

## Methodological notes

The complete mortality life tables for the Czech Republic and its NUTS-3 Regions are calculated from the third main group of demographic events (age-period specific), separately for males and females. The complete mortality life tables for regions have been processed for a two-year period to exclude random fluctuations.

### Complete life tables indicators:

**Death probability ( $q_x$ )** expresses the probability that an individual at the exact age of  $x$  will die in a given period, i.e. before the exact age of  $x+1$ . The input death probabilities are derived from the age-specific mortality rates ( $m_x$ ).

$$q_x = 1 - e^{-m_x}$$

**Probability of survival ( $p_x$ )** expresses the probability that an  $x$ -year-old individual will not die in a given period and will survive to the age of  $x+1$ . It complements the death probability up to 1.

$$p_x = 1 - q_x$$

**Table number of survivors ( $l_x$ )** is the hypothetical number of individuals alive at the exact age of  $x$  out of 100,000 live births (table root -  $l_0$ ), given the mortality conditions of the reference period.

$$l_{x+1} = p_x * l_x$$

**Table number of deaths ( $d_x$ )** is the hypothetical number of individuals who die at the age of  $x$ .

$$d_x = l_x - l_{x+1}$$

**Table number of person-years ( $L_x$ )** expresses the hypothetical number of individuals alive at the completed age of  $x$ . It is calculated as the average of two subsequent table numbers of survivors, except for the age of 0.

$$L_x = \frac{l_x + l_{x+1}}{2}$$

$$L_0 = l_0 - \alpha * d_0$$

Table number of person-years at the age of 0 is derived from the frequency distribution of infant deaths during the analysed period by the year of born. The coefficient " $\alpha$ " informs what is the ratio of infant deaths born in the previous year from all infant deaths in analysed year. In the life tables for regions the coefficient reflects the situation of the whole Czech Republic during two analysed years (age-cohort specific deaths from the first main group of demographic events).

**Auxiliary indicator ( $T_x$ )** expresses the number of years of life to be lived by the table generation (not of an individual) at a given age. It is the accumulation of  $L_x$  from the highest age of the table ( $\omega-1$ ) to the age of  $x$ .

$$T_x = T_{x+1} + L_x$$

$$T_x = \sum_{\omega-1}^x L_x$$

**Life expectancy ( $e_x$ )** indicates the expected remaining life duration with presumption of unchangeable mortality conditions of a given year. It is a synthetic indicator displaying mortality conditions of a given year through all age groups.

$$e_x^0 = \frac{T_x}{l_x}$$

**Calculation of input death probability for the complete life table:**

Input data:

- the number of deaths (D) by age and sex during an analysed year t (years t, t-1 for Regions)
- the number of deaths at the age of 0 in the Czech Republic by the year of born
- the number of people (P) by age and sex as of 1<sup>st</sup> July of a year in the whole Czech Republic and as of 1<sup>st</sup> January of a given year t in NUTS-3 Regions
- the number of live-births (N<sup>v</sup>) by sex for the whole CR and all Regions

Method:

1. Calculation of the age-specific mortality rates (m<sub>x</sub>) for age x ≥ 1 from an empirical data.

$${}_t m_x^{CR} = \frac{{}_t D_x}{1.7.t P_x} \qquad {}_{t-1,t} m_x^{Reg} = \frac{{}_{t-1} D_x + {}_t D_x}{1.1.t P_x} / 2$$

2. Calculation of the death probabilities based on continuous function  $q_x = 1 - e^{-m_x}$ . The death probability at the age of 0 is equal to infant mortality rate.

$${}_t q_0^{CR} = \frac{{}_t D_0}{{}_t N^v} \qquad {}_{t-1,t} q_0^{Reg} = \frac{{}_{t-1} D_0 + {}_t D_0}{{}_{t-1} N^v + {}_t N^v}$$

3. In order to exclude random fluctuations the death probabilities are equalized at the age of 4 and more.

$$q_x^{equal} = [105 * q_x + 90 * (q_{x-1} + q_{x+1}) + 45 * (q_{x-2} + q_{x+2}) - 30 * (q_{x-3} + q_{x+3})] / 315$$

4. After equalization the function (for the older ages) is further adjusted by Gompertz-Makeham formula  $\log p_x = a + b * c^x$  with undermentioned input:

$$\ln p_x^{equal} = \ln(1 - q_x^{equal})$$

The Czech Statistical Office uses King-Hardy method of extrapolation where d is the interval length and x<sub>0</sub> is the start age of the first interval (here x<sub>0</sub> = 60 and d = 8).

$$R_1 = \sum_{i=x_0}^{x_0+d-1} \ln p_i^{equal} \qquad R_2 = \sum_{i=x_0+d}^{x_0+2d-1} \ln p_i^{equal} \qquad R_3 = \sum_{i=x_0+2d}^{x_0+3d-1} \ln p_i^{equal}$$

5. Calculation of constants included in Gompertz-Makeham formula according to:

$$c^d = \frac{R_3 - R_2}{R_2 - R_1} \qquad c = \sqrt[d]{c^d} \qquad b = \frac{(c-1) * (R_2 - R_1)}{c^{x_0} * (c^d - 1)^2} \qquad a = \left[ R_1 - \frac{(R_2 - R_1)}{(c^d - 1)} \right] / d$$

6. Calculation of model probability of survival  $r_x = \exp(a + b * c^x)$  for the age x ≥ 71. Find the age y (y ≥ 75) where  $|p_x^{equal} - r_x|$  gains its minimum. From y onwards the death probability is best expressed by function q<sub>x</sub><sup>GM</sup> that complements r<sub>x</sub> up to 1. The transition is smoothed for the ages z = (y - 4), ..., (y + 4):

$$q_z^{GM} = 1 - \left[ \left(1 - \frac{z - y + 5}{10}\right) * p_z^{equal} + \frac{z - y + 5}{10} * r_z \right]$$

Summary: The input death probabilities for the mortality life tables are:

- at the age of 0.....infant mortality rate
- at the age of 1, 2, 3 .....probabilities based on age-specific mortality rates
- at the age of 4 to (y - 5) .....equalized probabilities
- at the age of (y - 4) to ω-1.....equalized and adjusted probabilities