Copula`:

$$^{Normal}TVaR_{0.95}(S) \approx 2\ 343\ 729\ . \tag{39}$$

Insurers' internal models make use of this risk measure to determine the capital required to cover unexpected claims, respectively losses. For our purpose we will interpret it as the highest possible loss which may arise with probability greater than 0.95, i.e.

$$P(S \leq {}^{Normal}TVaR_{0.95}(S)) > 0.95\ . \tag{40}$$

We will now consider an innovative approach to aggregating risk using the survival Clayton copula. As opposed to the classical approach we will purposefully model a given risk level scenario for the tail dependency such that a guarantee of covering the highest possible loss is achieved even in the case of the concurrent occurrence of extreme values of the aggregate risk. Their occurrence can have fatal consequences for the insurer leading even to insolvency. We will in the conclusion compare the results from the two approaches.

The conditional quantile exceedance probability *cqep* is an important indicator of the level of tail dependence. Given the aggregation of risks present in insurance we concentrate on analysing its upper version $cqep_U$, Formulas (23) and (33).

In the case of the survival Clayton copula the value $^{SCLC}cqep_U(v)$ depends also on its parameter θ. We show this dependence also in the context of dependence on the level of the quantile v in Figure 7.

From this visualisation we get some very interesting information, namely that for $\theta \geq 3$ the value $cqep_U(v)$ is the same for all levels of the quantile.

Remark 2: The graph of $cqep_U$ in Figure 7 for $v = 0.99$ is shown below for $\theta < 3$.

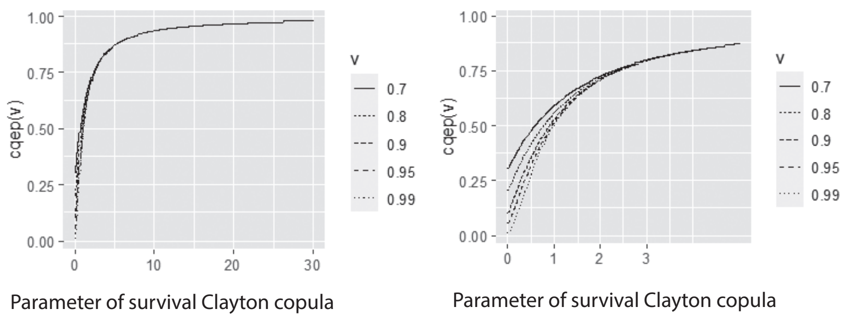
To illustrate the results shown we give the coordinates of some of the points in the graph [θ; $cqep_U(v)$], for example [3; 0.794], [5; 0.871], [10; 0.933], [15; 0.955], [30; 0.9772].

It is clear that as the value of θ for the survival Clayton copula increases so also does the value $cqep_U(v)$. We recall that the generated joint distribution has the same value for this indicator as the copula which we used in the simulation. For $\theta = 3$ we have $^{SCLC}cqep_U(v) = {}^{JD}cqep_U(v) = 0.794$ for all levels of the quantile v. We can interpret this as meaning that the probability of exceeding the quantile $F_{X_2}^{-1}(v) = {}^2x_v$

of the random variable X_2 assuming that the quantile $F_{X_1}^{-1}(v) = {}^1x_v$ of the random variable X_1 is exceeded is equal to 0.794.

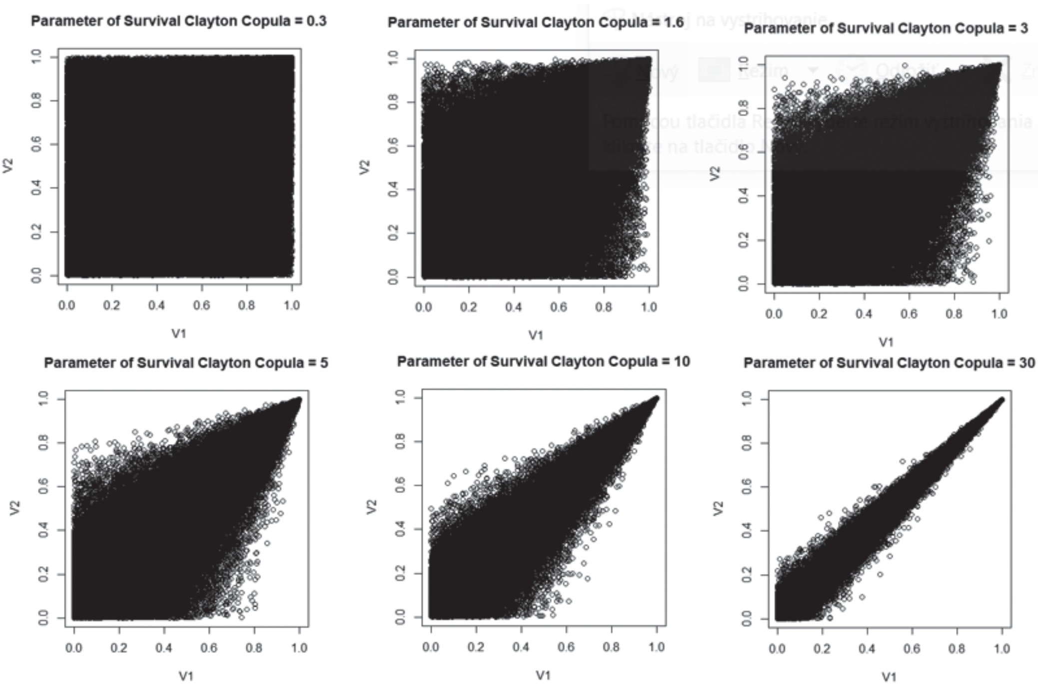
If we consider the points in the scatter plot of the joint distribution where the quantile ${}^1x_v$ of the random variable X_1 was exceeded, then for 79.4% of these the quantile ${}^2x_v$ of the random variable X_2 was

Figure 7 Graphical representation of the dependence of $cqep_U(v)$ on the parameter of the survival Clayton copula for various levels of the quantile x_v of the marginal distributions



Source: Own construction, customized in R

Figure 8 Scatter plot of the survival Clayton copula for various values of its parameter

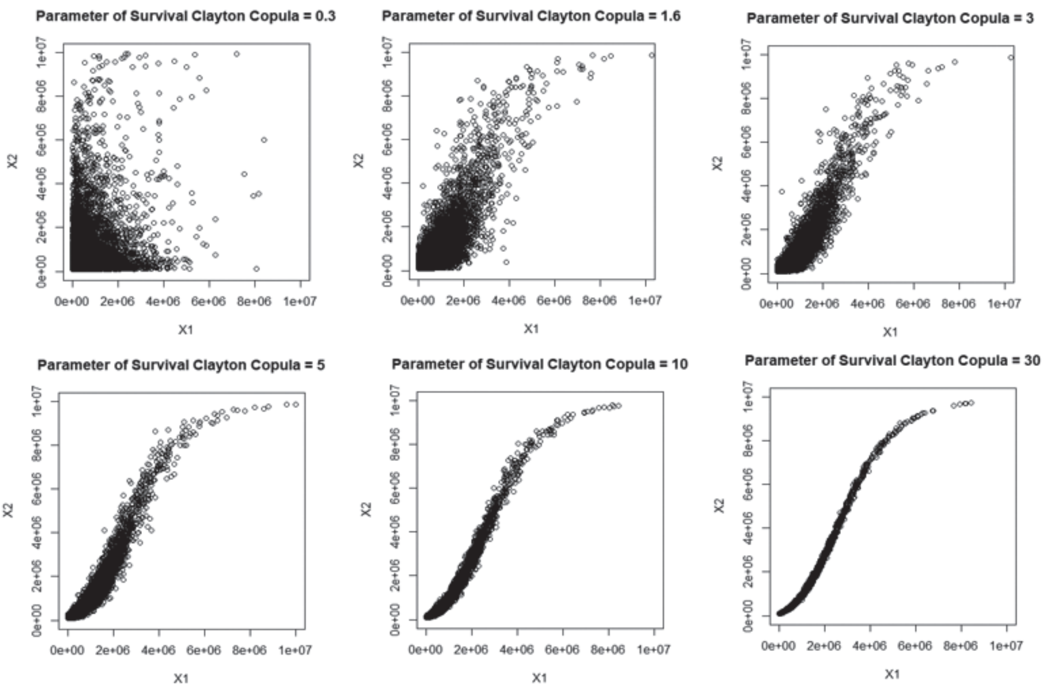


Source: Own construction, customized in R also exceeded. This indicator together with the parameter θ of the survival Clayton copula represent a particular risk scenario for modelling the tail dependency of the aggregated risks.

For clarity we show a scatter plot of the survival Clayton copula according to the value of its parameter θ, Figure 8. Information as to the tail dependency hidden in this copula function is passed to the generated joint distribution.

Figure 9 shows scatter plots for the joint distribution in respect of each copula function in Figure 8.

Figure 9 Scatter plots of the joint distribution generated by the survival Clayton copula



Source: Own construction, customized in R

It is obvious that as the parameter of the copula increases the modelled tail dependency is demonstrably more evident.

Remark 3: It is recommended that the visualisation on the scatter plot be seen in the context of the characteristics of the marginal distributions, respectively in the context of the range of both axes.

By choosing a suitable value of the parameter of the survival Clayton copula we can purposefully model a considered risk scenario for the aggregate distribution of given risks. As we have already mentioned, we will measure the effect of applying a given risk scenario using the risk measure $TVaR_p(S)$. One can assume that with an increase in the value of the parameter of the survival Clayton copula θ, respectively with an increase in the conditional quantile exceedance probability ${}^{JD}cqep_U(v)$, the value $TVaR_p(S)$ of the aggregate distribution will also increase.

The values of $TVaR_p(S)$ for $p \in \{0.9; 0.95; 0.99\}$ are shown in Table 1.

The dependence of $TVaR_p(S)$ on the parameter of the survival Clayton copula θ is shown graphically in Figure 10.

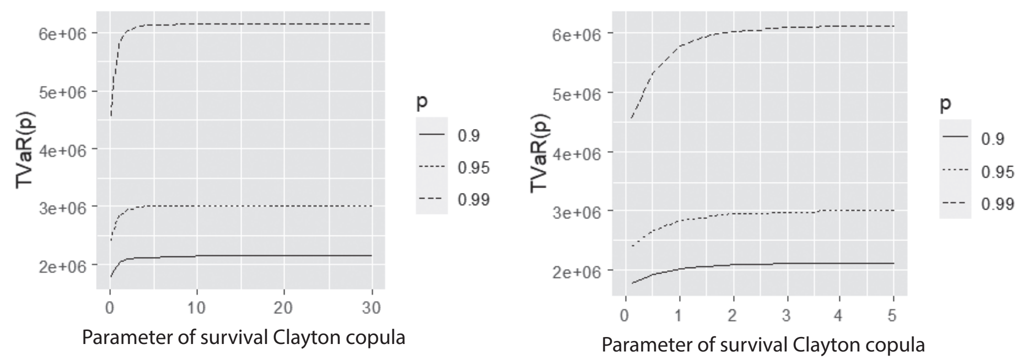
Table 1 Values of $TVaR_p(S)$ obtained by aggregation using the survival Clayton copula

θ	**0.1**	**1**	**3**	**5**	**10**	**30**
$TVaR_{0.9}(S)$	1 793 981	2 021 891	2 118 868	2 132 683	2 139 379	2 141 596
$\boldsymbol{TVaR_{0.95}(S)}$	**2 419 258**	**2 837 231**	**2 986 344**	**3 007 149**	**3 015 927**	**3 017 989**
$TVaR_{0.99}(S)$	4 566 745	5 779 918	6 096 432	6 127 033	6 145 136	6 149 955

Source: Own construction

It is clear from the presented results that the value $TVaR_p(S)$ is, for values of the parameter of the survival Clayton copula $\theta \geq 3$, at a "stable" level. If $\theta = 3$ the largest possible aggregate loss ${}^{SCC}TVaR_{0.95}(S)$, which can occur with probability greater than 0.95, is 2 986 344. We obtained the values shown by carrying out simulations of the values of the joint distribution of the marginal risks, respectively of the aggregate variable using the survival Clayton copula. To get more accurate results we carried out this series of simulations 5 000 times and the resulting value of ${}^{SCC}TVaR_p(S)$ was taken as the average of the values obtained from each series.

Figure 10 Values of $TVaR_p(S)$ for $p \in \{0.9; 0.95; 0.99\}$ depending on the parameter of the survival Clayton copula



Source: Own construction, customized in R

CONCLUSION

This paper has presented an innovative approach to the aggregation of risks using a survival Clayton copula. Specific modelling of a given risk scenario, dependent on the parameter of the copula, is important particularly from the point of view of the occurrence of extreme values of the aggregate risk. The upper conditional quantile exceedance probability $cqep_U(v)$ is an authoritative indicator of the simulated tail dependence. We used the risk measure $TVaR_p(S)$ to measure the effect of the achieved aggregation. It represents the largest possible aggregate loss which can occur with probability greater than p. By setting up a capital requirement at this level it would be possible to guarantee the fulfilment of the insurer's liabilities at the chosen significance level p.

Given the results obtained for $TVaR_p(S)$ we can assert that the parameter of the survival Clayton copula $\theta = 3$ secures a risk scenario model with an "adequate" tail dependence. If we compare the value ${}^{SCC}TVaR_{0.95}(S) \approx 2\,986\,344$ obtained this way with that using the classical approach ${}^{Normal}TVaR_{0.95}(S) \approx 2\,343\,729$ we see that there is a significant difference. By specifically modelling larger (extreme) values arising in the aggregated pairs of the joint distribution we have produced a prediction of the possible occurrence of larger of the largest aggregate losses $TVaR_p(S)$.

If instead we look at the value obtained using the parameter $\theta = 0.1$ for the survival Clayton copula, namely ${}^{SCC}TVaR_{0.95}(S) \approx 2\,419\,258$ we could assert that it is comparable with the value obtained using the classical approach. As the value of the parameter $\theta \to 0$ the survival Clayton copula tends towards the position of the independent copula. This is in the context of assuming use of the Normal copula estimated from data using pseudo-observations. The choice of the level of the modelled scenario is up to experts, or it may follow from legislative requirements concerning own appraisal and the management of risk in an insurer's internal models. For the latter use is made of their own copulas for aggregating risk, whereby the risk measure $TVaR_p(S)$ is, as already mentioned, also used to determine the capital required to cover unexpected losses. For some risks it is common practice to show their values in the form of profits and losses, i.e. they appear as negative and positive values. Our presented approach can of course also be applied to such models.

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References

ABAN, I. B., MEERSCHAERT, M. M., PANORSKA, A. K. (2006). Parameter estimation for the truncated Pareto distribution. *Journal of the American Statistical Association*, 101(473): 270–277.

ADÈS, M., PROVOST, S., ZANG, Y. (2024) Four Measures of Association and Their Representations in Terms of Copulas [online]. *AppliedMath*, 4(1): 363–382. <https://doi.org/10.3390/appliedmath4010019>.

ALLEN D., MCALEER M., SINGH A. (2017). Risk Measurement and Risk Modelling Using Applications of Vine Copulas [online]. *Sustainability*, 9(10): 1762. <https://doi.org/10.3390/su9101762>.

BARGÉS, M., COSSETE, H., MARCEAU, E. (2009). TVaR-based capital allocation with copulas [online]. *Hal open science*, 32. <https://hal.archives-ouvertes.fr/hal-00431265>.

CIPRA, T. (2015) *Riziko ve financích a pojišťovnictví.* Prague: Ekopress. ISBN 978-80-8786-524-8

CUEVAS, C., YELA, J., ACHCAR, J. (2019). A Method to Select Bivariate Copula Functions [online]. *Revista Colombiana de Estadistic*a, 42: 61–80. <https://doi.org/10.15446/rce.v42n1.71780>.

CUYPERS, F. (2020). *Risk Dependencies* [presentation]. PrimeRe Solutions.

DE LUCA, G., RIVIECCIO, G. (2012). Multivariate Tail Dependence Coefficients for Archimedean Copulae [online]. *Advanced Statistical Methods for the Analysis of Large Data-Sets.* <https://doi.org/10.1007/978-3-642-21037-2_26>.

DI BERNARDINO, E., PRIEUR, C. (2018). Estimation of the multivariate conditional tail expectation for extreme risk levels [online]. *Environmetrics*, 29(2). <https://doi.org/10.1002/env.2510>.

EMBRECHTS, P., LINDSKOG, F., MCNEIL, A. (2001). *Modelling Dependence with Copulas and Applications to Risk Management* [online]. ETH Zurich, Department of Mathematics, 48. <https://people.math.ethz.ch/~embrecht/ftp/copchapter.pdf>.

ELING, M., JUNG, K. (2020). Risk aggregation in non-life insurance [online]. *Mathematics and Economics*, 95: 183–198. <https://doi.org/10.1016/j.insmatheco.2020.09.003>.

GHOSH, D. I., CHAKRABORTY, S., WATTS, D. (2022). Modelling Bivariate Dependency in Insurance Data via Copula: a Brief Study [online]. *Journal of Risk and Financial Management*, 15. <https://doi.org/10.3390/jrfm15080329>.

GIJBELS, I., KIKA, V., OMELKA, M. (2020). Multivariate Tail Coefficients: Properties and Estimation [online]. *Entropy*, 22: 728. <https://doi.org/10.3390/e22070728>.

GIJBELS, I., VERAVERBEKE, N., OMELKA, M. (2011). Conditional copulas, association measures and their applications [online]. *Computational Statistics & Data Analysis*, 55: 1919–1932. <https://doi.org/10.1016/j.csda.2010.11.010>.

HOFERT, M., KOJADINOVIC, I., MAECHLER, M., YAN, J. (2023). *Copula: Multivariate Dependence with Copulas* [online]. R package version 1.1-3. <https://CRAN.R-project.org/package=copula>.

HOFERT, M., KOJADINOVIC, I., MAECHLER, M., YAN, J. (2018). *Elements of Copula Modelling with R. Springer*, 254. ISBN 978–3–319–89634–2

HUA, L., JOE, H. (2011). Tail order and intermediate tail dependence of multivariate copulas [online]. *Journal of Multivariate Analysis*, 102: 1454–1471. <https://doi.org/10.1016/j.jmva.2011.05.011>.

CHARPENTIER, A., SEGERS, J. (2009). Tails of multivariate Archimedean copulas [online]. *Journal of Multivariate Analysis*, 100: 1521–1537. <https://doi.org/10.1016/j.jmva.2008.12.015>.

CHEN, S., HUANG, T. (2007). Nonparametric Estimation of Copula Functions For Dependence Modelling [online]. *Canadian Journal of Statistics*, 35: 265–282. <https://doi.org/10.1002/cjs.5550350205>.

CHERUBINI, U., LUCIANO, E., VECCHIATO, W. (2004). *Copula methods in finance.* England: John Wiley & Sons Ltd., 293. ISBN 0–470–86344–7

JOO, K., SHIN, J., HEO, J. (2020). Modified Maximum Pseudo Likelihood Method of Copula Parameter Estimation for Skewed Hydrometeorological Data [online]. *Water*, 12(4): 1182. <https://doi.org/10.3390/w12041182>.

MARRI, F., MOUTANABBIR, K. (2022). Risk aggregation and capital allocation using a new generalized Archimedean copula [online]. *Insurance: Mathematics and Economics*, Elsevier, 102(C): 75–90. <https://doi.org/10.1016/j.insmatheco.2021.11.007>.

MILEK, J. (2020). *Multivariate B11 copula family for risk capital aggregation* [online]. Talks in Financial and Insurance Mathematics, ETH Zurich, Department of Mathematics. <https://www2.math.ethz.ch/t3/fileadmin/math/ndb//00014/04970/ETH.March.6.2014.Zurich.Milek.v.3.pdf>.

MILEK, J. (2020). *Quantum Implementation of Risk Analysis-relevant Copulas* [online]. <https://www.researchgate.net/publication/339350691_Quantum_Implementation_of_Risk_Analysis-relevant_Copulas>.

McNEIL, A. J., FREY, R., EMBRECHTS, P. (2015). *Quantitative Risk Management: Concepts, Techniques and Tools.* Princeton University Press. ISBN 978-0-691-16627-8

MUCHA, V. (2023). Survival Clayton Copula Simulation. Trends in the Education of Stu-dents in the Study Program of the Actuarial Science. *Reviewed Monographic Collection of Research Papers*, H.R.G., 48–54. ISBN 978–80–7490–312–0

MUCHA, V., ŠKROVÁNKOVÁ, L. (2022). Risk Aggregation Using B11 Copula. Determination of the Capital Requirement Cover Selected Catastrophic Risks in Life and Non-Life Insurance. *Reviewed Monographic Collection of Research Papers*, H.R.G., 47–56. ISBN 978–80–7490–283–3

NAGLER, T., SCHEPSMEIER, U. et al. (2023). *VineCopula: Statistical Inference of Vine Copulas* [online]. R package version 2.5.0. <https://github.com/tnagler/VineCopula>.

NICOLAS, M. GARCIN, M. (2021). *Nonparametric estimator of the tail dependence coefficient: balancing bias and variance* [online]. <https://doi.org/10.48550/arXiv.2111.11128>.

NELSEN, B. R. (2006). *An Introduction to Copulas.* 2nd Ed. New York: Springer series in statistics. ISBN 978–0387–28659–4

NGUYEN, T., MOLINARI, R. (2011). Risk Aggregation by Using Copulas in Internal Models [online]. *Journal of Mathematical Finance*, 1(3): 50–57. <https://doi.org/10.4236/jmf.2011.13007>.

PFEIFER, D., STRASSBURGER, D., PHILIPPS, J. (2020). *Modelling and simulation of dependence structures in nonlife insurance with Bernstein copulas* [online]. <https://doi.org/10.48550/arXiv.2010.15709>.

REMILLARD, B., GENEST, C., BEAUDOIN, D. (2009). Goodness-of-fit tests for copulas: A review and a power study [online]. *Insurance: Mathematics and Economics*, 44: 199–213. <https://doi.org/10.1016/j.insmatheco.2007.10.005>.

PINDA, Ľ., MUCHA, V., SMAŽÁKOVÁ, L. (2022). *Risk management using the extreme value theory and alternative risk transfer.* H.R.G. ISBN 978–80–7490–251–2

YANG, L. (2022). Nonparametric Copula Estimation for Mixed Insurance Claim Data [online]. *Journal of Business & Economic Statistics*, 40(2): 537–546. <https://doi.org/10.1080/07350015.20.1835668>.

ICT Development Index and Its Role in the FDI – Growth Nexus: Evidence from G20 Economies

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Abstract

The collective GDP of the G20 nations constitutes over 80% of the global GDP, making them pivotal recipients of significant economic investments in various forms including foreign direct investments. This study delves into the dynamic interplay of ICT development and foreign direct investment (FDI) and GDP nexus among the G20 economies. A comprehensive index is constructed using PCA to gauge ICT development across economies. The study further examines the relationships among FDI, ICT development, and GDP using panel data spanning from 2000 to 2019. The study finds that in the absence of interaction, FDI alone does not exhibit a statistically significant impact on GDP. However, considering the interaction between FDI and ICT, a nuanced pattern emerges. The study discerns that the influence of FDI on GDP is contingent upon the maturity of a country's ICT sector. This suggests the need for policymakers to adopt a more focused approach, tailoring strategies to leverage the interdependence of FDI and ICT for optimal economic growth.

Keywords	***DOI***	***JEL code***
FDI, ICT, G20, Growth	*https://doi.org/10.54694/stat.2023.59*	*O10, O49, O50*

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INTRODUCTION

As per UNESCO Institute for Statistics (2009), ICT refers to "a diverse set of technological tools and resources used to transmit, store, create, share or exchange information. These technological tools and resources include computers, the Internet, live broadcasting technologies, recorded broadcasting technologies and telephony". Information and communication technology (ICT) and IT-enabled services are considered as a revolution which involves the creation, codification, and dissemination of knowledge (Kirkman et al., 2002), with its effective use having a positive impact on economic growth and competitiveness (Hanna, 2009).

ICT has the potential to enhance productivity and catalyse human development. Accordingly, it has been suggested that policy makers use ICT penetration as a target instrument to achieve higher levels of development (Asongu and Roux, 2017). ICT, as highlighted by the Digital Opportunity Initiative (2001)[6] significantly enhances sustainable environmental management, improves monitoring, and addresses issues like aging, poverty, health, and education. World Bank Group (2012) envisages that governments employ ICT to revolutionize public service delivery at the national and local levels in the areas of health, education, social protection, justice, agriculture, energy, and transportation. The extension of choices available to society with respect to health, education and other components of living standards made possible due to the widespread use of ICT are considered as key parameters of human development (Yakunina and Bychkov, 2015). The participatory aspect of ICT establishes new relationships that stimulate and support innovation, allowing new ideas and beliefs to be integrated. It is capable of unleashing a social transformation and modernizing the economy (Saith and Vijayabaskar, 2005).

The advancement of ICT expands human freedom in numerous ways which according to Sen's human capability approach leads to an improvement in the quality of life (Sen, 2010). Also, increasingly ICT use is no longer a matter of choice, rather it is an essential element in the daily lives of the active populations of countries across the development spectrum. It commands this power due to its ability to facilitate the optimization of limited resources. ICT development has tremendous scope as it influences social behaviour, democratic processes, and innovation.

For the reasons mentioned above, active and effective cooperation for the promotion of ICT enabled digital global economy is high on the G20[7] agenda. The consideration of the G20 economies stems from the substantial role this group plays in global economic dynamics. The G20 is a platform where the leaders commit to working with developing countries, particularly those with low incomes, to help them implement policies and priorities that are based on their national needs in order to achieve international development goals, particularly the Millennium Development Goals (MDGs), and to reaffirm their commitment to standstill. In order to support growth and development, the G20 provides policy coherence, analysis, and practical tools with a broad objective to ensure financial stability and promote growth. Further, as developing countries are getting more integrated into the global economy, the phenomenon contributes to the G20's objective of strong, sustainable, balanced and inclusive global growth. The 2030 Agenda for Sustainable Development too sets an ambitious, transformative and universal agenda for sustainable development efforts.

The Digital Economy Working Group (DEWG), currently chaired by India, is deliberating on plans to foster cooperation among the member countries for the equitable progress of digitalization. The GDP

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[6] Digital Opportunity Initiative is a unique public-private partnership between Accenture, the Markle Foundation and the United Nations Development Programme (UNDP).

[7] G20 is a strategic forum which brings together the world's major developed and developing economies. The members of the G20 are: Argentina, Australia, Brazil, Canada, China, France, Germany, India, Indonesia, Italy, Japan, Republic of Korea, Mexico, Russia, Saudi Arabia, South Africa, Turkey, the United Kingdom, the United States, and the European Union.

of the G20 countries cumulatively exceeds 80% of the global GDP, 75% of global commerce, and 60% of the global population.[8] Therefore, the G20 can play a crucial role, strategically, in the path towards future global prosperity and economic progress. The widespread use of the ICT infrastructure and the consequent digitalization is essential to create conditions that can potentially allow developing countries to leapfrog their way into economic prosperity. Due to the digital economy's expanding influence on the G20 economies and its ability to impact both the levels and rates of change of employment and production, there is a pressing need for new data, indicators, and measuring tools.

The objective of the paper is twofold. Firstly, we construct an ICT Development Index (IDI) using data for selected G20 economies over the period 2000–2019 using the technique of Principal Components Analysis (PCA). The variables have been carefully chosen to cover the three broad dimensions of ICT identified in the literature i.e. access, usage and capability. The index aims to quantify and then compare the ICT performance of these countries vis-a-vis each other and suggest recommendations based on learnings from leading countries. This index is an important tool to quantify the ICT development in a comprehensive manner. Secondly, the paper seeks to empirically verify the hypothesis that the ICT development may catalyse the association between FDI and economic growth. A certain amount of the ICT development may be a prerequisite for FDI to make an impact on economic growth. It enhances a country's readiness to harness technology spillovers and speed up human capital formation and thereby contribute to greater economic growth. Using the constructed ICT Development Index, the study evaluates if the ICT development augments the impact of FDI on economic growth.

1 REVIEW OF LITERATURE

The relationship between FDI and economic growth has been extensively examined in the extant literature (Li and Liu, 2005) and provides mixed evidence on the relationship between FDI and economic growth. Although some studies have identified a positive association between foreign direct investment (FDI) and economic growth (Sunde, 2017), there are others that have empirically established the impact of FDI on growth to be negative (Benzaim et al., 2023) and also those that have reported no causal impact of FDI on economic growth (Shimul, 2009; Tabassum and Ahmed, 2014). Hudea and Stancu (2012) for instance, reports a significant and positive impact of FDI on economic growth, both in the short and long terms, thereby reducing the technological disparity. The study also uncovered a bidirectional relationship, demonstrating causality not solely from FDI to economic growth but also in the converse direction, suggesting that FDI precipitates a chain reaction effect.

Alfaro (2003) challenges the commonly held notion that foreign direct investment (FDI) generates significant benefits uniformly for host countries. A closer examination makes it evident that there are significant sectoral effects. Moreover, the study finds that the overall impact of total FDI on growth remains inconclusive.

In this context, a relevant argument stems from an understanding of how FDI would influence the growth. While FDI is primarily a source of external capital for the host country and in that capacity relieves the resource constraint on growth and development, there is another indirect channel that reinforces this effect. FDI is an important channel for the transfer of technology and the host country needs to be equipped to fully tap its potential. A certain level of absorptive capacity may in fact be considered essential. So, when a company decides to set up a factory in a foreign land, it relies for some inputs on the host country, most important being the labour. For the investment to be productive and hence profitable the labour needs to be efficient. There can be little opposition to the proposition that the higher the digital literacy and more extensive the ICT use in the host country the more productive its labour and

[8] Source: OECD.

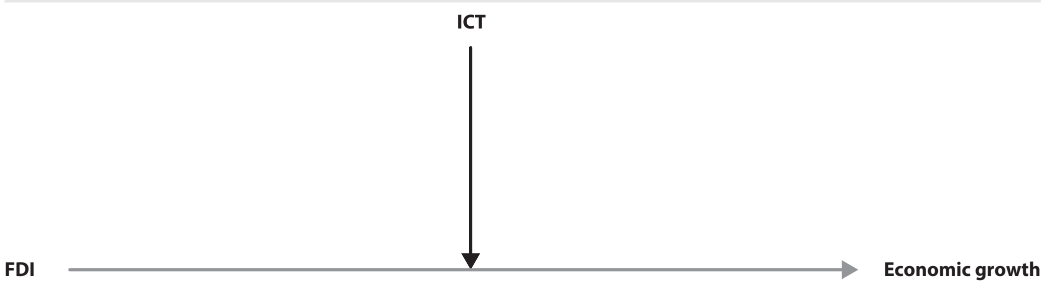
the more conducive the environment to generate profits. It may then be argued that a positive impact of FDI on economic growth may be conditional on the extent of the ICT use in the economy.

Similar findings have been reported by Gholami et al. (2010) who argue that the effects of ICT are not automatic and can vary depending on factors such as sector-specific implementation. Alfaro et al. (2004) asserted that FDI constituted a substantial determinant of economic growth across 20 countries belonging to the Organization for Economic Cooperation and Development (OECD). Dinh et al. (2019) reports significant role of FDI in stimulating long-term economic growth but found a negative impact in the short run. Similarly, Kallal et al. (2021) find empirical evidence supporting the positive long-term impact of ICT on Tunisia's economic growth, while attributing the observed negative short-run effect to the presence of investment bias.

Numerous studies have extensively explored the relationship between ICT and economic growth (Pohjola, 2001; Oulton, 2002; Kim et al., 2008; Toader et al., 2018). The potential consequences of ICT are diverse, making it a critical factor in promoting social development and economic advancement (Dimelis and Papaioannou, 2011). Notably, several studies emphasize the particular significance of ICT in fostering economic growth in developed countries (Nair et al., 2020; Kurniawati, 2020; Myovella et al., 2020).

Dewan and Kraemer (2000) contribute to the discourse by uncovering a statistically significant positive impact of ICT investment in West African Economic and Monetary Union (WAEMU) countries. Pohjola (2001) highlights the substantial role of ICT in driving economic activities and growth within developed countries. Kim et al. (2008) employs a knowledge management and resource-based perspective to analyse the influence of hardware, software, and internal spending as dimensions of IT investment on GDP. The study provides empirical evidence that supports the inconclusive nature of the relationship between the value of IT investment and GDP. Kurniawati (2020) highlights the potential of well-established ICT infrastructure to enhance economic growth in OECD economies. Myovella et al. (2020) provides empirical evidence demonstrating the positive impact of digitalization on economic growth in two distinct groups of countries, namely OECD (Organization for Economic Cooperation and Development) and SSA (Sub-Saharan Africa). Nair et al. (2020) highlights that in order to achieve sustained economic growth, policymakers in the OECD economies should implement an integrated framework that incorporates co-development policies related to R&D investment, ICT diffusion, and initiatives that enhance economic growth. However, despite the extensive body of research, the findings regarding the impact of ICT on economic growth have presented a complex picture, with studies yielding mixed results.

Figure 1 Proposed model



Source: Authors' compilation

It is widely acknowledged that the impact of FDI on economic growth is contingent upon various complementary factors. A holistic understanding of the interplay among FDI, and such factors is crucial for analysing the FDI-growth nexus (Benetrix et al., 2023). Gönel and Aksoy (2016) emphasizes the significance of considering additional mechanisms, such as technology-upgrading progress, among other factors implemented by the host country, in order to assess the potential positive impact of foreign direct investment (FDI) on economic growth. Similarly, Silajdzic and Mehic (2016) suggests that FDI has a positive impact on economic growth, particularly in technologically advanced transition economies. However, these studies emphasize that the relationship between FDI and economic growth is contingent upon various contextual factors and mechanisms adopted by the host country, or the presence of sufficient absorptive capacity within the host countries, suggesting that a comprehensive analysis incorporating these factors is crucial for an understanding of the FDI-growth nexus.

On the basis of literature reviewed, the current study aims to study the impact of the ICT development on the relationship between FDI and economic growth using a PCA constructed the ICT Index as displayed in Figure 1. The role of ICT in the growth FDI nexus is far from settled. This paper contributes to the literature by providing evidence based on the experience of the G20 nations, an economic grouping that represents a significant amount of global economic activity. It also contributes by constructing an index that provides a means to numerically evaluate the spread of ICT and digitalisation.

2 DATA AND METHODOLOGY

Given the importance and relevance of the G20 countries, it is imperative to have a dedicated index of ICT development of the member countries in order to analyse the performance of these countries vis-a-vis each other and how learnings from each other can help them perform better. In this regard, the next section discusses the relevant data sources and methodology adopted for constructing the IDI index for the G20 countries.

2.1 Variables and data sources

An ICT Development Index (IDI) for the G20 countries is constructed and used to rank them on the basis of their mean index score. Table 1(a) lists the six variables that are used to capture the three broad dimensions of ICT i.e., access, usage and capability. Table 1(b) lists the other variables taken for the study. The data used covers the G20 economies[9] over the period 2000–2019.

Table 1(a) Variable description and data sources for IDI construction

Dimensions	Description	Variables	Source
ICT access	Measures the readiness of network infrastructure and access to ICT	Mobile cellular subscription (per 100 people)	International Telecommunication Union (ITU) World Telecommunication/ ICT Indicators Database (26th Edition)
		Fixed telephonic subscription (per 100 people)	
ICT usage	Captures the extent to which ICTs are used in society, as well as the intensity with which they are used	Fixed broadband subscription (per 100 people)	
		Individuals using the internet (% of population)	
ICT capability	Identifies the competence or skills of efficient and effective ICT use as important input indicators	ICT service exports (% of service exports, BoP)	
		Research and development expenditure (% of GDP)	

Source: Author's compilation

9 South Korea has been excluded from analysis owing to data unavailability.

Table 1(b) Variable description and data sources

Variables		Description	Source
GDP	Gross Domestic Product	Annual percentage growth rate of GDP at market prices based on constant local currency	World Development Indicators, the World Bank
FDI	Foreign Direct Investment as a % of GDP	The sum of equity capital, reinvestment of earnings, other long-term capital, and short-term capital as shown in the balance of payments	
GFCF	Government Final Consumption Expenditure as a % of GDP	Includes all government current expenditures for purchases of goods and services (including compensation of employees	
Trade	Trade as a % of GDP	The sum of exports and imports of goods and services measured as a share of gross domestic product	
GFCE	Gross Fixed Capital Expenditure as a % of GDP	Includes land improvements (fences, ditches, drains, and so on); plant, machinery, and equipment purchases; and the construction of roads, railways, and the like, including schools, offices, hospitals, private residential dwellings, and commercial and industrial buildings	
EDU	Education Index	Component of the Human Development Index, measuring the educational attainment	UNDP's Human Development Report

Source: Author's compilation

2.2 Empirical methodology

This section is divided into two parts: the first part explains the methodology used to construct the ICT development index (IDI) while the second explains the panel data models employed to empirically understand the role of ICT development in explaining the FDI- growth relation. After the first step of compiling the data for the several variables to be used (explained in Section 2.1), the next step is to construct an ICT Development Index for the G20 economies. This is followed by a panel data analysis for assessing the role of ICT in FDI – Growth nexus. The following two subsections explain each of these steps in requisite detail.

2.2.1 ICT index construction

The study uses Principal Component Analysis (PCA) for constructing the IDI index. PCA is a widely used dimension reduction technique and recommended as the appropriate methodology for constructing composite indices (Asongu et al., 2018; Malik and Kaur, 2020). It merges standardized variable values and extracts vital information by reducing dimensionality, enhancing interpretability, and minimizing information loss (Abdi and Williams, 2019; Jolliffe and Cadima, 2016). Besides, utilizing this approach for constructing a composite index can be advocated as it is well-suited for highly correlated datasets (Asongu et al., 2018).

Before applying PCA to construct an index, the variables are standardized using Cumulative Distribution Functions (CDFs), where ranks are assigned to individual observations after organizing the dataset in ascending order such that $x_{[1]} \leq x_{[2]} \leq x_{[3]} \leq \ldots\ldots x_{[n]}$. The transformed data is represented by z_t such that:

$$z_t = \begin{cases} {}^{r}/_{n} \; for \; x_{[n]} \leq x_t, r = 1, 2, \ldots., n-1, \\ 1 \; for \; x_t \geq x_{[n]} \end{cases} \tag{1}$$

where: x_t, z_t, r, and n represent original series, transformed series, the assigned rank of x_t and the total observations respectively.

Bartlett's test of sphericity and Kaiser Meyer Olkin (KMO) test of sampling adequacy are applied to test for the suitability of the dataset for PCA. To enhance the orthogonality of sub-indicators and reveal a distinct loading pattern, the obtained components are subjected to varimax rotation (Issah and Antwi, 2017). Varimax rotation serves to simplify the representation of a specific sub-space by emphasizing only the major items within it. It is important to note that the actual coordinate system remains unchanged; instead, it's the orthogonal basis that undergoes rotation to align with those coordinates. Factors having the highest eigenvalues are chosen as PCs. Kaiser (1960) recommends considering PCs which have eigenvalues higher than one. Using the weights derived by the weighted average of eigenvalue with indicator loadings of each variable, the final index is constructed using the following equation.

Assignment of weights using PCA:

$$W_i = \sum_{i=1}^{n} |L_{ij}| E_j \,, \tag{2}$$

where: W_i is the weight of the i^{th} indicator; E_j is the eigenvalue of the j^{th} factor; L_{ij} is the loading value of the i^{th} unit of grouping on j^{th} factor; i and j represents indicators and PCs respectively.

After obtaining the weights, Index (I) is created as:

$$I = \frac{\sum_{i=1}^{n} z_i W_i}{\sum_{i=1}^{n} W_i} \,, \tag{3}$$

where: z_i and W_i represent the normalized value and the weight of the i^{th} indicator respectively.

2.2.2 Panel data analysis

The dataset used for the study is a panel spanning nineteen countries over the period 2000–2020, hence the need to employ the appropriate panel data model.

Consider the following equation that may be used to model the dataset available:

$$Y_{it} = \beta_1 + \sum_{j=2}^{k} \beta_j X_{jit} + \alpha_i + \varepsilon_{it} \,, \tag{4}$$

where: Y_{it} is the dependent variable for the ith cross-sectional unit for the time period t, X_{jit} is the jth independent variable for the same observation, α_i is the individual specific unobserved effect for the ith cross-sectional unit and ε_{it} is the error term. The study uses flow chart provided by Dougherty (2011) that summarizes the decision-making process for the choice of the appropriate panel data model. To check for the random effects for individual heterogeneity i.e., the presence of random effects, the study uses the Breusch–Pagan test. The Hausman test is then used to choose between the Random and Fixed effects models. In this study, Hausman test indicates that a fixed effects model is appropriate. In Fixed effects model, variables are expressed as mean-corrected values, and Ordinary Least Squares on this de-meaned equation is applied (Gujarati et al., 2019). The final equation estimated for fixed effects model is as below:

$$y_{it} = \sum_{j=2}^{k} \beta_j x_{jit} + u_{it} \,, \tag{4a}$$

where: the dependent, independent and the error terms represent the de-meaned values.

The IDI index constructed using the methodology described earlier is then used to examine empirically whether ICT can serve as a catalyst in the FDI growth relation. A static panel data analysis for G20 countries using data over the period 2000 to 2020 is conducted. The specification of the estimated model, Model 1 is given as:

$$GDP_{it} = \alpha_0 + \alpha_1 FDI_{it} + \alpha_2 IDI_{it} + \alpha_3 GFCE_{it} + \alpha_4 Trade_{it} + \alpha_5 GFCF_{it} + \alpha_6 EDU_{it} + \mu_i + \epsilon_{it}, \tag{5}$$

where: *GDP* is GDP growth; *FDI* is Foreign Direct Investment as a % of GDP; *IDI* is the ICT Development Index (IDI) constructed using PCA; *GFCE* is the Government Final Consumption Expenditure as a % of GDP; *Trade* has been taken as a % of GDP; $GFCF_{it}$ is the Gross Fixed Capital Expenditure as a % of GDP; *EDU* is Education Index.

We start the analysis by estimating the regression in Model 1. This model assesses the marginal impact of FDI on GDP after controlling for ICT and other factors. In line with the objective of the study, i.e., whether the role of FDI on GDP is conditional on the ICT index, we introduce an explanatory variable, FDIIDI, which is an interaction of FDI and the IDI. The same is represented in Model 2 as:

$$GDP_{it} = \alpha_{i0} + \alpha_1 FDI_{it} + \alpha_2 ICT_{it} + \alpha_3 GFCE_{it} + \alpha_4 Trade_{it} + \alpha_5 GFCF_{it} + \alpha_6 EDU_{it} + \alpha_7 FDIICT_{it} + \varepsilon_{it}. \tag{6}$$

The coefficient term α_7 measures the significance of the interaction term on the GDP growth. It signifies whether the interaction term of FDI and ICT amplifies or distorts the impact of FDI. Therefore, the conditional effect of FDI on growth can be calculated as:

$$\frac{\partial GDP}{\partial FDI} \; \alpha_1 + \alpha_7. \tag{7}$$

If $\alpha_1 < 0$ and $\alpha_7 > 0$, then it denotes that ICT reduces/ increases the impact of FDI on GDP growth.

3 RESULTS

After applying PCA to construct the index, the G20 economies have been ranked on the basis of the mean index score calculated in the process. Table 2 gives an overview of the index scores of the G20 countries along with their mean index score values. This is followed by ranking of countries based on their mean index score.

While most of the extant literature on determinants of GDP have concluded a positive relation between FDI and growth (Abbes et al., 2015; Soylu et al., 2023, for instance), some have also established the reasons for a negative impact (Susilo, 2018; Dinh et al., 2019, for instance). FDI slows down growth especially at the initial stages but shows a positive effect in the long run (Dinh et al., 2019). The authors believe that to negate the initial effect and to make FDI enable growth, it's important to interact it with other significant variables like ICT index. The empirical results in this study conform to the results in the vast literature on the nexus between FDI and growth.

The results of the panel data analysis are discussed below. Both Bruesch Pagan test and the Hausmann test give results in favour of the Fixed Effects model. We can see from Table 3 that the coefficient of variable FDI is negative, which implies that a percentage increase in FDI leads to a .04% decrease in the growth of GDP. However, the coefficient is insignificant. However, in the model with interaction, see Table 4, both FDI variable and the interaction term become significant. This calls for increasing FDI in the ICT sector to increase the conditional effect on growth.

The conditional effect of FDI can be written as:

$$\frac{\partial GDP}{\partial FDI} \; -.39 + .67.$$

Table 2 Estimated Mean Index Score based on PCA

Year	Indonesia	Mexico	India	South Africa	Turkey	Saudi Arabia	Brazil	China	Argentina	Russia	Australia	Italy	Japan	United States	Canada	European Union	UK	France	Germany
2000	0.03	0.19	0.21	0.11	0.16	0.08	0.15	0.12	0.2	0.2	0.44	0.39	0.45	0.51	0.57	0.41	0.45	0.47	0.5
2001	0.03	0.17	0.21	0.13	0.17	0.09	0.21	0.12	0.23	0.23	0.44	0.46	0.48	0.54	0.61	0.45	0.47	0.52	0.56
2002	0.06	0.17	0.22	0.14	0.18	0.1	0.21	0.16	0.23	0.24	0.49	0.44	0.5	0.55	0.61	0.48	0.52	0.57	0.59
2003	0.1	0.17	0.22	0.15	0.19	0.12	0.27	0.23	0.24	0.27	0.51	0.49	0.5	0.57	0.63	0.51	0.57	0.58	0.62
2004	0.15	0.19	0.23	0.17	0.22	0.13	0.26	0.27	0.27	0.29	0.54	0.51	0.52	0.58	0.62	0.53	0.61	0.59	0.65
2012	0.31	0.29	0.31	0.36	0.34	0.48	0.46	0.49	0.59	0.58	0.65	0.66	0.68	0.66	0.71	0.75	0.74	0.76	0.84
2013	0.32	0.28	0.31	0.39	0.36	0.47	0.49	0.52	0.61	0.6	0.66	0.67	0.71	0.66	0.69	0.76	0.75	0.76	0.85
2014	0.33	0.27	0.32	0.4	0.36	0.48	0.53	0.54	0.6	0.62	0.65	0.67	0.73	0.68	0.68	0.77	0.76	0.78	0.83
2015	0.33	0.31	0.32	0.42	0.36	0.48	0.53	0.57	0.61	0.64	0.64	0.67	0.74	0.7	0.68	0.76	0.77	0.77	0.83
2016	0.35	0.31	0.34	0.41	0.38	0.47	0.52	0.58	0.62	0.64	0.65	0.67	0.76	0.74	0.68	0.77	0.77	0.77	0.86
2017	0.37	0.32	0.34	0.43	0.39	0.48	0.51	0.6	0.62	0.66	0.65	0.67	0.77	0.75	0.69	0.78	0.76	0.77	0.87
2018	0.31	0.33	0.34	0.43	0.42	0.56	0.53	0.63	0.63	0.65	0.67	0.69	0.78	0.76	0.7	0.79	0.75	0.79	0.87
2019	0.35	0.35	0.36	0.45	0.42	0.56	0.52	0.66	0.6	0.66	0.67	0.69	0.8	0.77	0.71	0.8	0.75	0.78	0.86
Mean Index Score	0.25	0.26	0.29	0.3	0.32	0.35	0.38	0.42	0.47	0.5	0.6	0.61	0.64	0.65	0.67	0.67	0.68	0.69	0.77
Rank based on Mean Index Score	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

Source: The authors

Table 3 Fixed-Effects Model

GDP growth	Coefficients	S.E.	t-value	p-value	[95% conf.	interval]	Significance
FDI	–.044	.087	–0.51	.61	–.216	.127	
ICT	–5.3	2.102	–2.52	.012	–9.435	–1.164	**
GFCE	–.609	.134	–4.55	0	–.872	–.345	***
Trade	.072	.021	3.49	.001	.031	.113	***
EDU	.154	4.684	0.03	.974	–9.06	9.368	
GFCF	.256	.066	3.87	0	.126	.386	***
Constant	6.495	3.689	1.76	.079	–.762	13.751	*
Model statistics							
Mean dependent var	3.097		**SD dependent var**		3.401		
R-squared	0.216		**Number of obs.**		359		
F-test	15.421		**Prob > F**		0.000		
Akaike crit. (AIC)	1 651.466		**Bayesian crit. (BIC)**		1 678.650		

Note: *** p<.01, ** p<.05, * p<.1.
Source: Authors' compilation

Table 4 Interaction Model

GDP growth	Coefficients	S.E.	t-value	p-value	[95% conf.	interval]	Significance
FDI	–.393	.221	–1.78	.077	–.828	.043	*
ICT	–6.336	2.182	–2.90	.004	–10.628	–2.044	***
GFCE	–.623	.134	–4.66	0	–.886	–.36	***
Trade	.074	.021	3.61	0	.034	.115	***
EDU	–.411	4.682	–0.09	.93	–9.621	8.8	
GFCF	.278	.067	4.15	0	.146	.411	***
FDIICT	.674	.394	1.71	.088	–.101	1.449	*
Constant	7.052	3.693	1.91	.057	–.212	14.316	*
Model statistics							
R-squared	0.223		**Observations**		359		
F-test	13.713		**Prob > F**		0.000		
Akaike crit. (AIC)	1 650.332		**Bayesian crit. (BIC)**		1 681.399		

Note: *** p<.01, ** p<.05, * p<.1.
Source: Authors' compilation

The results are robust by amplifying the significance of ICT index on FDI in its effect on growth. The interaction has a positive effect on growth i.e., effect of FDI on growth is conditional on the magnitude of the interaction term.

We expect all coefficients to be positive. i.e., FDI, ICT, Trade, Expenditures, Investment, Education to boost economic growth. Even though several studies have exhibited a positive relation between FDI and growth, a negative relation is also a potential outcome (Abbes et al., 2015).

DISCUSSION AND CONCLUSION

The study attempts to assess the dynamic nature of the FDI-GDP growth nexus, emphasizing the role of ICT as a contextual determinant. In terms of the mean IDI score values, Germany has the highest mean score of 0.77 among all the G20 nations. Germany is a leading force in innovation, demonstrated by its top ranking in the World Economic Forum's 2018 Global Competitiveness Report. The country's emphasis on ICT is evident through the government's prioritization of this sector in the BMWi's Digital Agenda. Germany boasts one of Europe's largest ICT industries, with a significant software market and a substantial number of IT businesses and workers. With the mean IDI score of 0.69, the digital divide in France is closing as more households acquire computers and internet connections. The ICT market in France reached US$112.07 billion in 2021 and is projected to grow at a CAGR of 7.3% to US$159.25 billion by 2026.[10] France's strong ICT usage is recognized in the Global Innovation Index 2021, where it ranks 10th. The French government is fully committed to the ICT growth, exemplified by the strategic implementation of the French National Plan for Digital Inclusion (September 2018), promoting digital transformation in businesses and the establishment of a secure, people-centric digital society. UK retains its position as one of the world's largest ICT markets, ranking second in ICT spending per capita.[11] With a digital technology turnover of over $240 billion in 2018, the UK houses approximately 100 000 software companies and serves as the top destination for U.S. ICT businesses in Europe.

The EU and Canada both have a mean index score of 0.67. In the past four years, the EU has experienced significant growth in various aspects of the ICT infrastructure, access, and usage, leading globally in terms of internet access, with an estimated 85% of households having access in 2019, compared to the global average of 57.4%.[12] Similarly, Canada's technology sector serves as a major economic driver, surpassing much of the country's overall economy. In recent years, Canada has become a hub for tech entrepreneurship, laying the foundation for the ICT expansion. The United States, with a mean IDI score of 0.65, has the most advanced software and IT services industry in the world. The industry accounts for $1.8 trillion of U.S. value-added GDP (more than 10% of the national economy) and 11.8 million jobs.

Next is Japan with a mean IDI score of 0.64. Japan's ICT market, which has developed through the spread of telecommunications services and the advancement of telecommunications networks, holds a 6.4% share of the global market.[13] The ICT sector is playing an increasingly strategic role in Italy, which has the mean IDI score of 0.61, as it now provides fundamental contributions to all other sectors of the economy. R&D spending by Italian ICT companies reached $2.29 billion in 2017, 7.5% more than in 2016 and amounting to 10.6% of total R&D spending across all sectors. With a mean IDI score of 0.60, Australia serves as a strategic site for various ICT activities, attracting global and regional attention due to its robust research infrastructure, skilled workforce, and technology-driven clientele. It has seen notable examples of renowned companies viz. Avaya, Canon, and IBM leveraging Australia's ICT resources. While Russia aims to control internet content, its government is also attempting to improve the use of modern technologies. The ICT regulatory framework has shown significant development with the adoption of various laws, adoption of the Telemedicine Law (2018), the Law on Critical Infrastructure (2017), the Online Cash Register Law (2017), amendments to the Public-Private Partnerships Law (2018).

Argentina, China, Brazil, Saudi Arabia, Turkey, South Africa, India, Mexico and Indonesia have mean IDI scores less than 0.50. Argentina's long-term growth performance remained susceptible to macro-fiscal crises. Inadequate human capital, difficulty in obtaining financing for innovation, particularly for start-ups, weak links between relevant players, and insufficient incentives for public research and technology

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[10] Source: Global Data – France ICT Market Size and Forecast (by IT Solution Area, Size Band and Vertical), 2022–2026.
[11] Source: International Trade Administration – United Kingdom Information and Communications Technology.
[12] Source: ITU, Based on ITU WTI Database.
[13] Source: Japan External Trade Organization, Digitalization of society brought about by 5G and Beyond 5G, based on data from Statista.

institutions have been additional barriers to innovation. The landscape of the ICT policy in China is not easy. China's approach to the issue balances on concerns of commerce and national social security. Furthermore, although the protection of intellectual property has improved over time, piracy remains a major issue. Brazil has been recently making progress in the right direction. The Brazilian Information and Communication Technology market (ICT) was valued at US$ 49.5 billion in 2020. It has implemented the Brazil More Digital (Brasil Mais Digital), an online education initiative. Saudi Arabia is just as keen as the other nations to use IT to reap larger benefits. Saudi Arabia's transition to an informational society, however, is being hampered by a lack of knowledge, time, and trust in new systems. Regulations in Turkey on social media platforms, a new tax on digital services, and requirements for local content make it more challenging for foreign businesses to operate there. The economy has, nevertheless, exhibited developments recently. South Africa has demonstrated technological superiority in the areas of mobile software, security software, and online banking services in the past decade and has shown a tremendous growth. Using ICT to promote socioeconomic fairness and inclusion, boosting competitiveness and preparing the country for the digital industrial revolution (fourth industrial revolution) is at the core of the priorities of the South African government. The bold ICT vision of India is constantly articulated and persuasively outlined in Rebooting India. India has shown a consistent growth over the years in terms of ICT development. Mexico's ICT sector has seen increased competition and investment since 2013's landmark regulatory reform, which created the Federal Institute of Telecommunications (IFT). While Mexico has advanced telecommunications regulation in Latin America according to ITU regulatory tracker, it lacks a clear roadmap for ICT public policy or a national digital strategy. Despite encouraging developments in Internet usage, Indonesia still has significant difficulties, such as an unequal population distribution.

The results of the panel data analysis carry noteworthy implications for policy development, with a specific emphasis on the interaction between ICT and FDI. In the Fixed-Effects Model without interaction terms, the non-significant coefficient of FDI suggests that, on average, FDI alone does not exert a statistically significant impact on the GDP growth. However, the unexpected negative sign introduces a new dimension – an incremental percentage increase in FDI coincides with a 0.04% reduction in the GDP growth, though this is not statistically significant. Of significance is the conspicuously important negative coefficient associated with the ICT variable, indicating a noteworthy adverse effect on the GDP growth.

The results become more intricate in the model incorporating interaction terms. The emergence of a statistically significant negative coefficient for FDI within the context of its interaction with ICT (FDIICT) introduces a paradoxical scenario – an increased FDI correlates with a 0.39% decrement in GDP growth. This apparent contradiction challenges established paradigms and underscores the conditional nature of FDI's impact contingent upon the developmental stage of ICT. Contrarily, the positive and significant coefficient affiliated with the interaction term (FDIICT) suggests that the amalgamation of FDI and ICT engenders a positive effect on the GDP growth. This provides empirical credence to the proposition that directing attention toward the ICT sector to attract FDI may amplify its constructive repercussions on economic growth.

The anticipation of positive coefficients across all variables, including FDI, ICT, Trade, Expenditures, Investment, and Education, aligns with prevailing economic theories emphasizing their favourable contributions to economic growth. However, the unique findings of this study, particularly the conditional nature of FDI's impact in the presence of ICT, emphasize the exigency for context-specific policy considerations. The ostensibly discordant negative correlation between FDI and the GDP growth serves as a poignant reminder that the influence of FDI is contingent upon a multitude of contextual factors.

In order to realize the objective of empirical analysis of the role of ICT in the economic growth-FDI nexus, the study constructs an ICT index. While the advantage of constructing the index lies in a comprehensive covering of the various facets of ICT development, it may be a matter of concern that the principal components constructed lack economic interpretability. Also, while panel data controls for any spurious relationships between ICT and growth, it might not account for all variables that vary across time for different countries. The analysis does not deal with plausible endogeneity issue and hence is a limitation of this study. Exploring other econometric techniques that overcome these limitations are suggested as avenues for future research.

In conclusion, these findings advocate for a discerningly targeted policy paradigm. Governments, in formulating strategies, should contemplate substantial investments in ICT infrastructure and the creation of an environment conducive to FDI within the ICT sector. Such an approach is poised to harness the latent synergies between FDI and ICT for optimal economic growth. This study thus challenges reductionist interpretations of the FDI-GDP growth nexus, accentuating the indispensability of bespoke policies attuned to the specific conditions characterizing individual countries. As the G20 nations continue to shape the global economic landscape, understanding and harnessing the potential of ICT development becomes imperative for sustainable and inclusive growth.

References

ABBES, S. M., BELMOKADDEM, M., SEGHIR, G., ZAKARYA, G. Y. (2015). Causal Interactions between FDI, and Economic Growth: Evidence from Dynamic Panel Co-integration [online]. *Procedia, Economics and Finance*, 23: 276–290. <https://doi.org/10.1016/s2212-5671(15)00541-9>.

ALFARO, L., CHANDA, A., KALEMLI-OZCAN, S., SAYEK, S. (2004). FDI and economic growth: the role of local financial markets [online]. *Journal of International Economics*, 64(1): 89–112. <https://doi.org/10.1016/s0022-1996(03)00081-3>.

ASONGU, S. A., EFOBI, U., TCHAMYOU, V. S. (2018). Globalisation and governance in Africa: a critical contribution to the empirics [online]. *International Journal of Development Issues*, 17(1): 2–27. <https://doi.org/10.1108/ijdi-04-2017-0038>.

ASONGU, S. A., ROUX, S. L. (2017). Enhancing ICT for inclusive human development in Sub-Saharan Africa [online]. *Technological Forecasting and Social Change*, 118: 44–54. <https://doi.org/10.1016/j.techfore.2017.01.026>.

BENETRIX, A., PALLAN, H., PANIZZA, U. (2023). *The elusive link between FDI and economic growth* [online]. World Bank Blogs. <https://blogs.worldbank.org/developmenttalk/elusive-link-between-fdi-and-economic-growth#:~:text=While%20a%20few%20papers%20exist,between%20FDI%20and%20economic%20growth>.

BENZAIM, S., FTITI, Z., KHEDHAOURIA, A., DJERMANE, R. (2023). US foreign investments: Technology transfer, relative backwardness, and the productivity growth of host countries [online]. *The Quarterly Review of Economics and Finance*, 87: 275–295. <https://doi.org/10.1016/j.qref.2021.03.012>.

DEWAN, S., KRAEMER, K. L. (2000). Information Technology and Productivity: Evidence from Country-Level Data [online]. *Management Science*, 46(4): 548–562. <https://doi.org/10.1287/mnsc.46.4.548.12057>.

DIMELIS, S. P., PAPAIOANNOU, S. K. (2011). ICT growth effects at the industry level: A comparison between the US and the EU [online]. *Information Economics and Policy*, 23(1): 37–50. <https://doi.org/10.1016/j.infoecopol.2010.03.004>.

DINH, T., VO, D. H., NGUYEN, T. T. (2019). Foreign Direct Investment and Economic Growth in the Short Run and Long Run: Empirical Evidence from Developing Countries [online]. *Journal of Risk and Financial Management*, 12(4): 176. <https://doi.org/10.3390/jrfm12040176>.

DOUGHERTY, C. (2011). Introduction to Econometrics [online]. <https://ideas.repec.org/b/oxp/obooks/9780199567089.html>.

GHOLAMI, R., HIGÓN, D. A., HANAFIZADEH, P., EMROUZNEJAD, A. (2010). Is ICT the key to development? [online]. *Journal of Global Information Management*, 18(1): 66–83. <https://doi.org/10.4018/jgim.2010091104>.

GÖNEL, F., AKSOY, T. (2016). Revisiting FDI-led growth hypothesis: the role of sector characteristics [online]. *Journal of International Trade & Economic Development*, 25(8): 1144–1166. <https://doi.org/10.1080/09638199.2016.1195431>.

GUJARATI, D., PORTER, D., PAL, M. (2019). *Basic Econometrics*. Mc. Graw Hill.

HANNA, N. K. (2009). Government Transformation: tools and challenges [online]. *Innovation, technology and knowledge management*, 281–309. <https://doi.org/10.1007/978-1-4419-1185-8_12>.

ISSAH, M., ANTWI, S. (2017). Role of macroeconomic variables on firms' performance: Evidence from the UK [online]. *Cogent Economics & Finance*, 5(1): 1405581. <https://doi.org/10.1080/23322039.2017.1405581>.

KAISER, H. F. (1960). The application of electronic computers to factor analysis [online]. *Educational and Psychological Measurement*, 20(1): 141–151. <https://doi.org/10.1177/001316446002000116>.

KALLAL, R., HADDAJI, A., FTITI, Z. (2021). ICT diffusion and economic growth: Evidence from the sectorial analysis of a periphery country [online]. *Technological Forecasting and Social Change*, 162: 120403. <https://doi.org/10.1016/j.techfore.2020.120403>.

KIM, Y. J., KANG, H., SANDERS, G., LEE, S. T. (2008). Differential effects of IT investments: Complementarity and effect of GDP level [online]. *International Journal of Information Management*, 28(6): 508–516. <https://doi.org/10.1016/j.ijinfomgt.2008.01.003>.

KIRKMAN, G. S., CORNELIUS, P. K., SACHS, J. D., SCHWAB, K. (2002). The Global Information Technology Report 2001–2002. Readiness for the Networked World [online]. *World Economic Forum*, Oxford University Press.

KURNIAWATI, M. A. (2020). ICT infrastructure, innovation development and economic growth: a comparative evidence between two decades in OECD countries [online]. *International Journal of Social Economics*, 48(1): 141–158. <https://doi.org/10.1108/ijse-05-2020-0321>.

LI, X., LIU, X. (2005). Foreign Direct Investment and Economic Growth: An Increasingly Endogenous Relationship [online]. *World Development*, 33(3): 393–407. <https://doi.org/10.1016/j.worlddev.2004.11.001>.

MALIK, S., KAUR, S. (2020). Multi-dimensional public–private partnership readiness index: a sub-national analysis of India [online]. *Transforming Government: People, Process and Policy* (ahead-of-print). <https://doi.org/10.1108/tg-06-2020-0107>.

MYOVELLA, G. A., KARAÇUKA, M., HAUCAP, J. (2020). Digitalization and economic growth: A comparative analysis of Sub-Saharan Africa and OECD economies [online]. *Telecommunications Policy*, 44(2): 101856. <https://doi.org/10.1016/j.telpol.2019.101856>.

NAIR, M., PRADHAN, R. P., ARVIN, M. B. (2020). Endogenous dynamics between R&D, ICT and economic growth: Empirical evidence from the OECD countries [online]. *Technology in Society*, 62: 101315. <https://doi.org/10.1016/j.techsoc.2020.101315>.

OULTON, N. (2002). ICT and Productivity Growth in the United Kingdom [online]. *Oxford Review of Economic Policy*, 18(3): 363–379. <https://doi.org/10.1093/oxrep/18.3.363>.

POHJOLA, M. (2000). Information Technology and Economic Growth: A Cross-Country Analysis [online]. *AgEcon Search*, WP No. 173. <https://doi.org/10.22004/ag.econ.295500>.

SAITH, A., VIJAYABASKAR, M. (2005). ICTs and Indian economic development: Economy, work, regulation [online]. *SAGE Publications eBooks.* <http://ci.nii.ac.jp/ncid/BA73535964>.

SEN, A. (2010). The Mobile and the World. *Information Technologies and International Development*, 6th spec. Ed.

SHIMUL, S. N. (2009). *An examination of FDI and growth nexus in Bangladesh: engle granger and bound testing cointegration approach* [online]. <http://hdl.handle.net/10361/455>.

SILAJDZIC, S., MEHIC, E. (2015). Absorptive Capabilities, FDI, and Economic Growth in Transition Economies [online]. *Emerging Markets Finance and Trade*, 52(4): 904–922. <https://doi.org/10.1080/1540496x.2015.1056000>.

SOYLU, Ö. B., ADELEYE, B. N., OKUR, F., KÖNEL, M. E. (2023). Tourism and FDI-Growth nexus in Upper-Middle income countries [online]. *Tourism*, 71(1): 162–177. <https://doi.org/10.37741/t.71.1.10>.

SUNDE, T. (2017). Foreign direct investment, exports and economic growth: ADRL and causality analysis for South Africa [online]. *Research in International Business and Finance*, 41: 434–444. <https://doi.org/10.1016/j.ribaf.2017.04.035>.

TABASSUM, N., AHMED, S. U. (2014). Foreign Direct Investment and Economic Growth: Evidence from Bangladesh [online]. *International Journal of Economics and Finance*, 6(9). <https://doi.org/10.5539/ijef.v6n9p117>.

TOADER, E., FIRTESCU, B., ROMAN, A., ANTON, S. G. (2018). Impact of Information and Communication Technology Infrastructure on Economic Growth: An Empirical Assessment for the EU Countries [online]. *Sustainability*, 10(10): 3750. <https://doi.org/10.3390/su10103750>.

UNESCO INSTITUTE FOR STATISTICS. (2009). *Guide to measuring information and communication technologies (ICT) in Education* [online]. UNESCO Digital Library (UIS/TD/09-04). <https://unesdoc.unesco.org/ark:/48223/pf0000186547>.

WORLD BANK GROUP. (2012). *Information and Communication Technology (ICT) for greater development impact: World Bank Group strategy for ICT* [online]. Board Report 73236, The World Bank Group. <http://documents.worldbank.org/curated/en/105121468149370524/Information-and-Communication-Technology-ICT-for-greater-development-impact-World-Bank-Group-strategy-for-ICT>.

YAKUNINA, R., BYCHKOV, G. (2015). Correlation analysis of the components of the Human Development Index across countries [online]. *Procedia. Economics and Finance*, 24: 766–771. <https://doi.org/10.1016/s2212-5671(15)00692-9>.

APPENDIX

Table A1 Hausman test results

	Coefficients			
	(b) Fixed	**(B) Random**	**(b-B) Difference**	**sqrt(diag(V_b-V_B)) S.E.**
Foreigndir~i	–.0444572	.0852562	–.1297134	.0326429
ICTINDEX	–5.299693	–6.46908	1.169387	1.112262
Generalgov~p	–.6085715	–.1958342	–.4127373	.115851
TradeofGDP	.0719963	.0473668	.0246296	.0161303
Educationi~x	.1541575	3.024796	–2.870639	3.575626
Grossfixed~n	.2557294	.2290673	.0266621	.0538752
b = consistent under Ho and Ha; obtained from xtreg				
B = inconsistent under Ha, efficient under Ho; obtained from xtreg				
Test: Ho: difference in coefficients not systematic				
chi2(6) = (b-B)'[(V_b-V_B)^(-1)](b-B) = 42.72 Prob>chi2 = 0.0000				

Source: Authors' compilation

K-Medoids and Support Vector Machine in Predicting the Level of Building Damage in Earthquake Insurance Modeling

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Abstract

Yogyakarta, an Indonesian province prone to earthquakes, frequently suffers extensive damage to buildings, necessitating insurance coverage to mitigate potential losses. This study aims to forecast earthquake insurance premiums by predicting building damage levels resulting from earthquakes. Utilizing data from buildings affected by the June 30, 2023, earthquake in Yogyakarta, we employ K-Medoids Clustering and Support Vector Machine (SVM) to predict two categories of building damage: minor (labelled as 1) and heavy (labelled as 2). The total premiums for minor damage range from approximately USD 86.55 to USD 288.50, while for heavy damage, they range from USD 120.05 to USD 400.18 using the K-Medoids algorithm. Meanwhile, premiums for minor damage range from USD 83.14 to USD 277.13, and for heavy damage, they range from USD 223.67 to USD 745.55 using the SVM algorithm.

Keywords	DOI	JEL code
Clustering, disaster, earthquake, Yogyakarta, insurance, premium	*https://doi.org/10.54694/stat.2024.13*	*C10, C38, C55, G22*

INTRODUCTION

Disasters are events that occur suddenly and unpredictably resulting in great losses to human life and the environment. Disasters can occur naturally or because of human activity (Makwana, 2019). Disasters that occur naturally include tornadoes, landslides, earthquakes, tsunamis, and erupting mountains. While disasters caused by humans include floods, pollution, and leakage of factory waste. According to McFarlane et al. (2006), disasters are phenomena that can cause trauma to individuals, starting from critical and time-limited conditions and occurring because of nature, technology, and even humans. Indonesia is not spared from natural disasters that threaten such as earthquakes. There are many reasons why Indonesia often experiences earthquakes. According to the United States Geological Survey (USGS),

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most earthquakes and volcanic eruptions do not occur randomly, but occur in certain regions, namely the Pacific ring of fire. This area is a confluence between the Pacific plate and various tectonic plates around it, so this area is a very seismically and volcanically active. The fact that Indonesia has a complex and active tectonic arrangement and has a geographical location at the confluence of four tectonic plates supports that Indonesia will not be separated from earthquakes. These tectonic plates include the Pacific Plate, Eurasian Plate, Philippine Sea Plate, and Indo-Australian Plate (National Earthquake Study Center (Indonesia), & Research and Development Center for Housing and Settlements (Indonesia), 2021).

According to Meteorology Climatology and Geophysics Council of Indonesia (BMKG) in 2022, data states that within three years earthquakes in Indonesia have increased. A total of 8 264 earthquakes in 2020, increased in 2021 to 10 519 earthquakes and continued to increase until 2022 to 10 792 earthquakes. The location of this earthquake spread across all provinces in Indonesia from Sabang to Merauke. Of all provinces in Indonesia, Yogyakarta is one of the provinces with a high frequency of earthquakes. In 2006, precisely on May 26, a large earthquake measuring 6.3 on the Richter scale shook Yogyakarta which had a depth of 11.3 km. This earthquake resulted in a lot of losses and casualties. Recently, there has been another major earthquake in Yogyakarta on June 30, 2023. According to the report of the Head of the Regional Disaster Management Agency (BPBD) of the Special Region of Yogyakarta, a tectonic earthquake occurred at 19.57 Indonesian Time with the center located in the South Indian Ocean of Java with a depth of 67 km. All kinds of natural disasters certainly cause losses both material and non-material. This study specifically discusses the earthquake that occurred in Yogyakarta on June 30, 2023, and its impact, especially on buildings. Based on this information, one way to minimize the risk of loss from building damage caused by an earthquake is the use of insurance. Referring to Article 246 of the Commercial Law Code (KUHD) states that insurance is a contract in which the insurer promises to the insured through the collection of premiums, to compensate for losses, damages, and even loss of profits caused by an uncertain event (Santri, 2017). One part of insurance is loss insurance which contains disaster insurance. At present, seismic risk is a pressing issue for both public authorities and private organizations due to the potential for numerous fatalities and considerable economic losses resulting from a seismic event (Hofer et al., 2022).From 2000 to 2016, the average direct economic loss in the form of damage to buildings and supporting objects caused by natural disasters that occurred in Indonesia reached around IDR 22.8×10^{12} or USD 1 414 585 096.80 and the possibility of losses due to natural disasters will increase in the future if efforts to reduce, prepare, and transfer risks are not carried out (Fiscal Policy Agency, Ministry of Finance of the Republic of Indonesia, 2018). Risk reduction, setup, and transfer efforts can be run through insurance. Therefore, a good disaster insurance model, especially earthquake insurance is needed in Indonesia.

The purpose of this research is to provide an overview of the calculation of earthquake insurance premiums that result in material losses, especially for affected buildings, by employing k-medoids and Support Vector Machine in determining the level of building damage. In this study, we simulate the calculation of premiums that must be paid by customers to the insurer regarding loss insurance from earthquakes based on actual data from the Regional Disaster Management Agency of Bantul, Yogyakarta, which includes variables such as impact, urban village, latitude, longitude, damage, and Peak Ground Acceleration (PGA). Premium calculation simulation is carried out by predicting the level of damage to buildings due to earthquakes in advance. There are many statistical methods that can be used in predicting data, one of which is k-medoids and *Support vector machine* (SVM). K-medoids is the process of grouping data into certain classes that have the same characteristics, while SVM is one of the statistical methods that is usually used to classify and predict by finding hyperplane or a delimiter to separate two sets of data from two different classes (Octaviani et al., 2014). The k-medoids and SVM methods have differences. In the k-medoids method, there is no need for class labels, but in SVM there are class labels that are used to build models in prediction process. This study compares the two methods for classification of the level of building damage. We then simulate the calculation of premiums caused by the level of damage resulting from each method.

1 REVIEW OF LITERATURE

The 2018 study by Agustian Noor titled "Comparison of Ordinary Support Vector Machine and Particle Swarm Optimization-Based Support Vector Machine Algorithms for Earthquake Prediction" utilized earthquake data from South Sumatra spanning from 2014 to 2020. The research aimed to analyze earthquake occurrences in North Sumatra by comparing the SVM and SVM-PSO methods, with their performance measured using Root Mean Square Error (RMSE). The results of this study showed that the RMSE value of SVM was 9.720, which was lower compared to the RMSE value of SVM-PSO, which was 37.685 (Noor, 2018).

The 2018 study by Devni Prima Sari et al. titled "Application of Bayesian Network Model in Determining the Risk of Building Damage Caused by Earthquakes" utilized variables from data on building damage caused by the 2009 earthquake in West Sumatra. The data included three independent variables: building structure, PGA (Peak Ground Acceleration), and soil type, as well as one dependent variable: the level of building damage. This research aimed to minimize potential building losses due to earthquakes by predicting the likelihood of building damage at a specific location using a Bayesian network model. The results of this study indicated a 33% probability for light and severe building damage and a 34% probability for moderate building damage, with an accuracy rate of 66% (Wibowo & Institute of Electrical and Electronics Engineers, n.d.).

Previous research in 2019 by Devni Prima Sari et al., titled "K-means and Bayesian Networks to Determine Building Damage Levels," used 7 variables consisting of 4 dependent variables including construction, landslide risk, PGA, and damage, and 3 independent variables including close to faults, slope, and epicenter distance from building unit data of the 2009 West Sumatra earthquake. This research aimed to determine the level of building damage due to earthquakes. The results of this study showed that the levels of light, moderate, and high building damage were 35.46%, 35.14%, and 29.4%, respectively, with an accuracy rate of 70% (Sari et al., 2019).

Research in 2019 by Mariana Yusoff et al., titled "Hybrid backpropagation neural network-particle swarm optimization for seismic damage building prediction," used data obtained from IDARC-2D software from 35 buildings across Malaysia, including 1-story to 35-story buildings. This study used 7 variables including age, number of bays, height, length of seismic zone, natural period, ground acceleration, and building damage index. This research aimed to predict earthquake damage to buildings using hybrid backpropagation neural network and particle swarm optimization (BPNN-PSO). The results of this study showed that BPNN-PSO demonstrated better results with an accuracy of 89% compared to backpropagation neural network with only 84% (Yusoff et al., 2019).

2 METHODS

Cluster analysis or group analysis is one of the statistical research methods that help us to easily classify a set of objects based on information from data into different small clusters. However, objects in each cluster have an affinity or similar characteristics. The groups formed have high internal homogeneity and high external heterogeneity. It can also be interpreted that group analysis maximizes distance between objects and minimizes similarities between groups. Each object is classified according to the distance or proximity of objects to one another, while each variable is classified according to the size of its correlation (Harnanto et al., 2017). This means that grouping is done based on the proximity of the distance between data and the correlation between data variables. High variable correlation allows data to be of the same class. But the absence of correlation between variables will increase the results of grouping. The definition of cluster in data mining is a grouping of several data or objects from clusters (groups) so that each cluster contains information that is as similar as possible and different from objects in other clusters. In general, clustering methods are divided into two types, namely hierarchical and non-hierarchical ones (Sahriman et al., 2019). Clustering algorithms aim to identify groups of objects that are similar based on their attribute values (Harikumar and Surya, 2015). This study used K-Medoids or can be called Partitioning Around Medoids (PAM), that is a method of grouping n partition objects into k clusters. This grouping uses an object in a set of objects that can represent the cluster. These objects are called medoids which are centrally located in a cluster that is formed. Cluster formation is done by calculating the proximity between medoids objects and non-medoids objects (Musfiani, 2019). K-medoids is a clustering algorithm like k-means, but it is more tolerant to outliers. The main idea of k-medoids is to identify the center of a cluster using k clusters generated randomly (Mohemad et al., 2022). The algorithm of k-medoids is as follows (Mohemad et al., 2022):

1. Determine the number of clusters.
2. Determine k cluster centers randomly.
3. Calculate the distance of each object to the nearest cluster using the gower distance method,

$$d(x_i, x_j) = \frac{\sum_{k=1}^{p} w_k \cdot d_k(x_i, x_j)}{\sum_{k=1}^{p} w_k}, \tag{1}$$

where $d(x_i, x_j)$ is the distance between two objects x_i and x_j, p is the number of variables, w_k is the weight for each variable k, d_k (x_i, x_j) is the distance between two objects x_i dan x_j for variable k.

4. Randomly select non medoid objects in each cluster as new medoid members.
5. Calculate the distance of each non medoid object to the new medoids and assign each non medoid object to the nearest medoid member, then calculate the total distance.
6. Calculate the total deviation *S*, if the new total deviation is less than the old total deviation, change the position of the new medoid, then make it the new medoid.
7. Repeat steps 4–6 until the medoid does not change (Hermansyah et al., 2024).

After k-medoids are clustered, the results are used as labels in Support Vector Machine (SVM) analysis. SVM is one of the learning methods in machine learning (Wang et al., 2024). Machine learning utilizes past data to build models for predicting future data. Learning, an essential component of artificial intelligence, encompasses diverse statistical, probabilistic, and optimization techniques like logistic regression, artificial neural networks (ANN), K-nearest neighbor (KNN), decision trees (DT), and Naive Bayes (Huang et al., 2018). SVM known for their computational power in supervised learning, are extensively employed in addressing classification, clustering, and regression tasks (Nayak et al., 2015). The Support Vector Machine has demonstrated its effectiveness as a powerful tool for supervised classification (Wang et al., 2024; Huang et al., 2018; Nayak et al., 2015). Vapnik in 1998 introduced the method SVM as one of the methods for classification which basically works in finding the boundary between two classes with the maximum distance of the best closest data through the formation of hyperplane (limit). This limit is obtained by measuring the margin hyperplane and looking for the maximum point. The margin represents the distance between the closest point of each class and hyperplane. This point is commonly referred to as SVM (Achmad Rizal et al., 2019). SVM can handle both linear and nonlinear data. In linear data, hyperplane is easy to find, whereas in nonlinear data, data is simulated into three dimensions first using a function called a kernel. The kernel used is a function used in grouping low-dimensional data into high-dimensional (Mase et al., 2018). Some kernels that can be used are as follows:

- Kernel linear:

$$K(x, y) = x \cdot y, \tag{2}$$

where x is training data and y is testing data.

- Kernel polynomial:

$$K(x, y) = (x \cdot y + c)^d, \tag{3}$$

where x is the training data and y is the testing data. While d is the degree of polynomial.

- Kernel Gaussian RBF (Radial Basis Function):

$$K(x, y) = exp\left(\frac{-||x - y||^2}{2 \cdot \sigma^2}\right), \tag{4}$$

where x is training data and y is testing data.

- Kernel sigmoid:

$$K(x, y) = \tanh(\sigma(x \cdot y) + c), \tag{5}$$

where x is training data and y is testing data. While c is a coefficient.

- Inverse multiquadric kernel:

$$K(x, y) = \frac{1}{\sqrt{||x - y||^2 + c^2}}, \tag{6}$$

where x is training data and y is testing data. While c is a coefficient.

The result of SVM is in the form of a confusion matrix. Confusion matrix contains prediction data and actual data to illustrate how the actual classes and their predictions differ.

Table 1 Confusion matrix

		Actual Value	
		Positive	**Negative**
Predicted Value	Positive	TP	FP
	Negative	FN	TN

Source: Mase et al. (2018)

There are 4 terms as a representation of the results of the classification process in the confusion matrix. True Positive (TP) is true data predicted positive, True Negative (TN) is true data predicted negative, False Positives (FP) is data that is incorrectly predicted to be positive, and False Negative (FN) is data that is incorrectly predicted to be negative.

Based on the results obtained from the confusion matrix, then calculated each value using accuracy, precision, recall, and F-1 score.

a. Accuracy is the value of the accuracy of the model in the classification.

$$accuracy = \left(\frac{(BP + BN)}{(BP + SP + BN + SN)}\right) X\ 100\%. \tag{7}$$

b. Precision is the accuracy between data and prediction results.

$$precision = \left(\frac{(BP)}{(BP + SP)} \right) X\ 100\%\,. \tag{8}$$

c. Recall is a value that indicates the success of the model in finding information.

$$recall = \left(\frac{(BP)}{(BP + SN)} \right) X\ 100\%\,. \tag{9}$$

d. F-1 score is a comparison between *precision and recall values*.

$$f1 - score = \left(\frac{2 \times precision \times recall}{precision + recall} \right) X\ 100\%\,. \tag{10}$$

After applying both methods in classifying and predicting the level of building damage, the results will be used to simulate premium calculations in earthquake disaster insurance following these steps (Yucemen, 2005).

- Defining the probability of damage for each level of building damage $i(P_i(DB))$:

$$P_i(DB) = \frac{N_i(DB)}{N(DB)}\,, \tag{11}$$

with $N_i(DB)$ is the number of damaged buildings in the earthquake area with type i = 1, 2, 3, where 1, 2, and 3 represent minor, moderate, and heavy damage, respectively, and $N(DB)$ is the total number of buildings that suffered damage caused by earthquake.

- Calculate the mean damage ratio for each level of building damage $i(MDR_i)$:

$$MDR_i(M) = \Sigma_{DB}\, P_i(DB) \times CDR_{DB}\,, \tag{12}$$

where CDR_{DB} is the corresponding central damage ratio or ratio of the number of damaged buildings and the overall buildings in the earthquake area.

- Calculating the expected annual damage ratio for level of building damage $i(EADR_i)$:

$$EADR_i = \Sigma_M\, MDR_i(M) \times AP_M\,, \tag{13}$$

where $MDR_i(M)$ is the mean damage ratio for the level of building damage i that experienced an earthquake with intensity M and AP_M is the annual probability of an earthquake with intensity M occurs in an area.

- Calculating the pure risk premium for level of building damage $i(PRP_i)$:

$$PRP_i = EADR_i \times BIV\,, \tag{14}$$

then the pure risk premium can be calculated based on the building insured value (BIV).

- Calculating the total earthquake insurance premium for level of building damage $i(TP_i)$:

$$TP_i = \frac{PRP_I}{1 - LF}, \tag{15}$$

where LF is load factor which is defined as hidden uncertainties such as administrative expenses, business taxation, and benefits for the Insurance Company.

3 RESULTS AND DISCUSSION

3.1 Clustering

The solution for clustering analysis using R Studio generally uses the PAM method and there will be three steps taken, namely determining the distance between observations using gower distance because the data used are numerical and categorical mixed data, determining the number of clusters, and clustering. The gower distance will compare the data pair on a scale of 0 to 1. If the two data compared are close to 0 then the data are close together. Conversely, if the two data compared are close to 1 then the data are far apart. In R Studio, the calculation of gower distance is contained in cluster packages with the daisy() function and in Table 2 and 3 are examples of the closest and most distant data.

Table 2 The most nearby data

Data	Impact	Urban village	Latitude	Longitude	Epicenter distance	Damage	PGA
38	House	Srigading	–7.808	110.456	19.91480	Cracked wall	3.268858
37	House	Srigading	–7.807	110.455	19.91475	Cracked wall	3.268874

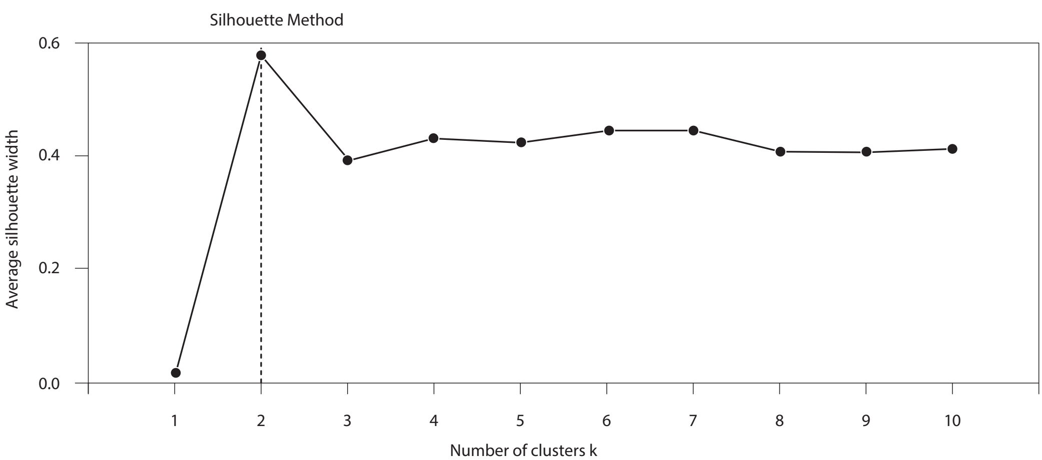
Source: Own elaboration

Table 3 The most distant data

Data	Impact	Urban village	Latitude	Longitude	Epicenter distance	Damage	PGA
69	House	Srigading	–8.005	110.266	19.94268	4-point cracked wall	3.261004
24	Stall	Parangtritis	–6.404	106.817	19.57868	Roof collapsed	3.365682

Source: Own elaboration

Figure 1 Optimal number of clusters



Source: Own elaboration

Based on Table 2 and Table 3, data 37 and 38 are very close together with a minor difference in numerical observations of 0.00005 in the epicenter distance variable compared to data 69 and 24 which are very far apart because there are many differences in each numerical variable, such as the epicenter distance variable which has a difference of 0.364. Furthermore, the determination of number of clusters using the silhouette method with the help of R Studio produces a graph in Figure 1.

Figure 1 above shows that as many as 2 clusters are the best. Then the next step is to group the data into 2 groups with the PAM method using the pam() function in R Studio. This function produces two cluster centers that can be used to divide the data into two clusters.

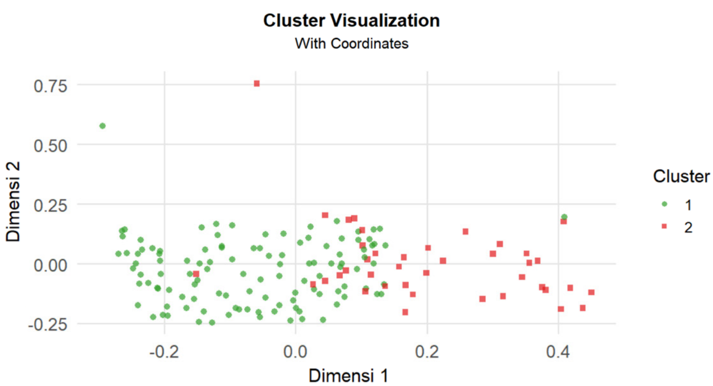
Table 4 Cluster center

Cluster	Data	Impact	Urban village	Latitude	Longitude	Epicenter distance	Damage	PGA
1	41	House	Srigading	−7.811	110.459	19.91497	Cracked walls	3.268811
2	131	Educational facilities	Parangtritis	−7.990	110.316	19.93937	Classroom	3.261934

Source: Own elaboration

Based on the results of the functions that have been executed in Table 4, each cluster center is at data 41 for cluster 1 and data 131 for cluster 2, the results of clustering are listed in the Figure 2.

Figure 2 Cluster results graph



Source: Own elaboration

Based on Figure 2, cluster 1 has 114 data members and cluster 2 has 40 data members. To see which clusters are classified as minor damage and which clusters are classified as heavy damage, profiling is carried out. Profiling is done by calculating the average for numerical data for epicenter and PGA distance variables.

Based on the results in Table 5, cluster 2 has smaller epicenter distances than cluster 1, meaning that the area of earthquake-affected buildings close to the epicenter is the affected building area classified in cluster 2. Therefore, buildings in cluster 1 are minorly affected, while buildings in cluster 2 are heavily affected.

According to the level of damage grouped into 3, namely minor, moderate, and heavy damage. Cluster 1 has a higher percentage of minor damage than cluster 2. While cluster 2 has a higher percentage of moderate and heavy damage than cluster 1 even though the different percentage of level of heavy damage

Table 5 Epicenter and PGA distance variable cluster profiling

	1	2
Epicenter distance	19.9203238	19.9199753
PGA	3.2673206	3.26745224

Source: Own elaboration

Table 6 Percentage of damage from K-Medoids

Type of damage buildings		Cluster 1		Cluster 2	
Minor wall crack	Minor	45.6%	53.5%	12.5%	12.5%
Broken pipelines		0.9%		-	
Roof collapsed		7.0%		-	
Broken tiles and cracked walls	Moderate	25.4%	31.6%	22.5%	72.5%
Sloping and cracking walls		4.4%		-	
Some points of the wall are broken		1.8%		50.0%	
Heavy wall cracks	Heavy	6.1%	14.9%	2.5%	15.0%
Wall collapsed		8.8%		12.5%	

Source: Own calculations

is only 0.1%. Therefore, cluster 1 members can be called earthquake-affected observations with minor damage and cluster 2 members can be called earthquake-affected observations with heavy damage.

3.2 Support vector machine

SVM analysis uses data labels to train its model. Labeling in this case is obtained from the results of clustering k-medoids. The data used was resampling data from the original data to 9 000 from 154 data. Resampling data is conducted to improve the accuracy of SVM. The data uses numerical variables from the original data, namely latitude, longitude, epicenter distance, PGA, and cluster as labeling. SVM is done by dividing data into training and testing data in a ratio of 70:30. The results of the classification and prediction of the level of damage to buildings due to the June 30, 2023, Bantul earthquake are described in Table 7.

Table 7 Confusion matrix SVM

	Data training		Data testing	
	Actual data classes		Actual data classes	
Prediction class	Minor damage rate	Heavy damage rate	Minor damage rate	Heavy damage rate
Minor damage rate	4 621 (74.24%)	1 603 (25.76%)	2 017 (75.74%)	646 (24.26%)
Heavy damage rate	35 (46.67%)	41 (53.33%)	20 (54.05%)	17 (45.95%)

Source: Own calculations

Table 7 shows that in the training data there is 74.24% data that is predicted correctly as data with the level of minor building damage and in the testing data there is 75.74% data that is predicted correctly as data with the level of minor building damage. Furthermore, in the training data there is 53.33% data that is predicted correctly as data with the level of heavy building damage and in the testing data there is 45.95% data that is predicted correctly as data with the level of heavy building damage. The accuracy obtained in training data is 74% and testing data is 75%, meaning that the model built can predict large parts of the data correctly. Then from the prediction results using SVM, each level of damage is grouped to get a percentage of damage, namely prediction 1 is the buildings that are predicted to have a minor level of damage and prediction 2 is the buildings that have a heavy level of damage. The details of the damage percentages are explained in Table 8.

Table 8 SVM result damage percentage

	Prediction 1	Prediction 2
Minor	47.9%	0.0%
Moderate	39.8%	51.3%
Heavy	12.1%	48.6%

Source: Own calculations

Based on Table 8, SVM predicts that none of the buildings suffered minor damage in prediction class 2.

3.3 Building damage insurance premium calculation simulation

From the results of the classification of building damage levels based on k-medoids and SVM, both can be used as a reference in the calculation of simulated building damage premiums caused by the Bantul earthquake on June 30, 2023, see Table 9.

Table 9 Probability of damage

Probability of damage $P_i(DB)$	K-Medoids		SVM	
	1	2	1	2
Minor damage	0.535	0.125	0.479	0
Moderate damage	0.316	0.725	0.398	0.513
Heavy damage	0.149	0.150	0.121	0.486

Source: Own calculations

Before calculating the average damage ratio, an appropriate central damage ratio (CDR_{DB}) is required and refers to the average damage ratio of the Yogyakarta earthquake in 2006 of 5.9 Scale of Richter (SR) (Arrie and Amin, 2018), see Table 10.

Table 10 Average damage ratio of 2006 Yogyakarta earthquake

Damage rate	Damage ratio of the 2006 Yogyakarta earthquake
Minor damage	0.0121
Moderate damage	0.1399
Heavy damage	0.5790

Source: Arrie and Amin (2018)

Then calculate the average damage ratio ($MDR_i(M)$), see Table 11.

Table 11 Average damage ratio

$MDR_i(M)$	K-Medois		SVM	
	Cluster 1	Cluster 2	1	2
Minor damage	0.0064735	0.0015125	0.0057959	0
Moderate damage	0.0440685	0.1014275	0.0556802	0.0717687
Heavy damage	0.086271	0.08685	0.070059	0.281394
Sum	0.136813	0.18979	0.1315351	0.3531627

Source: Own calculations

Calculating the expected annual damage ratio ($EADR_i$) to AP_M is the annual probability of an earthquake with intensity M occurring in a region. During 2023, there have been 49 earthquakes around Bantul, based on BMKG data. Earthquakes of magnitude 6.4 or more occur only once a year. Then $AP_{6.4SR} = 0.02041$. The resulting $EADR_i$ are given in Table 12.

Table 12 Annual damage ratio

$EADR_i$	K-Medois		SVM	
	Cluster 1	Cluster 2	1	2
	0.00279	0.00387	0.00268	0.00721

Source: Own calculations

Then it can be calculated pure risk premium by Formula (15), with BIV is the value of the building insured, for example IDR 300 000 000 (USD 18 612.96), IDR 500 000 000 (USD 31 021.60), IDR 750 000 000 (USD 46 532.40), and IDR 1 000 000 000 (USD 62 043.21), see Table 13.

Table 13 Pure risk premium

BIV	PRP_i			
	K-Medoids		SVM	
	Cluster 1	Cluster 2	1	2
IDR 300 000 000 (USD 18 612.96)	IDR 837 000 (USD 51.93)	IDR 1 161 000 (USD 72.03)	IDR 804 000 (USD 49.88)	IDR 2 163 000 (USD 134.20)
IDR 500 000 000 (USD 31 021.60)	IDR 1 395 000 (USD 86.55)	IDR 1 935 000 (USD 120.05)	IDR 1 340 000 (USD 83.14)	IDR 3 605 000 (USD 223.67)
IDR 750 000 000 (USD 46 532.40)	IDR 2 092 500 (USD 129.83)	IDR 2 902 500 (USD 180.08)	IDR 2 010 000 (USD 124.71)	IDR 5 407 500 (USD 335.50)
IDR 1 000 000 000 (USD 62 043.21)	IDR 2 790 000 (USD 173.10)	IDR 3 870 000 (USD 240.11)	IDR 2 680 000 (USD 166.28)	IDR 7 210 000 (USD 447.33)

Source: Own calculations

After getting a pure risk premium by assuming that the building values are IDR 300 000 000 (USD 18 612.96), IDR 500 000 000 (USD 31 021.60), IDR 750 000 000 (USD 46 532.40), and IDR 1 000 000 000 (USD 62 043.21), the total premium described in Table 14 can be calculated according to (15). In this work, we assume that the load factor is 0.4.

Table 14 Total premium

BIV	*TP$_i$*			
	K-Medoids		SVM	
	Cluster 1	Cluster 2	1	2
IDR 300 000 000 (USD 18 612.96)	IDR 1 395 000 (USD 86.55)	IDR 1 935 000 (USD 120.05)	IDR 1 340 000 (USD 83.14)	IDR 3 605 000 (USD 223.67)
IDR 500 000 000 (USD 31 021.60)	IDR 2 325 000 (USD 144.25)	IDR 3 225 000 (USD 200.09)	IDR 2 233 333 (USD 138.56)	IDR 6 008 333 (USD 372.78)
IDR 750 000 000 (USD 46 532.40)	IDR 3 487 500 (USD 216.38)	IDR 4 837 500 (USD 300.13)	IDR 3 350 000 (USD 207.84)	IDR 9 012 500 (USD 559.16)
IDR 1 000 000 000 (USD 62 043.21)	IDR 4 650 000 (USD 288.50)	IDR 6 450 000 (USD 400.18)	IDR 4 466 667 (USD 277.13)	IDR 12 016 667 (USD 745.55)

Source: Own calculations

Based on Table 14, the total premium for building damage due to the Bantul earthquake on June 30, 2023, for minor to heavy damage from the k-medoids algorithm increases gradually as the *BIV* increased. The total premiums for cluster 2 (heavy-affected building) are higher than those of cluster 1 (minor-affected building). However, it does not have significantly different value ranges. The total building damage premium for predictions using the SVM algorithm also increases gradually as the *BIV* increased. However, for prediction 2 (heavy damage levels), there is a significant increase in premiums, which may be due to the absence of buildings classified or predicted to have minor damage. All building units are classified as moderate and heavy types.

CONCLUSIONS

Using the k-medoids algorithm in clustering and Support Vector Machine (SVM) in prediction produces two types of building damage: minor damage as cluster 1 (prediction 1) and heavy damage as cluster 2 (prediction 2). From these two methods, the amount of earthquake disaster insurance premiums that must be paid can be simulated. For the k-medoids algorithm, the premium amount for minor and heavy damage levels does not differ significantly. This is because both cluster 1 and cluster 2 contain buildings with minor, moderate, and heavy damage levels even though with different percentages. Meanwhile, for the SVM algorithm, the premium amount for minor and heavy damage levels differs significantly. This is because there are no buildings predicted to have minor damage. All buildings are predicted to have moderate and heavy damage levels. What can be further developed in research on this topic is the possibility of conducting simulation calculations for claims and improving the accuracy of the SVM method.

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References

ACHMAD RIZAL, R., SANJAYA GIRSANG, I., APRIYADI PRASETIYO, S. (2019). Face Classification Using Support Vector Machine (SVM) (in Indonesian). *Research and E-Journal of Computer Informatics Management*, 3(2).

ARRIE, M., AMIN, R. (2018). *Ratio of Residential Building Damage. Case Study: Aceh Earthquake on July 2, 2013* (in Indonesian).

FISCAL POLICY AGENCY, MINISTRY OF FINANCE OF THE REPUBLIC OF INDONESIA (2018). *Financing Strategy and Disaster Risk Insurance* (in Indonesian).

HARIKUMAR, S., SURYA, P. V. (2015). K-Medoid Clustering for Heterogeneous DataSets [online]. *Procedia Computer Science*, 70: 226–237. <https://doi.org/10.1016/j.procs.2015.10.077>.

HARNANTO, Y. I., RUSGIYONO, A., WURYANDARI, T. (2017). Application of Ward Cluster Analysis Method to Regencies/Cities in Central Java Based on Contraceptive Use [online] (in Indonesian). *GAUSSIAN Journal*, 6: 528–537. <http://ejournal-s1.undip.ac.id/index.php/gaussian>.

HERMANSYAH, M., AFREYNA FAUZIAH, D., SABILIRRASYAD, I., FAIZ FIRDAUSI, M., WAHID, A. (n.d.). *Comparison of K-Means and K-Medoids Algorithms in Students English Skill Clasterization.* 1(1): 1.

HOFER, L., ZANINI, M. A., FALESCHINI, F., PELLEGRINO, C. (2022). Expected losses vs earthquake magnitude curves, for seismic risk mitigation and for insurance purposes [online]. *Procedia Structural Integrity*, 44: 1824–1831. <https://doi.org/10.1016/j.prostr.2023.01.233>.

HUANG, S., NIANGUANG, C. A. I., PENZUTI PACHECO, P., NARANDES, S., WANG, Y., WAYNE, X. U. (2018). Applications of support vector machine (SVM) learning in cancer genomics [online]. In: *Cancer Genomics and Proteomics*, International Institute of Anticancer Research, 15(1): 41–51. <https://doi.org/10.21873/cgp.20063>.

MASE, J., FURQON, M. T., RAHAYUDI, B. (2018). Implementation of Support Vector Machine (SVM) [online] (in Indonesian). *Algorithm in Cat Disease Classification*, 2(10). <http://j-ptiik.ub.ac.id>.

MCFARLANE, A. C., NORRIS, F. H., GALEA, S., FRIEDMAN, M. J., WATSON, P. J. (2006). *Definitions and Concepts in Disaster Research.*

MOHEMAD, R., MUHAIT, N. N. M., NOOR, N. M. M., OTHMAN, Z. A. (2022). Performance analysis in text clustering using k-means and k-medoids algorithms for Malay crime documents [online]. *International Journal of Electrical and Computer Engineering*, 12(5): 5014–5026. <https://doi.org/10.11591/ijece.v12i5.pp5014-5026>.

MUSFIANI, M. (2019). Cluster Analysis Using Partition Method on Contraceptive Users in West Kalimantan [online] (in Indonesian). *Bimaster: Scientific Bulletin of Mathematics, Statistics, and Its Applications*, 8(4). <https://doi.org/10.26418/bbimst.v8i4.36584>.

NATIONAL EARTHQUAKE STUDY CENTER, RESEARCH AND DEVELOPMENT CENTER FOR HOUSING AND SETTLEMENTS. (2021). *Map of earthquake sources and hazards in Indonesia in 2017 in Indonesia.*

NAYAK, J., NAIK, B., BEHERA, H. S. (2015). A Comprehensive Survey on Support Vector Machine in Data Mining Tasks: Applications & Challenges [online]. *International Journal of Database Theory and Application*, 8(1): 169–186. <https://doi.org/10.14257/ijdta.2015.8.1.18>.

NOOR, A. (2018). *Comparison of Regular Support Vector Machine Algorithm and Particle Swarm Optimization-Based Support Vector Machine for Earthquake Prediction* (in Indonesian).

OCTAVIANI, P. A., WILANDARI, Y., ISPRIYANTI, D. (2014). Application of Support Vector Machine (SVM). Classification Method on Elementary School (SD) [online] (in Indonesia). *Accreditation Data in Magelang Regency*, 3(4): 811–820. <http://ejournal-s1.undip.ac.id/index.php/gaussian>.

SAHRIMAN, S., KALONDENG, A., KOERNIAWAN, V. (2019). Statistical Downscaling Modeling with Dummy Variables Based on Hierarchical and Non-Hierarchical Cluster Techniques for Rainfall Estimation [online] (in Indonesian). *Indonesian Journal of Statistics and Its Applications*, 3(3). <http://www.climatexp.knmi.nl>.

SANTRI, S. H. (2017). The Principle of Utmost Good Faith in Insurance Contracts [online] (in Indonesian). *UIR LAW REVIEW*, 1(1): 77. <https://doi.org/10.25299/ulr.2017.1.01.165>.

SARI, D. P., ROSADI, D., EFFENDIE, A. R., DANARDONO. (2019). K-means and bayesian networks to determine building damage levels [online] (in Indonesian). *Telkomnika (Telecommunication Computing Electronics and Control)*, 17(2): 719–727. <https://doi.org/10.12928/TELKOMNIKA.V17I2.11756>.

VALERO-CARRERAS, D., ALCARAZ, J., LANDETE, M. (2023). Comparing two SVM models through different metrics based on the confusion matrix [online]. *Computers and Operations Research*, 152. <https://doi.org/10.1016/j.cor.2022.106131>.

WANG, P., GUO, J., LI, L.-F. (2024). Machine learning model based on non-convex penalized huberized-SVM [online]. *Journal of Electronic Science and Technology*, 100246. <https://doi.org/10.1016/j.jnlest.2024.100246>.

WIBOWO, F. W. (n.d.). *2018 International Conference on Information and Communications Technology (ICOIACT)*, 6–7 March 2018, Institute of Electrical and Electronics Engineers.

YUCEMEN, M. S. (2005). Probabilistic assessment of earthquake insurance rates for Turkey [online]. *Natural Hazards*, 35(2): 291–313. <https://doi.org/10.1007/s11069-004-6485-8>.

YUSOFF, M., DNAJIB, F. M., ISMAIL, R. (2019). Hybrid backpropagation neural network-particle swarm optimization for seismic damage building prediction [online] (in Indonesian). *Indonesian Journal of Electrical Engineering and Computer Science*, 14(1): 360–367. <https://doi.org/10.11591/ijeecs.v14.i1>.

Household Surveys Integration: Household Budget Survey Methodology in Czechia

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Abstract

The article aims at mapping the history of the Czech version of the Household Budget Survey (HBS), i.e. Statistika rodinných účtů (SRÚ), and on the historical changes in its methodology. Statistika rodinných účtů is a survey focused mainly on private household expenditure in all the regions of the Czech Republic. The first official SRÚ survey was conducted in 1920 by the Statistical office of the former Czechoslovak Republic. This article offers a brief overview of the more than 100 years of the survey, which have been shaped significantly throughout the different eras it had underwent, both by the socio-political contexts and by the gradual technological progress. Last but not least, methodological changes which occurred in 2017 are discussed, as well as some important aspects of the present-day form of the survey.

Keywords	***DOI***	***JEL code***
Household Budget Survey (HBS), Statistika rodinných účtů (SRÚ), Czech Statistical Office (CZSO), household survey methodology, household consumption expenditure	*https://doi.org/10.54694/stat.2024.6*	*C81, C83, D10, G50*

INTRODUCTION

Household Budget Survey (HBS) is a survey carried out in all EU countries, which is focused primarily on private household expenditure. In the Czech Republic, the national survey is conducted under the name "Statistika rodinných účtů" (hereinafter referred to as SRÚ) in all Czech regions. It is aimed mainly at mapping the expenditure's value, as well as its structure, based on the information acquired from the households directly. In the Czech context, the survey has been first conducted in 1920, therefore it has a long tradition of over 100 years. Since 1957, the survey has been carried out systematically and annually (CZSO, 2023a). Until 2016, the SRÚ had been based on quota sampling (CZSO, 2023b). Since 2017, the survey has been newly integrated into the EU-SILC (Living Conditions – "Životní podmínky") survey – only the households previously selected for the EU-SILC can be chosen for the SRÚ survey (CZSO, 2023a).

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The survey unit consists of a private household previously successfully interviewed during the EU-SILC survey. The main objective is the expenditure of all members of the interviewed household supplemented by data regarding the composition of the household, data on the housing and furnishings, and on the potential household farming. Surveyed households record information on their members' expenditure for eight successive weeks in a special Diary which they receive from the CZSO interviewer. The interviewer visits each household three times in total, explains how to record the information in the Diary and, if needed, completes additional information about the household (first obtained from the EU-SILC survey).

The results of the SRÚ survey on household consumption expenditure are unique and cannot be obtained in any other way than by interviewing and by keeping expenditure diaries in the households. The survey results serve primarily as a basis for assessing the social and economic situation of households in the Czech Republic. They are also crucial for creation of the so-called consumer basket, which the Czech Statistical Office (CZSO) uses in order to calculate the consumer price index (inflation). The data are also one of the sources in the compilation of national accounts estimates for the household sector (CZSO, 2023a). The results of the SRÚ survey are published annually and can be freely accessed on the CZSO website.

1 HISTORY OF SRÚ SINCE 1920

The SRÚ survey has a long tradition in the Czech Republic dating back to the establishment of the National Statistical Office (Státní úřad statistický).[3] The first similar survey focused on household consumption was carried out by Professor Karel Engliš, at the time a prominent Czech economist and politician, in 1913/1914 on a sample of 65 teacher families in Moravia, which was aimed at mapping living conditions of Moravian teachers at the beginning of the First World War (Engliš, 1917).

The SRÚ itself, incorporated into the state survey system, was introduced in 1920 (CZSO, 2023a). During the first years of the survey, following the war and inter-war period, only households with a low standard of living were included in the sample, i.e. households whose head was a labourer, a clerk or a teacher. The SRÚ of that time therefore did not provide a fully representative picture of Czech households' social situation, nevertheless it offered important insights into the living conditions of lower-income families (Bezouška and Vytlačil, 1958).

Data obtained from working-class households during years 1931 and 1932 indicate that a significant proportion of household consumption expenditure went to the purchase of food and non-alcoholic beverages. In 46.1% of the labourers' households, more than 2 persons lived in one room, while by "rooms" were meant not only bedrooms or living rooms, but also kitchens (Státní úřad statistický, 1937).

In the inter-war period, the SRÚ survey was carried out until 1937, when it was discontinued, mainly due to financial and other reasons related to war (Bezouška and Vytlačil, 1958). After the Second World War, the survey was reestablished in 1947. A purposive sampling of non-agricultural households was carried out, again with a general focus on the lower-class households (Statistická ročenka republiky Československé, 1948).

In 1953, the sample was newly extended to other population groups, i.e. socially stronger households of professional or technical employees, and there was an effort to keep the representative sample stably at a minimum of 2 000 households. However, the survey faced a high percentage of declining households, mainly due to low remuneration (Bezouška and Vytlačil, 1958).

Due to a reorganization of the survey in 1956, the sample of households was increased again. Households were recruited for the survey by a combination of purposive and random sampling from business records and employment data, which was very labor demanding. A network of 'family account inspectors' was

3 Státní úřad statistický (SÚS; National Statistical Office) was a predecessor of the Czech Statistical Office (CZSO; Český statistický úřad).

put in place to maintain personal contact with households and to guide them methodically to complete the Diaries correctly. Today, this task is carried out by interviewers working within the interviewing network. In 1956, family account inspectors also processed the data, thus performing the role of today's compilers. In 1956, households whose head was a labourer, an employee, a pensioner, a member of a unified agricultural cooperative (in Czech "JZD"), an individual farmer, and a metalworker were surveyed (Státní úřad statistický, 1957).

2 THE BEGINNING OF THE MODERN SRÚ SURVEY

It can be argued that the modern history of the SRÚ dates back to 1957, when the survey underwent a reform and when the quota sampling was introduced. In the survey carried out in 1957, the unit of the survey was a household which consisted of a group of several persons living in a common dwelling and where its members run the household together (Bezouška and Vytlačil, 1958).

The sample size varied significantly between the 1950s and 1980s. During 1963 and 1965, it reached its highest point, with the sample size consisting of 6 600 households, when pensioners were included in the sample, too (CZSO, 1983). Even though the sample was more diverse than before, between 1957 and 1991, the only population groups that were interviewed on a stable basis were households of labourers, emloyees and members of JZDs; pensioner households were surveyed only occasionally. Data from this period show an increasing standard of living of households, which was reflected mainly in the changing structure of expenditures. However, it is important to note that due to the political and economic context of the period, some goods were not available, which also affected household consumption (CZSO, 1983; CZSO, 1992).

The quality and representativeness of these pre-1989 household surveys was regarded as higher in the former Czechoslovakia (and other central European countries) compared to some countries of different regions, e.g. former Soviet Union (Filer and Hanousek, 2002). Major social and economic changes after 1989 (i.e. the year of the Velvet Revolution and collapse of the communist regime) were also evident in the survey, regarding both data processing technology and changes in the consumption structure (connected to the profound socio-political changes) (CZSO, 2005).

Since 1991, households of families with children and households of non-working pensioners were newly included in the sample. Inclusion of these households made it possible to obtain data on people living near the subsistence level (CZSO, 1992). Starting in 1991, survey data have been collected on a monthly basis in a decentralised manner at the CZSO district offices on personal computers. Before that, individual items were processed manually in paper questionnaires and the data were then transferred to central processing centre (CZSO, 2005). From 1993, households of self-employed people (OSVČ) and unemployed people have been newly surveyed. Since 1993, tools for creating aggregated publication outputs have been implemented. Newly, all data were collected in a complex software in which not only income and expenditure were processed, but also the characteristics of households and their members (CZSO, 2005).

Since 1999, the coding and processing of expenditure items based on the Czech version of the international Classification of Individual Consumption by Purpose (COICOP) has been fully applied. The Czech version is named CZ-COICOP, and compared to the 14-section international classification, it had 12 sections (CZSO, 2005). Since 2006, the survey has incorporated all types of households, including households of economically active pensioners and economically inactive persons (CZSO, 2019).

In 2016, a profound methodological reform of the SRÚ survey took place. Its main goal was the transition to probability sampling of households and the saving of financial costs for the survey. The SRÚ has been newly integrated into the Living Conditions Survey (EU-SILC), meaning that the current sample of SRÚ households is based on a random sample of households for the EU-SILC.

Until 2016, the SRÚ had been based on quota sampling. The quota had determined, for example, how many households in the surveyed sample must be employees with lower education and with one child, with a net monthly income per person in a certain range (e.g. 8 001–11 000 CZK) and living in a family house in a municipality with a population of less than 50 thousand inhabitants (CZSO, 2019).

The surveyed sample of 3 000 households was constructed in such a way that its composition regarding chosen characteristics corresponded to the structure of households in the Czech Republic. The results of the so-called Micro-census (a random sample survey designed to obtain representative data on the level and structure of incomes and basic socio-demographic characteristics of Czech (and previously Czechoslovak) households) and the Census were previously used as a basis for setting the quotas. After that, the results of the Living Conditions survey (the national module of the EU-SILC) served as the basis for quota setting (CZSO, 2019).

3 METHODOLOGY UNTIL 2016 AND METHODOLOGICAL CHANGES AFTER 2017

Between 1997 and 2003, the CZSO started contemplating and testing the possibility of introducing a random sampling method for the SRÚ survey, in an effort to improve the representativeness of the results. During this time period, there have been thorough analyses of the results of the pilot surveys and numerous discussions with experts. However, it was decided that the quota sampling will remain for the time being, with the aim to further improve the sampling method and other aspects of the survey (CZSO, 2019).

Only in 2016, a profound methodological reform of the SRÚ survey took place. The main goal of the methodological reform was the harmonisation of the national SRÚ survey with the international methodology of the HBS, as well as saving of financial costs for the survey and reducing respondent's burden. The main change was the transition to probability sampling of households. Even though Eurostat's reference period for HBS is 5 years, it has been deciced that it will be more beneficial for SRÚ to be conducted as an annual survey. This ensured that the survey would deliver regular results, while maintaining stable interviewer network and more stable budget.

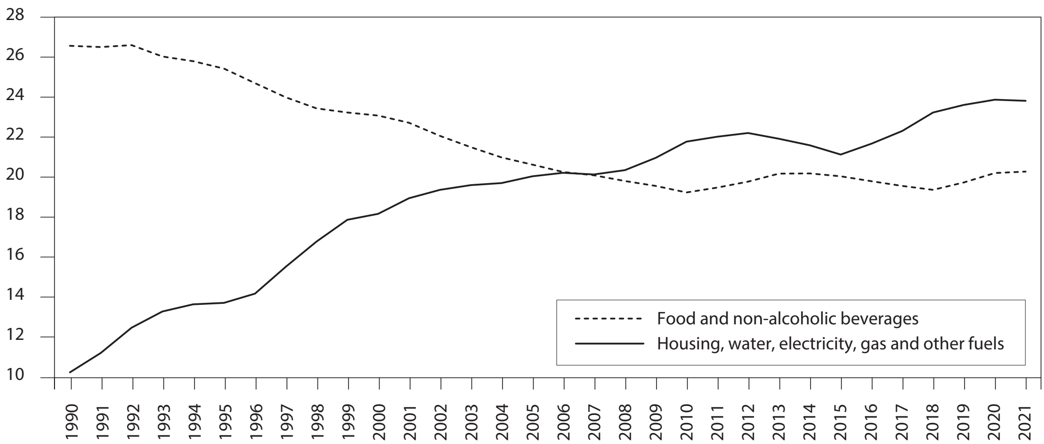
In addition, the SRÚ has been newly integrated into the EU-SILC/Living Conditions (Životní podmínky) survey, meaning that the sample of SRÚ households was newly based on a random sample of households previously selected for the EU-SILC. Thus, it was newly possible to contact the previously surveyed households again in the next reference period. For the first wave of SRÚ, all households from third wave of SILC are contacted. Households that did not refuse to participate in the SRÚ survey were subsequently contacted again in the following year (second wave of SRÚ, being equivalent to fourth wave of EU-SILC) in a different reference period (CZSO, 2023a).

It has been decided to integrate the SRÚ survey namely into EU-SILC mainly due to a certain overlap of questions in both surveys, regarding household composition, income, expenditure on housing and energy etc. During the data collection for the SRÚ, there is a short introductory interview with the household, which partly consists of updating the household composition and other relevant information. The reference period for the survey has been shortened; households report on their expenditures for eight weeks, meaning they keep two aforementioned Diaries regarding their expenditure, each for four weeks, while the survey consists of 26 reference weeks. The households document their regular payments, concerning housing, telecommunication, transport, insurance etc. Other types of spendings are also documented via the collection of receipts. The collected receipts are then transcribed at the CZSO by trained workers, however, it is intended that in the future the receipts might be scanned and automatically coded into the COICOP classification. During the methodological reform, the Diaries have been simplified, mainly by introducing chronological recording, to ensure easier manipulation for respondents.

Since the integration of the SRÚ into the EU-SILC, the response rate on the first wave (of SRÚ) is approximately 50%, while on the second wave it reaches about 90%. The survey is conducted annually on a sample of about 2 000 households. However, the results are published using 2-year moving averages, meaning the sample consists of approximately 4 000 households. The households receive a financial compensation for the participation in the survey and for completing the Diaries. Since 2017, it has been approximately 80 euros, which was subsequently increased to approximately 120 euros in 2023.

Due to the described major changes in methodology, the HBS time series was discontinued in 2016. Starting in 2017, a new time series ensued (CZSO, 2023c). However, the change in the SRÚ methodology between 2016 and 2017 has not caused significant differences in the time series as a whole. Figure 1 shows 3-year moving averages of shares of household expenditure on food and housing – in accordance with the updated international eCOICOP classification – in total expenditure. These expenses represented approximately 37% of total household expenditure in 1990, while in 2021 it was already almost 45%. In 1990, household expenditure on food was significantly predominant, however, the ratios were gradually changing, and since 2007 the housing and energy expenditures have started to dominate.

Figure 1 Shares of household expenditure on food and housing in total household expenditure in Czechia (1990–2021), in %



Source: CZSO

Eurostat's reference years for the HBS are at 5-year intervals, i.e. 2010, 2015 and 2020 (Eurostat, 2003). Since 2021, Household Budget Survey (on a European level) has been implemented under the Regulation (EU) 2019/1700 of the European Parliament and of the Council of 10 October 2019 (also known as the EU regulation 2019/1700 or IESS regulation) (Regulation (EU) 2019/1700 2019). The aim of this regulation is to establish a common framework for European statistics, and thus help with better comparability of the data across the surveyed countries (Eurostat, 2023). Together with this regulation, a new international classification COICOP 2018 is being introduced. According to this new regulation, the next reference year for the HBS will be 2026.

4 CONSUMPTION VS. EXPENDITURE

The aim of the HBS should be to collect and publish data on household expenditure on consumption, regardless of who, where or when consumed it. For instance, it is problematic to track consumption regarding gifts received or given between multiple households or other goods that members of one household bought and members of other household consumed. It can be argued that relevance for collection of consumption in HBS is lower than risk to collect incomparable data without explanatory ability.

Another issue is the question of collecting quantities in HBS. The Czech Statistical Office deciced not to collect the quantities of consumed goods in the SRÚ survey, for which there are several reasons. Firstly, collecting quantities of consumed goods can lead to additional respondents' burden. Furthermore, several goods can be documented in different units of measurement (e.g. fruit and vegetables in kilograms or in pieces), which makes it more challenging to accurately document household's consumption. Lastly, as stated above, in some cases it cannot be said which person from which household consumed which product.

The CZSO's Department of Household Surveys has been cooperating for several years with the National Accounts (NA) department. In this regard, it is important to note that microdata from household survey can never equal to NA macro aggregates. There are several reasons for this. Firstly, both departments base their results on a different population. In SRÚ (i.e. in EU-SILC), only private households and their members are interviewed. Therefore, the results of the survey do not account for individuals in institutional settings, such as retirement home or children's home. There are also methodological differences between social statistics and national accounts. Furthermore, imputed rent, which is a part of the NA, should not be included in social statistics – it has been collected in the EU-SILC survey before, however, the results showed poor quality and have never been published. In other words, imputed rent is relevant for the system of NA, however, not for social statistics where it can cause artificial increase in both incomes and expenditure, thus not corresponding with actual situation of the households. For similar reasons, social transfers in kind (STiKs) provided by government (e.g. on health or education), which are a part of the system of NA, should not be included in social statistics.

For these reasons, it can be argued that HBS (and thus SRÚ as well) should collect and publish data only on monetary household expenditure on consumption. This kind of data can be viewed as a feedback for the respondents, while the NA macro data are more often used by macro-economic analysts.

CONCLUSION

The main goal of the article was to map the history of the Czech version of the Household Budget Survey (HBS), i.e. Statistika rodinných účtů (SRÚ), with a special focus on the historical changes in its methodology. Even though the first official SRÚ survey was conducted more than 100 years ago, it can be argued that the modern history of the survey dates back to 1957 when the quota sampling was firstly introduced, which was then replaced in 2017 by the integration of the SRÚ into the EU-SILC based on random sampling. In addition to the transition to random sampling and reducing respondent's burden, the integration of SRÚ into EU-SILC has also enabled financial savings. It can be argued that this methodological change did not have a significant impact on the SRÚ time series. The long tradition of the survey thus provides a source of information which enables the data users to study and compare the consumption expenditures of different types of households throughout different eras.

These unique data of the SRÚ survey provide important information on household financial expenditures by different types of households (e.g. by activity status of head of household, by municipality size and by tenure of dwelling, by net money income per capita, or by number of dependent children). These outputs mainly show the structure of households' expenditure on their consumption. As this is a sample household survey, the results have certain limitations. Most importantly, household surveys do not include the richest nor the poorest households. SRÚ outputs thus report aggregate microdata of "middle class" households and serve as a mirror of consumption expenditures for the respondents. Secondly, it can be argued that households tend to undervalue some of their expenditures (such as on alcohol and tobacco products). These calculations are then supplemented at the macro-level in the National accounts within the Non-Observed Economy. Household final consumption expenditure in the balance sheet concept of the National accounts then provides data for further macroeconomic analysis.

References

BEZOUŠKA, J., VYTLAČIL, J. (1958). *Metoda a organisace statistiky rodinných účtů.* Prague: Státní úřad statistický.

CZSO. (2005). *Retrospektivní údaje statistiky rodinných účtů za období 1989–2003 v ČR* [online]. Prague: Czech Statistical Office. [cit. 27.10.2023]. <https://www.czso.cz/csu/czso/retrospektivni-udaje-statistiky-rodinnych-uctu-za-obdobi-1989-2003-n-r6nbs0a4kp>.

CZSO. (2019). *Statistika rodinných účtů (SRÚ) – Metodika do roku 2016* [online]. Prague: Czech Statistical Office. [cit. 6.11.2023]. <https://www.czso.cz/csu/czso/statistika-rodinnych-uctu-metodika>.

CZSO. (2023a). *Household Budget Survey (HBS)* [online]. Prague: Czech Statistical Office. [cit. 12.9.2023]. <https://www.czso.cz/csu/vykazy/household-budget-survey-hbs>.

CZSO. (2023b). *Household Consumption Expenditures – 2022 (Methodology section)* [online]. Prague: Czech Statistical Office. [cit. 12.9.2023]. <https://www.czso.cz/csu/czso/household-consumption-expenditures-2022>.

CZSO. (2023c). *Household Budget Survey (HBS) – Methodology since 2017* [online]. Prague: Czech Statistical Office. [cit. 6.11.2023]. <https://www.czso.cz/csu/czso/household-budget-survey-hbs-methodology>.

ENGLIŠ, K. (1917). *Theorie statistiky spotřebního hospodářství 65 učitelských rodin na Moravě z roku 1913–1914.* Brno: Ústřední spolek jednot učitelských na Moravě.

EUROSTAT. (2003). *Household Budget Surveys in the EU. Methodology and recommendations for harmonisation – 2003* [online]. Eurostat. [cit. 8.2.2024]. <https://ec.europa.eu/eurostat/cache/metadata/Annexes/hbs_esms_an1.pdf>.

EUROSTAT. (2023). *Household Budget Surveys. Legislation* [online]. Eurostat. [cit. 6.11.2023]. <https://ec.europa.eu/eurostat/web/main/home>.

FILER, R. K., HANOUSEK, J. (2002). Data Watch: Research Data from Transition Economies. *The Journal of Economic Perspectives*, 16(1): 225–240.

Peněžní příjmy, vydání a naturální spotřeba u domácností zpravodajského souboru SŽÚD v letech 1957–1982 v ČSR. Prague: Czech Statistical Office, 1983.

Příjmy a vydání domácností zpravodajského souboru statistiky rodinných účtů v České republice v roce 1991. Prague: Czech Statistical Office, 1992.

Regulation (EU) 2019/1700 of the European Parliament and of the Council of 10 October 2019 [online]. Official Journal of the European Union. [cit. 8.2.2024]. <https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32019R1700>.

Rozbor výsledků zpravodajských domácností statistiky rodinných účtů za rok 1956. Prague: Státní úřad statistický, 1957.

Spotřeba v rodinách dělnických, zřízeneckých a úřednických v roce 1931/1932. Prague: Státní úřad statistický, 1937.

Statistická ročenka republiky Československé 1948. Prague: Státní úřad statistický, 1948.

Current Issues in the Development of the National Accounts System

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On 20 and 21 June 2024, already the eighteenth conference of the National Accounting Association (Association de Comptabilité Nationale – ACN) took place in Paris in the premises of the OECD (Boulogne Billancourt). This international conference was traditionally held under the auspices of the French statistical office (Institut National de la Statistique et des Etudes Economiques – INSEE) and with the support of the OECD. The conference, which was attended by over 100 experts from ten countries representing statistical offices, universities, research institutes, and other national and supranational institutions (Eurostat, International Monetary Fund, OECD), was divided into five relatively separate thematic blocks. The organisers defined themes of individual breakout sessions very well, which led to a great interest of all participants, both in terms of the breadth of the topics presented and the richness of the ensuing discussion.

I would like to present some of the most important contributions from the conference, which was again extremely interesting and beneficial this year.

The conference was officially opened by Sarah Barahona (Head of the National Accounts Division, OECD) who underlined the importance of bringing together experts in the field of national accounts, especially in times of economic turbulences, when it is necessary to provide a unified view of economic development in European countries monitored through a harmonised methodology for national accounts. The uniform rules of this macroeconomic information system, which will be updated in the SNA 2025 or ESA 2028 standard, are a prerequisite for mutual understanding of experts in the field of national accounts, economics, statistics, economic policy, etc. As usual, formal and informal meetings of experts in these fields have provided a real atmosphere of mutual cooperation. The biennial conference of the ACN, an organisation of more than 800 experts from 70 countries all over the world, serves just such a purpose. She underlined that the 18th ACN conference would focus not only on the changes brought about by the SNA 2025 and ESA 2028 standards, but also on the potential of national accounts to capture phenomena such as the environment, the informal economy, the digital economy, and income inequalities.

The conference was chaired by the ACN Chairman (and former Director of the INSEE National Accounts Department and OECD National Accounts Division) Francois Lequiller. The first thematic session was devoted to changes posed to national accounts by changes in society. In *The Augmented National Accounts program at INSEE*, Sébastian Roux (INSEE) revisited the conclusions of the Stiglitz Commission and stressed that GDP is an indicator of "only" economic activity and that while there is (thanks to the Stiglitz Commission's conclusions) a set of indicators informing about other aspects of societal development, these are not coherent with the national accounts. Therefore, it is necessary for the national accounts to be also able to capture phenomena that are "above GDP" such as human capital, income inequalities, domestic work, sustainability, etc. The initiative to integrate environmental and inequality analysis into the national accounts was taken jointly by the OECD and Eurostat. This would

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involve building an information system to support the regular (annual) production of statistics on the environment and income distribution in line with national accounts. In November 2024, INSEE will publish carbon accounts for 2023 and the first detailed accounts of the distribution of household income for 2022.

In the longer term, INSEE will focus on building a system of synthetic indicators reflecting well-being and/or sustainability, on natural capital valuation (in the extent of the SNA and SEEA), and on other dimensions - human capital, health, well-being, etc.

The second thematic session was dedicated to changes that will be brought about by the transition to the new French national accounts base and the new SNA 2025 and ESA 2028 standards. The transition to the new French national accounts base was presented in two papers, the authors of which were Guillaume Houriez (INSEE) and Valérie Chauvin (Banque de France). The first contribution *The 2020 Base of French National Accounts* was devoted to changes that the new base will bring. The discussion before the change of the base was mainly around the choice of the base year itself – whether 2019 or 2020 (the first year of the covid pandemic) should be chosen. Analyses have shown that the choice of 2020 is suitable and that only some sub-areas (e.g. social insurance, research and development, imputed rent estimates) will see major changes when the time series is extrapolated into the past; GDP will change only slightly (a decrease by 0.2% for the year 2019 compared to the 2014 base). The paper *Calibration of National Accounts and Balance of Payments – 2020 Base Change* addressed the possibilities for convergence of data in the non-resident account and the balance of payments. The object of interest is the same – to best capture the movement of goods and services and income movements. The data sources are similar; however, there are still differences due to the specificities of these documents (INSEE emphasises consistency with employment statistics and the balance of resources and uses by product, the Banque de France focuses on the geographical breakdown and consistency with the financial account; moreover, the national accounts and the balance of payments are published at different times). However, the ESA 2010 standard and the BPM6 manual already represent a step forward in the convergence of the two documents.

Naturally, the biggest attention was focused on a contribution of John Verrinder (Eurostat) who presented the *SNA and ESA Novelties (Excluding Environmental Issues)*. In the beginning of his presentation, he summarised the process of preparing the revision of the SNA 2008 standard, i.e. the basic ideas in 2018, the start of work on the revision in 2020, the research work, recommendations for changes, consultations and testing in 2020–2023, and then in 2024 the preparation of the text of the standard and its consultation with the national statistical offices. The revised standard is to be approved by the Advisory Expert Group (AEG) and the Intersecretariat Working Group on National Accounts (ISWGNA) in October 2024 and adopted by the United Nations Statistical Commission (UNSC) as the revised standard SNA 2025 in March 2025. Concurrently with the work on the revision of the national accounts standard, the revision of the BPM6 balance of payments manual is underway; the new BPM7 manual is to be adopted by the International Monetary Fund also in March 2025.

The key issue areas that should be included in the revision of the SNA 2008 standard, which were already outlined in 2018, are globalisation, digitalisation, well-being and sustainability. Despite many suggestions and recommendations, rather few conceptual changes have been pushed through in the area of the impact of globalisation. They mainly relate to the breakdown of enterprises into foreign-controlled and domestically-controlled enterprises, the process of identifying intellectual property by multinational enterprises; the extension of supply and use tables is also on the table. The impact of digitalisation should be seen in particular in the recognition of data as a produced asset. Data is defined as information content that is produced by collecting, recording, organizing and storing observable phenomena in a digital format, which provides an economic benefit in productive activities. Data that is produced and used in production for more than one year is gross fixed capital formation (GFCF). Data should be classified to a new category which includes output associated with databases – separated from "software".

Other changes should concern the treatment of crypto assets (crypto assets without a corresponding liability will be treated as non-produced non-financial assets), the creation of additional digital supply and use tables, and a more detailed breakdown of technologically significant assets. The problem heading on well-being and sustainability should primarily focus on data for the household sector. More detailed tables should be produced on the distribution of household income, consumption, savings and assets, detailed labour accounts, and unpaid household work, education and training, and health should also be observed. Among other proposed changes, it has to be mentioned that the central bank output should be treated as non-market and recorded in the use as the central bank final consumption expenditure. That would, however, imply a shift of the central bank from the financial institutions sector to the general government sector. Following the changes that will be brought by the SNA 2025, the ESA 2010 standard will also be revised; it is to be approved in March 2028 (as the ESA 2028) and implemented in September 2029.

In a thematic session on the Environment, six presentations were made in total, of which I consider the one made by Bram Edens (OECD) and Sylvain Larrieu (INSEE) to be the most important. Bram Edens presented the recommendations of *The Expert Group on Natural Capital* for the SNA 2025 on natural capital. In particular, the classification of non-financial assets will be expanded to include a group of Natural capital (AN.3), which will include Natural resources (AN.31) and Ecosystem assets (AN.32). Natural resources will not only include Land and Non-renewable mineral and energy resources, but also Renewable energy resources (it means that Solar, Water, Wind, and Geothermal energy resources will have to be valued as economic assets); further, there will be a very detailed breakdown of Biological resources, Water resources, and Radio spectra and other natural resources. Depletion of mineral, energy, and biological resources will be recorded as a cost of production instead of other changes in volume, as it is now. This means, inter alia, that the value of gross domestic product will remain unchanged (other things being equal), but the value of net domestic product will change. It will not only be affected by the level of consumption of fixed capital but also by the depletion of natural capital. However, in the case of renewable energy sources, it will be necessary to decide and define what is a renewable energy asset as a separate asset category and what is not (e.g. only solar radiation captured by solar panels is an asset, whereas a river without hydroelectric generation is not an asset).

In his presentation *Capturing natural assets in the SNA 2025 – application in French national accounts*, Sylvain Larrieu discussed the implementation of the aforementioned changes in the French national accounts. He pointed out that currently natural resources are included in different asset categories and that their separation will contribute to a clearer classification. However, a number of issues would need to be addressed, e.g. natural resource rent, estimation of production costs of renewable energy sources, etc.

The fourth thematic block (session) concerned possibilities of capturing income and wealth inequalities in national accounts. The progress of work on this topic in an international context (it is a joint project of the OECD (lead), the ECB, Eurostat, IMF, UN, and World Bank) was presented by Jorrit Zwijnenburg (OECD) in his paper *Distributional national accounts: an update on international developments*. The revised SNA 2025 standard will include a specific part on the distribution of households by income and wealth, which will be conceptually consistent with national accounts. However, it is first necessary to develop a methodology using micro-data sources to inform on household income and wealth inequalities in line with national accounts concepts. A uniform methodology will then guarantee international comparability of results and provide consistent information on three dimensions of economic well-being, i.e. income, consumption, and wealth. A household will be a unit of analyses. Households will be divided into groups of 10% and 20% in terms of disposable income and net worth, respectively. Other characteristics (age, sex, level of education, housing status, etc.) will also be taken into account.

Mathias André (INSEE) presented a paper on the progress of works on capturing inequalities in the household sector in French national accounts. In his paper, *Production and diffusion of distributional economic accounts*, he underlined objectives and the importance of distributional accounts. It is mainly about answering the questions of how national income is distributed, who benefits from economic growth, what is the redistributive effect of public services, and how income and wealth inequalities of French households develop. The INSEE has already published a number of studies on the distribution of household income and wealth and envisages publishing annual household distributional accounts.

The last thematic session addressed the issues of capturing the digital economy, the non-observed economy, and the informal economy in national accounts. The keynote was a presentation by Jorrit Zwijnenburg (OECD) on the *OECD Handbook on Digital Supply and Use Tables*.[2] The concept of digital supply and use tables itself has three dimensions – the nature of the transaction (the "how"), digital products (the "what"), and digital industries (the "who").

In terms of the nature of transactions, it is necessary to distinguish between transactions:

- digitally ordered, which is the sale or purchase of goods or services made through computer networks by methods specifically designed for the purpose of receiving or placing orders (does not include orders placed by telephone, fax, or e-mail),
- digitally delivered, i.e. transactions that are delivered remotely over computer networks.

From a product perspective, it has to be taken into account that in current input-output tables, digital products are hidden in many product lines that include both digital and non-digital products. In digital input-output tables, digital products are aggregated and displayed separately in the categories of a) information and communication technology (ICT) products, b) digital services. In addition, two products of considerable political interest are shown separately, namely a) cloud computing services (CCS), b) digital intermediation services (DIS).

In this context, it was also necessary to add some new industries to the existing ones, according to different types of manufacturers. The following have to be distinguished:

- the digitally enabling industry, which includes units that produce goods and services such as IT equipment and software (e.g. Samsung),
- digital intermediation platforms, i.e. producers operating online interfaces that facilitate, for a fee, the direct interaction between multiple buyers and sellers, without them taking economic ownership of the goods/services that are sold/intermediated (e.g. Amazon; Uber),
- data- and advertising-driven digital platforms, i.e. platforms that generate revenue via other means, e.g. via selling advertising space or by analysing based on the data they produce (e.g. Google, Instagram),
- producers dependent on digital intermediation platforms, i.e. units that sell most of their goods or services via intermediation platforms,
- e-tailers, i.e. units, for which the majority of orders, in terms of value, are received digitally,
- financial service providers predominantly operating digitally,
- other producers only operating digitally (e.g. Netflix, YouTube).

Jorrit Zwijnenburg admitted that the actual compilation of the digital input-output tables will not be easy for the Member States. Therefore, the Informal Advisory Group (IAG) has set key indicators that will be desirable to monitor; they are as follows:

- expenditure split by nature of the transaction, includes estimates of digital trade,
- output and/or intermediate consumption of total ICT goods and digital services, cloud computing services (CCS), and digital intermediation services (DIS),
- digital industries' output, gross value added (GVA) and its components.

In conclusion, Jorrit Zwijnenburg underlined that digital input-output tables are not a cure-all for statistical capturing of the digital economy; they are only part of a broader attempt to better describe digitalisation. The *OECD Handbook on Digital Supply and Use Tables* offers a non-prescriptive framework

[2] This document is attached to the presentation at: <*https://www.insee.fr/fr/information/8191500*>.

for creating internationally comparable indicators consistent with national accounts and can retrospectively help improve national accounts. Chapter 22 (extended and thematic accounts) of the SNA 2025 standard will provide the basic principles of digitalisation accounts.

What is also undoubtedly worthy of attention is a paper entitled *Free services and hosting via online platforms: a mystery to be solved for national accounts?*, in which Alexandre Bourgeois (INSEE) summarised the INSEE experience. Regarding services provided free of charge, he stated that there is no need to change the basic framework of the SNA; however, the indirect methods of estimating their value need to be explicitly explained when including them in GDP. Introduction of a satellite account would be a suitable solution. As for providing dwelling services of Airbnb type, he considers it to be an appropriate solution to consider the price of this service as an accommodation corresponding to renting for 3 to 4 months in a year.

The conference was concluded with the ACN general assembly, which approved a report on activities of the ACN and on its financial management and discussed further possibilities for the development of this international organisation. In this context, we remind you that membership in the ACN at INSEE is voluntary and free of charge and that anyone interested in joining the association can get registered on the INSEE[3] website. All presentations delivered at the conference are available on the INSEE website: <*https://www.insee.fr/fr/information/8191500*>.

[3] <*https://www.insee.fr/fr/information/1894371*>.

Conferences

The ***26th Applications of Mathematics and Statistics in Economics Conference (AMSE 2024)*** took place **from 28th August to 1st September 2024** in **Wroclaw, Poland**. The conference aims to acquaint its participants with the latest mathematical and statistical methods that can be used in solving theoretical and practical problems and challenges of economics. More at: <*https://www.amse-conference.eu*>.

The ***32nd Interdisciplinary Information Management Talks (IDIMT 2024)*** was held **during 4–6 September 2024** in **Hradec Králové, Czechia**. IDIMT-conferences became interdisciplinary international forum for the exchange of concepts and visions in the area of software intensive systems, management and engineering of information and knowledge, social media, business engineering, and related topics. More at: <*https://idimt.org*>.

The ***18th International Days of Statistics and Economics (MSED 2024)*** took place **during 5–6 September 2024** in **Prague, Czechia**. The aim of the conference is to present and discuss current problems of statistics, demography, economics and management and their mutual interconnection. More at: <*http://msed.vse.cz*>.

The ***23rd biannual conference and joint summer school in statistics ROBUST 2024*** took place **from 8th to 13th September 2024** in **Bardějov, Slovakia**. This conference is organized by the Union of Czech Mathematicians and Physicists, Czech Statistical Society and Slovak Statistical and Demographic Society. Its programme is focused on theoretical and applied statistics, probability, optimization and data analytics. Special section will be focused on impact of AI and open sources like ChatGPT, Wikipedia on teaching statistics and work of statisticians. More at: <www.karlin.mff.cuni.cz/~antoch>.

The ***42nd International Conference on Mathematical Methods in Economics (MME 2024)*** was held **from 11th to 13th September 2024** in **Ústí nad Labem, Czechia**. The conference is a traditional meeting of professionals from universities and businesses interested in the theory and applications of operations research and econometrics. More at: <*https://mme2024.ujep.cz*>.

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We publish articles focused at theoretical and applied statistics, mathematical and statistical methods, conception of official (state) statistics, statistical education, applied economics and econometrics, economic, social and environmental analyses, economic indicators, social and environmental issues in terms of statistics or economics, and regional development issues.

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Discussion brings the opportunity to openly discuss the current or more general statistical or economic issues, in short what the authors would like to contribute to the scientific debate. Contribution shall have up to 6 000 words or up to 10 1.5-spaced pages.

In the ***Methodology*** section we publish articles dealing with possible approaches and methods of researching and exploring social, economic, environmental and other phenomena or indicators. Articles shall have up to 12 000 words or up to 20 1.5-spaced pages.

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