

Methodological notes

The complete mortality life tables for the Czech Republic and its NUTS-2 Areas and NUTS-3 Regions are calculated from the third main group of demographic events (age-period specific), separately for males and females. The complete mortality life tables for Areas and Regions have been processed for a two-year period to exclude random fluctuations.

Complete life tables indicators

- **Number of deaths (D_x)** states the absolute number of deaths by age (x) in a given area during the reference period.

- **Number of inhabitants (P_x)** states the absolute number of inhabitants as of 1 July of a given year in a given area (the sum of population as of 1 July of given years in a case of life tables for Areas and Regions, except for the period 2010-2011, when data to mid-period (as of 1 January 2011) were used, for the reason that it is a population stock, in which the definitive results of Population Census 2011 are reflected)

- **Death probability (q_x)** expresses the probability that an individual at the exact age of x will die in a given period, i.e. before the exact age of $x+1$. The input death probabilities are derived from the age-specific mortality rates (u_x).

$$q_x = 1 - e^{-u_x}$$

- **Table number of survivors (l_x)** is the hypothetical number of individuals alive at the exact age of x out of 100,000 live births (table root - l_0), given the mortality conditions of the reference period.

$$l_{x+1} = l_x \cdot (1 - q_x)$$

- **Table number of deaths (d_x)** is the hypothetical number of individuals who die at the age of x .

$$d_x = l_x - l_{x+1}$$

- **Table number of person-years (L_x)** expresses the hypothetical number of individuals alive at the completed age of x . It is calculated as the average of two subsequent table numbers of survivors, except for the age of 0.

$$L_x = \frac{l_x + l_{x+1}}{2}$$

$$L_0 = l_0 - \alpha \cdot d_0$$

Table number of person-years at the age of 0 is derived from the frequency distribution of infant deaths during the analysed period by the year of born. The coefficient " α " informs what is the ratio of infant deaths born in the given year from all infant deaths in analysed year. In the life tables for regions the coefficient reflects the situation of the whole Czech Republic during two analysed years (age-cohort specific deaths from the first main group of demographic events).

- **Auxiliary indicator (T_x)** expresses the number of years of life to be lived by the table generation (not of an individual) at a given age. It is the accumulation of L_x from the highest age of the table ($\omega-1$) to the age of x .

$$T_x = T_{x+1} + L_x$$

$$T_x = \sum_{\omega-1}^x L_x$$

- **Life expectancy (e_x)** indicates the expected remaining life duration with presumption of unchangeable mortality conditions of a given year. It is a synthetic indicator displaying mortality conditions of a given year through all age groups.

$$e_x^0 = \frac{T_x}{l_x}$$

Calculation of input death probability for the complete life table

Input data:

- number of deaths (D) by age (x) and sex during an analysed year t / years $t, t-1$
- number of deaths at the age of 0 in the Czech Republic by the year of born
- number of people (P) by age (x) and sex as of 1st July of a given year / years
- number of live-births (N^v) by sex

Method:

1. Calculation of the age-specific mortality rates (\dot{u}_x) for age $x \geq 1$ from an empirical data.

$${}_t \dot{u}_x^{CR} = \frac{{}_t D_x}{1.7.t P_x} \qquad {}_{t-1,t} \dot{u}_x^{reg} = \frac{{}_{t-1} D_x + {}_t D_x}{1.7.(t-1) P_x + 1.7.(t) P_x}$$

2. Calculation of the death probabilities based on continuous function $q_x = 1 - e^{-\dot{u}_x}$. The death probability at the age of 0 is equal to infant mortality rate.

$${}_t q_0^{CR} = \frac{{}_t D_0}{N^v} \qquad {}_{t-1,t} q_0^{reg} = \frac{{}_{t-1} D_0 + {}_t D_0}{{}_{t-1} N^v + {}_t N^v}$$

3. In order to exclude random fluctuations the death probabilities are equalized at the age of 4 and more.

$$q_x^{equal} = [105 \cdot q_x + 90 \cdot (q_{x-1} + q_{x+1}) + 45 \cdot (q_{x-2} + q_{x+2}) - 30 \cdot (q_{x-3} + q_{x+3})] / 315$$

4. After equalization the function (for the older ages) is further adjusted by Gompertz-Makeham formula $\log p_x = a + b \cdot c^x$ with undermentioned input:

$$\ln p_x^{equal} = \ln(1 - q_x^{equal})$$

The Czech Statistical Office uses King-Hardy method of extrapolation where d is the interval length and x_0 is the start age of the first interval (here $x_0 = 60$ and $d = 8$).

$$R_1 = \sum_{i=x_0}^{x_0+d-1} \ln p_i^{equal} \qquad R_2 = \sum_{i=x_0+d}^{x_0+2d-1} \ln p_i^{equal} \qquad R_3 = \sum_{i=x_0+2d}^{x_0+3d-1} \ln p_i^{equal}$$

5. Calculation of constants included in Gompertz-Makeham formula according to:

$$c^d = \frac{R_3 - R_2}{R_2 - R_1} \qquad c = \sqrt[d]{c^d} \qquad b = \frac{(c-1) \cdot (R_2 - R_1)}{c^{x_0} \cdot (c^d - 1)^2} \qquad a = \left[R_1 - \frac{(R_2 - R_1)}{(c^d - 1)} \right] / d$$

6. Calculation of model probability of survival $r_x = \exp(a + b \cdot c^x)$ for the age $x \geq 71$. Find the age y ($y \geq 75$) where $|p_x^{equal} - r_x|$ gains its minimum. From y onwards the death probability is best expressed by function q_x^{GM} that complements r_x up to 1. The transition is smoothed for the ages $z = (y - 4), \dots, (y + 4)$:

$$q_z^{GM} = 1 - \left[\left(1 - \frac{z - y + 5}{10}\right) \cdot p_z^{equal} + \frac{z - y + 5}{10} \cdot r_z \right]$$

Summary: The input death probabilities for the mortality life tables are:

- at the age of 0.....infant mortality rate
- at the age of 1, 2, 3probabilities based on age-specific mortality rates
- at the age of 4 to $(y - 5)$ equalized probabilities
- at the age of $(y - 4)$ to $\omega-1$equalized and adjusted probabilities