Predictive Estimation of Finite Population Mean Using Exponential Estimators

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Abstract

This paper suggested the ratio-type and product-type exponential estimators of the population mean of a study variable through predictive approach using Bahl and Tuteja (1991) ratio-type and product-type exponential estimators as a predictor of the mean of the unobserved units of the population. Properties of the suggested estimators are studied up to first order of approximation in simple random sampling using information on an auxiliary variable. The theoretical conditions under which the suggested estimators are less biased and more efficient than the usual unbiased, ratio, product estimators and estimators due to Srivastava (1983) and Bahl and Tuteja (1991) have been obtained. In support of the theoretical study numerical illustration is also given and determined that the suggested estimators showed also an improvement over the classical estimators empirically.

Keywords	**JEL code**
Predictive approach, auxiliary information, population mean, exponential estimators, bias, mean squared error	*C13, C83*

INTRODUCTION

Sample surveys are widely used as a cost effective apparatus of data collection and for making valid inference about population parameters. Since in sample surveys the sample is only a part of the whole, extrapolation inevitably leads to errors. The main aim of survey statisticians is to reduce the errors either by devising suitable sampling schemes or by formulating efficient estimators of the parameters, see Singh and Solanki (2012, 2013) or both. To detract the errors various researchers have attempted to use additional information, which is correlated to the information under the study and about which the information is available before start of the survey known as auxiliary information. The literature on survey sampling describes a great variety of techniques/approaches including design based and model based methods for using auxiliary information to obtain more efficient estimators.

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In the predictive approach a model is specified for the population values and is used to predict the non-sampled values. Prediction theory for sample surveys (or model-based theory) can be con¬sidered as a general framework for statistical inferences on the character of finite population. Well known estimators of population parameters encountered in the classical theory, as expansion, ratio, regression, another estimators can be predictors in the general prediction theory under some special model. Several authors have applied the predictive approach either to form new predictive estimators or to examine the existing estimators from the predictive viewpoint. It is observed that the use of usual unbiased, ratio and regression estimators as a predictor for the mean of the unobserved units of the population result in the corresponding customary (usual) estimators of the mean of whole population.

Srivastava (1983) has shown that if the usual product estimator is used as a predictor for the mean of the unobserved units of the population, the resulting estimator of the mean of the whole population is different from the customary (usual) product estimator. Biradar and Singh (1998), Agrawal and Roy (1999) and Nayak and Sahoo (2012) provided some predictive estimators for finite population variance. Sahoo and Panda (1999) developed the regression type estimator for two stage sampling procedure. Sahoo and Sahoo (2001) and Sahoo et al. (2009) introduced a class of estimators for the finite population mean availing information on two auxiliary variables in two stage sampling. Ahmed (2004) proposed some estimators for finite population mean in two stage sampling using multivariate auxiliary information. Saini (2013) proposed a class of predictive estimators for two stage design consisting especially of two estimators namely ratio and regression.

In the preset study we attempt to examine the existing Bahl and Tuteja (1991) exponential estimators as predictor of the mean of the unobserved units of the population using the information of observed units in sample. Remaining part of the paper is organized as follows: Section 1 defines some notations and discusses some existing estimators of population mean. We suggest estimators with their properties in Section 2. We perform the theoretical comparison among different estimators in the Sections 3 and 4. In Section 5, the real data sets are used to observe the performance of various estimators numerically. Finally, last section provides some concluding remarks.

1 THE NOTATIONS

Much literature has been produced on sampling from finite populations to address the issue of the efficient estimation of the mean (or total) of a survey variable when auxiliary variables are available. Our analysis refers to simple random sampling without replacement (SRSWOR) and considers, for brevity, the case when only a single auxiliary variable is used.

Consider a finite population $U = (U_1, U_2, ..., U_N)$ of N units on which the study (survey) variable y and auxiliary variable x are defined, which take values $y1$ and $y2$ respectively for the unit U_1 of $U\,(1 \leq i \leq N)$. We are interested in estimating the population mean:

$$\overline{Y} = \frac{\sum_{i=1}^{N} y_i}{N},$$

of the study variable y on the basis of observed values of y on the units of a sample taken from finite population U. Any ordered subset of U is called a sample from U. Let S denote the collection of all possible samples from U. For any given $s \in S$, let $\vartheta(s)$ denote its effective sample size (the number of distinct units in (s) and $\bar{s}$ denote the set of all those units of U which are not in s. We designate:

$$\overline{Y}_s = \frac{1}{\vartheta(s)} \sum_{i \in s} y_i ,$$

$$\overline{Y}_{\bar{s}} = \frac{1}{(N - \vartheta(s))} \sum_{i \in \bar{s}} y_i .$$

For any given $s \in S$, we can write:

$$\overline{Y} = \left[\frac{\vartheta(s)}{N} \, \overline{Y}_s + \frac{(N - \vartheta(s))}{N} \, \overline{Y}_{\bar{s}} \right]. \tag{1}$$

In the representation of population mean $\overline{Y}$ at (1), the sample mean $\overline{Y}$ is known because it is based on the units of the sample s whose y values have been observed. Therefore, the statisticians should attempt a prediction of the mean $\overline{Y}_{\bar{s}}$ of the unobserved units of the population U on the basis of observed units in s. While admitting that a decision-theorist might object to making the choice of estimator after looking at the data, Basu (1971) nevertheless considered such an approach to represent the "heart of the matter" in estimating the finite population mean [see Cessal et al. (1977, p. 110)].

For a simple random sampling procedure with sample size n (*i.e.* $\vartheta(s) = n$) and the sample mean:

$$\bar{y} = \frac{1}{n} \sum_{i \in \bar{s}} y_i \,, \ (i.e. \ \overline{Y}_s = \bar{y}).$$

We can write (1) as:

$$\overline{Y} = \left[\frac{n}{N} \, \overline{Y}_s + \frac{(N - n)}{N} \, \overline{Y}_{\bar{s}} \right]. \tag{2}$$

From (2), an estimator of population mean $\overline{Y}$ can be written as:

$$t = \left[\frac{n}{N} \, \bar{y} + \frac{(N - n)}{N} \, T \right],$$

where T is considered as a predictor of $\overline{Y}_{\bar{s}}$.

Srivastava (1983) has shown that if we adopt the prediction approach described earlier, use of:

$$\bar{y} = \frac{1}{n} \sum_{i \in \bar{s}} y_i \qquad \text{(mean per unit estimator)},$$

$$\bar{y}_{\overline{lr}} = [\bar{y} + b(\overline{X}_{\bar{s}} - \bar{x})] \quad \text{(the regression estimator)},$$

$$\bar{y}_{\overline{R}} = \overline{X}_{\bar{s}} \left(\frac{\bar{y}}{\bar{x}} \right) \qquad \text{(the ratio estimator)},$$

for predicting the mean $\overline{Y}_{\bar{s}}$ of the unobserved units of the population result in the corresponding customary:

$$\bar{y} = \frac{1}{n} \sum_{i \in \bar{s}} y_i \qquad \text{(mean per unit estimator)},$$

$$\bar{y}_{lr} = [\bar{y} + b(\overline{X} - \bar{x})] \quad \text{(the regression estimator)},$$

$$\bar{y}_R = \bar{X}\left(\frac{\bar{y}}{\bar{x}}\right) \qquad \text{(the ratio estimator),}$$

of the population mean $\bar{Y}$, [*i.e.* if $T = \bar{y}$, $t = \bar{y}$; $T = \bar{y}_{\bar{R}}$, $t = \bar{y}_R$; $T = \bar{y}_{\bar{lr}}$, $t = \bar{y}_{lr}$], where b is the regression coefficient estimated from the sample s and:

$$\bar{x} = \frac{1}{n}\sum\nolimits_{i \in s} x_i ,$$

$$\bar{X} = \frac{1}{n}\sum\nolimits_{i=1}^{N} x_i ,$$

$$\bar{X}_{\bar{s}} = \frac{1}{(N-n)}\sum\nolimits_{i \in \bar{s}} x_i = \frac{(N\bar{X} - n\bar{x})}{(N-n)} .$$

However, if the product estimator:

$$\bar{y}_{\bar{P}} = \bar{y}\left(\frac{\bar{x}}{\bar{X}_{\bar{s}}}\right)$$

is used with such an approach, the resulting estimator of population mean $\bar{Y}$ is not the customary product estimator:

$$\bar{y}_P = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right),$$

i.e. if:

$$T = \bar{y}_{\bar{P}},\ t = \frac{n\bar{X} + (N-2n)\bar{x}}{(N\bar{X} - n\bar{x})} = t_P .$$

To the first degree of approximation the biases and mean squared errors (*MSE*s) of the estimators $\bar{y}_R$, $\bar{y}_P$ and t_P are respectively given by:

$$Bias(\bar{y}_R) = \theta\bar{Y}C_x^2(1 - C), \tag{3}$$

$$Bias(\bar{y}_P) = \theta\bar{Y}C^2\, C, \tag{4}$$

$$Bias(t_P) = \theta\bar{Y}C_x^2\,[C + f(1-f)^{-1}], \tag{5}$$

$$MSE(\bar{y}_R) = \theta\bar{Y}C^2\,[C_y^2 + C_x^2(1 - 2C)], \tag{6}$$

$$MSE(\bar{y}_P) = MSE(t_P) = \theta\bar{Y}^2\,[C_y^2 + C_x^2(1 + 2C)], \tag{7}$$

where:

$\theta = (1 - f)^{-1}, f = (n/N), C_y^2 = (S_y^2 / \bar{Y}^2), C_x^2 = (S_x^2 / \bar{X}^2), S_y^2 = (N-1)^{-1} \sum_{i=1}^{N} (y_i - \bar{Y})^2$,

$S_x^2 = (N-1)^{-1} \sum_{i=1}^{N} (x_i - \bar{X})^2$, $C = \rho(C_y / C_x), \rho = S_{yx}/(S_y S_x))$ and

$S_{yx} = (N-1)^{-1} \sum_{i=1}^{N} (y_i - \bar{Y})(x_i - \bar{X})$.

Bahl and Tuteja (1991) suggested the ratio-type and product- type exponential estimators of the population mean $\bar{Y}$ respectively as:

$$\bar{y}_{Re} = \bar{y}\, exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right),$$
$$\bar{y}_{Pe} = \bar{y}\, exp\left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}}\right).$$

To first degree of approximation, the biases and mean squared errors of $\bar{y}_{Re}$ and $\bar{y}_{Pe}$ are respectively given by:

$$Bias(\bar{y}_{Re}) = \frac{\theta}{8} \bar{Y} C_x^2 (3 - 4C), \tag{8}$$

$$Bias(\bar{y}_{Pe}) = \frac{\theta}{8} \bar{Y} C_x^2 (4C - 1), \tag{9}$$

$$MSE(\bar{y}_{Re}) = \theta \bar{Y}^2 \left[C_y^2 + \frac{C_x^2}{4}(1 - 4C)\right], \tag{10}$$

$$MSE(\bar{y}_{Pe}) = \theta \bar{Y}^2 \left[C_y^2 + \frac{C_x^2}{4}(1 + 4C)\right]. \tag{11}$$

In the following Sec. 2 we have suggested alternative ratio-type and product-type exponential estimators of population mean $\bar{Y}$ by using $\bar{y}_{Re}$ and $\bar{y}_{Pe}$ as a predictor T of $\bar{Y}_{\bar{s}}$ of the unobserved units of the population U on the basis of observed units in s. The biases and mean squared errors of suggested ratio-type and product-type exponential estimators up to first order approximation have obtained.

2 THE SUGGESTED PREDECTION APPROACH

In case, information on an auxiliary variable x positively correlated with the study variable y is available and one intends to use this in the form of Bahl and Tuteja (1991) ratio-type exponential estimator $\bar{y}_{Re}$, an obvious choice for T is:

$$\bar{y}_{\overline{Re}} = \bar{y}\, exp\left(\frac{\bar{X}_s - \bar{x}}{\bar{X}_s + \bar{x}}\right).$$

For this choice of T:

$$t = t_{Re} = \left[\frac{n}{N}\bar{y} + \left(\frac{N-n}{N}\right)\bar{y}\, exp\left(\frac{\bar{X}_{\bar{s}} - \bar{x}}{\bar{X}_{\bar{s}} + \bar{x}}\right)\right] = \left[\frac{n}{N}\bar{y} + \left(\frac{N-n}{N}\right)\bar{y}\, exp\left(\frac{N(\bar{X} - \bar{x})}{N(\bar{X} - \bar{x}) - 2n\bar{x}}\right)\right], \tag{12}$$

which is not the Bahl and Tuteja (1991) ratio-type exponential estimator $\bar{y}_{Re}$.

If the auxiliary variable x is negatively correlated with the study variable y and one wants to use this in the form of Bahl and Tuteja (1991) product-type exponential estimator $\bar{y}_{Pe}$ an obvious choice for T is:

$$\bar{y}_{\overline{Pe}} = \bar{y}\, exp\left(\frac{\bar{x} - \bar{X}_{\bar{s}}}{\bar{x} + \bar{X}_{\bar{s}}}\right).$$

For this choice of T:

$$t = t_{Pe} = \left[\frac{n}{N}\bar{y} + \left(\frac{N-n}{N}\right)\bar{y}\, exp\left(\frac{\bar{x} - \bar{X}_{\bar{s}}}{\bar{x} + \bar{X}_{\bar{s}}}\right)\right] = \left[\frac{n}{N}\bar{y} + \left(\frac{N-n}{N}\right)\bar{y}\, exp\left(\frac{N(\bar{x} - X)}{N\bar{X} + (N-2n)\bar{x}}\right)\right], \quad (13)$$

which is not the Bahl and Tuteja (1991) product-type exponential estimator $\bar{y}_{Pe}$.

To obtain the biases and MSEs of t_{Re} and t_{Pe}, we define:

$$e_0 = \left(\frac{\bar{y} - \bar{Y}}{\bar{Y}}\right) \text{ and } e_1 = \left(\frac{\bar{x} - \bar{X}}{\bar{X}}\right),$$

such that:

$$E(e_0) = E(e_1) = 0\,,$$

and up to first degree of approximation:

$$E(e_0^2) = \theta C_y^2\,,$$

$$E(e_1^2) = \theta C_x^2\,,$$

$$E(e_0 e_1) = \theta C C_x^2\,,$$

Expressing (12) in terms of e's, we have:

$$\begin{aligned} t_{Re} &= \bar{Y}(1+e_0)\left[\frac{n}{N} + \left(\frac{N-n}{N}\right) exp\left(\frac{Ne_1}{2(N-n) + (N-2n)e_1}\right)\right] \\ &= \bar{Y}(1+e_0)\left[f + (1-f) exp\left(-\frac{e_1}{2(1-f) + (1-2f)e_1}\right)\right] \\ &= \bar{Y}(1+e_0)\left[f + (1-f) exp\left\{-\frac{e_1}{2(1-f)}\left(1 + \frac{(1-2f)}{2(1-f)}e_1\right)^{-1}\right\}\right]. \end{aligned} \quad (14)$$

Expanding the right hand side of (14), multiplying out and neglecting terms of e's having power greater than two we have:

$$t_{Re} \approx \bar{Y}\left[1 + e_0 - \frac{e_1}{2} - \frac{e_0 e_1}{2} + \frac{e_1^2}{8}(3-4f)\right],$$

or: $$(t_{Re} - \bar{Y}) \approx \bar{Y}\left[e_0 - \frac{e_1}{2} - \frac{e_0 e_1}{2} + \frac{e_1^2}{8}(3-4f)\right], \quad (15)$$

Taking expectation of both sides of (15), we get the bias of t_{Re} to the first degree of approximation as:

$$Bias(t_{Re}) = \frac{\theta}{8}\bar{Y}C_x^2[3 - 4(C+f)]. \quad (16)$$

Squaring both sides of (15) and neglecting terms of e's having power greater than two, we have:

$$(t_{Re} - \bar{Y})^2 \approx \bar{Y}^2\left(e_0^2 + \frac{e_1^2}{4} - e_0 e_1\right). \tag{17}$$

Taking expectation of both sides of (17) we get the MSE of t_{Re} to the first degree of approximation as:

$$MSE(t_{Re}) = \theta\bar{Y}^2\left[C_y^2 + \frac{C_x^2}{4}(1 - 4C)\right], \tag{18}$$

which equals to the *MSE* of Bahl and Tuteja (1991) ratio-type exponential estimator $\bar{y}_{Re}$.

Now expressing t_{Pe} in terms of e's, we have:

$$t_{Pe} = \bar{Y}(1 + e_0)\left[\frac{n}{N} + \left(\frac{N-n}{N}\right) exp\left(\frac{Ne_1}{2(N-n) + Ne_1}\right)\right]$$

$$= \bar{Y}(1 + e_0)\left[f + (1-f) exp\left(-\frac{e_1}{2(1-f) + e_1}\right)\right]$$

$$= \bar{Y}(1 + e_0)\left[f + (1-f) exp\left(\frac{e_1}{2(1-f)}\left(1 + \frac{e_1}{2(1-f)}\right)^{-1}\right)\right]. \tag{19}$$

Expanding the right hand side of (19), multiplying out and neglecting terms of e's having power greater than two we have:

$$t_{Pe} \approx \bar{Y}\left[1 + e_0 + \frac{e_1}{2} + \frac{e_0 e_1}{2} - \frac{e_1^2}{8(1-f)}\right]$$

or: $$(t_{Pe} - \bar{Y}) \approx \bar{Y}\left[e_0 + \frac{e_1}{2} + \frac{e_0 e_1}{2} - \frac{e_1^2}{8(1-f)}\right]. \tag{20}$$

Taking expectation of both sides of (20), we get the bias of t_{Pe} to the first degree of approximation as:

$$Bias(t_{Pe}) = \frac{\theta}{8}\bar{Y}C_x^2\left(4C - \frac{1}{(1-f)}\right). \tag{21}$$

Squaring both sides of (20) and neglecting terms of e's having power greater than two we have:

$$(t_{Pe} - \bar{Y})^2 \approx \bar{Y}^2\left(e_0^2 + \frac{e_1^2}{4} + e_0 e_1\right). \tag{22}$$

Taking expectation of both sides of (22), we get the MSE of t_{Pe} to the first degree of approximation as:

$$MSE(t_{Pe}) = \theta\bar{Y}^2\left[C_y^2 + \frac{C_x^2}{4}(1 + 4C)\right], \tag{23}$$

which equals to the MSE of Bahl and Tuteja (1991) product-type exponential estimator $\bar{y}_{Pe}$.

3 BIAS COMPARISON

In this Section we have compared the absolute biases of the different estimators of the population mean $\bar{Y}$. The relevant conditions are given in which the proposed ratio-type exponential estimator $\bar{y}_{Re}$ (product-type exponential estimator t_{Pe}) is less biased to usual ratio estimator $\bar{y}_R$ (product estimator $\bar{y}_P$) and Bahl and Tuteja (1991) ratio-type exponential estimator $\bar{y}_{Re}$ (product-type exponential estimator $\bar{y}_{Pe}$).

3.1 Bias comparison of ratio type estimators

From (3), (8) and (16), we have:

(i) $|Bias(t_{Re})| < |Bias(\bar{y}_R)|$,if:

$$\frac{1}{8}\,|3-4(C+f)| < |1-C|\,,$$

i.e. if:

$$[48C^2 - 104C - 16f^2 + 24f - 32Cf + 55] > 0\,. \tag{24}$$

(ii) $|Bias(t_{Re})| < |Bias(\bar{y}_{Re})|$,

if:

$$3-4(C+f)| < |3-4C|\,,$$

i.e. if:

$$C < \frac{1}{4}\,(3-2f)\,. \tag{25}$$

If the conditions (24) and (25) are satisfied the proposed alternative ratio-type exponential estimator t_{Re} is less biased respectively to customary ratio estimator $\bar{y}_R$ and Bahl and Tuteja (1991) ratio-type exponential estimator $\bar{y}_{Re}$.

3.2 Bias comparison of product type estimators

From (4), (5), (9) and (21), we have:

(i) $|Bias(t_{Pe})| < |Bias(\bar{y}_P)|$,

if:

$$\frac{1}{8}\left|4C - \frac{1}{(1-f)}\right| < |C|\,,$$

i.e. if:

$$\left[48C^2 + \frac{8C}{(1-f)} - \frac{1}{(1-f)^2}\right] > 0\,. \tag{26}$$

(ii) $|Bias(t_{Pe})| < |Bias(t_P)|$

If:

$$\frac{1}{8}\left|4C - \frac{1}{(1-f)}\right| < \left|C + \frac{f}{(1-f)}\right|,$$

i.e. if:

$$\left[48C^2 + \frac{(8f-1)(8f+1)}{(1-f)^2} - \frac{2C(4+f)}{(1-f)}\right] > 0\,. \tag{27}$$

(iii) $|Bias(t_{Pe})| < |Bias(\bar{y}_{Pe})|$

If:

$$\left|4C - \frac{1}{(1-f)}\right| < |4C-1|\,,$$

i.e. if:

$$C > \frac{(2-f)}{8(1-f)}\,. \tag{28}$$

If the conditions (26), (27) and (28) are satisfied the proposed alternative product-type exponential estimator t_{Pe} is less biased respectively to customary product estimator $\bar{y}_P$, Srivastava (1983) product estimator t_P and Bahl and Tuteja (1991) product-type exponential estimator $\bar{y}_{Pe}$.

4 EFFICIENCY COMPARISON

In this Section we have obtained the conditions under which the proposed ratio-type exponential estimator t_{Re} and Bahl and Tuteja (1991) ratio-type exponential estimator $\bar{y}_{Re}$ [product-type exponential estimator t_{Pe} and Bahl and Tuteja (1991) product-type exponential estimator $\bar{y}_{Pe}$] are better than the usual unbiased estimator $\bar{y}$ and ratio estimator $\bar{y}_R$ (product estimator $\bar{y}_P$).

It is very well known under simple random sampling without replacement that the:

$$MSE(y) = Var(y) = \theta Y^2 C_y^2\,. \tag{29}$$

4.1 Efficiency comparison of ratio type estimators

From (6), (10), (18) and (29), we have:

$$\text{(i)}\quad MSE(\bar{y}_R) < MSE(\bar{y}) \text{ if } C > \frac{1}{2}\,. \tag{30}$$

$$\text{(ii)}\quad [MSE(t_{Re}) = MSE(\bar{y}_{Re})] < MSE(\bar{y}) \text{ if } C > \frac{1}{4}\,. \tag{31}$$

$$\text{(iii)}\quad [MSE(t_{Re}) = MSE(\bar{y}_{Re})] < MSE(\bar{y}_R) \text{ if } C < \frac{3}{4}\,. \tag{32}$$

Thus from (31) and (32) it follows that if the condition:

$$\left(\frac{1}{4} < C < \frac{3}{4}\right), \tag{33}$$

is satisfied the proposed estimator t_{Re} and Bahl and Tuteja (1991) estimator $\bar{y}_{Re}$ are better than the usual unbiased estimator $\bar{y}$ and usual ratio estimator $\bar{y}_R$.

4.2 Efficiency comparison of product type estimators

From (7), (11), (23) and (29), we have:

$$\text{(i)} \quad [MSE(\bar{y}_P) = MSE(t_P)] < MSE(\bar{y}) \text{ if } C < -\frac{1}{2}. \tag{34}$$

$$\text{(ii)} \quad [MSE(t_{Pe}) = MSE(\bar{y}_{Pe})] < MSE(\bar{y}) \text{ if } C < -\frac{1}{4}. \tag{35}$$

$$\text{(iii)} \quad [MSE(t_{Pe}) = MSE(\bar{y}_{Pe})] < [MSE(\bar{y}_P) = MSE(t_P)] \text{ if } C > -\frac{3}{4}. \tag{36}$$

Thus from (35) and (36) it follows that the condition:

$$\left(-\frac{3}{4} < C < -\frac{1}{4}\right), \tag{37}$$

is sufficient for the proposed estimator t_{Pe} and Bahl and Tuteja (1991) estimator $\bar{y}_{Pe}$ are better than the usual unbiased estimator $\bar{y}$ and usual product estimator $\bar{y}_P$ and Srivastava (1983) estimator t_P.

5 EMPIRICAL STUDY

To judge the merits of the suggested estimators t_{Re} and t_{Pe} over the estimators $\bar{y}$, $\bar{y}_R$, $\bar{y}_P$, $\bar{y}_{Re}$, $\bar{y}_{Pe}$ and t_P we have considered four natural population data sets. The description of the population data sets are given in Table 1.

Table 1 The population data sets

Population	N	n	C_x	C_y	ρ	C
I: Steel and Torrie (1960, p. 282) y: Log of leaf burn in sec. x: Chlorine percentages	30	6	0.7493	0.7000	0.4996	0.4667
II: Murthy (1967, p. 228) y: Output x: Fixed capital	80	20	0.7507	0.3542	0.9413	0.4441
III: Das (1988) y: The number of agricultural laborers for 1961 x: The number of agricultural laborers for 1971	278	60	1.6198	1.4451	0.7213	0.6435
IV: Cochran (1977) y: The number of persons per block x: The numbers of rooms per block	20	8	0.1281	0.1445	0.6500	0.7332

Source: Own construction

To examine the biasedness of various estimators of population mean $\bar{Y}$ we have computed the following quantities:

$$Q_{R1} = \left|\frac{Bias\,(\bar{y}_R)}{\theta \bar{Y} C_x^2}\right| = |1 - C|\,,\; Q_{R2} = \left|\frac{Bias\,(\bar{y}_{Re})}{\theta \bar{Y} C_x^2}\right| = \frac{1}{8}|3 - 4C|\,,\; Q_{R3} = \left|\frac{Bias\,(t_{Re})}{\theta \bar{Y} C_x^2}\right| = \frac{1}{8}|3 - 4(C + f)|\,,$$

$$Q_{P1}=\left|\frac{Bias\,(\bar{y}_P)}{\theta\bar{Y}C_x^2}\right|=|C|\,,\;Q_{P2}=\left|\frac{Bias\,(t_P)}{\theta\bar{Y}C_x^2}\right|=\left|C+\frac{f}{(1-f)}\right|,\;Q_{R3}=\left|\frac{Bias\,(\bar{y}_{Pe})}{\theta\bar{Y}C_x^2}\right|=\frac{1}{8}\,|4C-1)|\,,$$

$$Q_{P4}=\left|\frac{Bias\,(t_{Pe})}{\theta\bar{Y}C_x^2}\right|=\frac{1}{8}\left|4C-\frac{1}{(1-f)}\right|,$$

and findings are shown in Table 2.

Table 2 Values of the quantities $Q_{Ri}\,(i=1,2,3)$ and $(j=1,2,3,4)$

	Quantities						
Population	Q_{R1}	Q_{R2}	Q_{R3}	Q_{P1}	Q_{P2}	Q_{P3}	Q_{P4}
I	0.5333	0.1416	**0.0416**	0.4667	0.7167	0.1084	0.0771
II	0.5559	0.1529	**0.0279**	0.4441	0.7775	0.0971	0.0554
III	0.3565	**0.0532**	0.0547	0.6435	0.9174	0.1967	0.1625
IV	0.2668	**0.0084**	0.1916	0.7332	1.3998	0.2416	0.1583

Note: Bold numbers indicate the least biased value in the relevant data set.
Source: Own construction

It is observed from Table 2 that:

(i) The proposed ratio-type exponential estimator t_{Re} is less biased than the customary ratio estimator $\bar{y}_R$ (i.e. $Q_{R3} < Q_{R1}$) for all population data sets I–IV because the condition (24) is satisfied for all the data sets.

(ii) The proposed ratio-type exponential estimator t_{Re} is less biased than the Bahl and Tuteja (1991) ratio-type exponential estimator $\bar{y}_R$ (i.e. $Q_{R3} < Q_{R2}$) only for data sets I and II because the condition (25) is not fulfill in data sets III and IV.

(iii) The proposed product-type exponential estimator t_{Pe} is less biased than the customary product estimator $\bar{y}_P$, Srivastava (1983) product estimator t_P and Bahl and Tuteja (1991) product-type exponential estimator $\bar{y}_{Pe}$ (i.e. $Q_{P4} < Q_{Pj}$, $j = 1, 2, 3$) for all the population data sets I-IV because the conditions (26), (27) and (28) are satisfied in all of the data sets.

To see the relative performances of different estimators of the population mean $\bar{Y}$ we have computed the percent relative efficiencies (*PRE*s) of the estimators with respect to the usual unbiased estimator $\bar{y}$ by following formulae:

$$PRE(\bar{y}_R,\bar{y})=\frac{MSE(\bar{y})}{MSE(\bar{y}_R)}=\frac{C_y^2}{[C_y^2+C_x^2(1-2C)]}\times 100,$$

$$PRE(\bar{y}_P,\bar{y})=\frac{MSE(\bar{y})}{MSE(\bar{y}_P)}=\frac{C_y^2}{[C_y^2+C_x^2(1+2C)]}\times 100=PRE(t_P,\bar{y}),$$

$$PRE(\bar{y}_P,\bar{y})=\frac{MSE(\bar{y})}{MSE(\bar{y}_P)}=\frac{C_y^2}{[C_y^2+C_x^2(1+2C)]}\times 100=PRE(t_P,\bar{y}),$$

$$PRE(\bar{y}_{Pe},\bar{y})=\frac{MSE(\bar{y})}{MSE(\bar{y}_{Pe})}=\frac{C_y^2}{\left[C_y^2+\frac{C_x^2}{4}\,(1+4C)\right]}\times 100=PRE(t_{Pe},\bar{y}),$$

and finding are shown in Table 3.

Table 3 The *PRE*s of different estimators with respect to $\bar{y}$

Population	$PRE(\bar{y}_R, \bar{y})$	$PRE(\bar{y}_{Re}, \bar{y}) = PRE(t_{Re}, \bar{y})$	$PRE(\bar{y}_P, \bar{y}) = PRE(t_P, \bar{y})$	$PRE(\bar{y}_{Pe}, \bar{y}) = PRE(t_{Pe}, y)$
I	92.9156	**133.0374**	31.1004	54.9076
II	66.5810	**781.3982**	10.5463	24.2836
III	156.3967	**197.7846**	25.8171	47.1121
IV	157.8695	**161.2267**	34.0327	56.4111

Note: Bold numbers indicate the largest *PRE* in relevant data set.
Source: Own construction

It is observed from Table 3 that:

(i) The proposed ratio-type estimator t_{Re} and Bahl and Tuteja (1991) ratio estimator $\bar{y}_R$ both have the largest percent relative efficiency than the usual unbiased estimator and usual ratio estimator $\bar{y}_R$ in all the population data sets I-IV because the condition (33) has been satisfied by all data sets.

(ii) The suggested product-type estimator t_{Pe} and Bahl and Tuteja (1991) product estimator $\bar{y}_{Pe}$ are superior to the usual product estimator $\bar{y}_P$ and Srivastava (1983) estimator t_P because the condition (36) is satisfied by all data set I–IV but inferior to the usual unbiased estimator $\bar{y}$ because of dissatisfied condition (35).

(iii) The suggested ratio-type exponential estimator t_{Re} has maximum percent relative efficiency (= **781.3982**) in population II as well as least bias (= **0.0279**). Therefore, the proposed ratio-type exponential estimator t_{Re} appears to be the best in the sense of having largest percent relative efficiency as well as least bias in Population II.

CONCLUSION

We have utilized Bahl and Tuteja (1991) ratio-type and product-type exponential estimators as a predictor of the mean of the unobserved units of the population and observed that the resulting ratio-type and product-type exponential estimators of the mean of the whole population are different from the customary Bahl and Tuteja (1991) ratio and product estimators. The biases and mean squared errors of suggested ratio-type and product-type exponential estimators, up to first order approximation are obtained and observed that the mean squared errors of suggested ratio-type and product-type exponential estimators are equal to the Bahl and Tuteja (1991) ratio-type and product-type exponential estimators respectively. The theoretical conditions under which the proposed estimators are less biased and more efficient than the usual unbiased, ratio, product estimators and estimators due to Srivastava (1983) and Bahl and Tuteja (1991) have been obtained. It has been also found empirically that the suggested estimators are less biased and more efficient than other existing estimators if the theoretical conditions under which the proposed estimators are less biased and more efficient are satisfied. Thus we recommend the use of the proposed estimators in practice. However this conclusion cannot be extrapolated due to limited empirical study.

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