Interregional Flows for the Czech Economy

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Abstract

The paper brings both the methodology and data on the construction of regional flows in the interregional model. We focus on the comparison of the entropy method and the commonly used gravity method. The entropy method is based on minimizing import distances at the determined rate of entropy of the interregional flows of intermediaries. The gravity method is used in its standard form with an additional factor for adjusting the warehouses, and its parameters are estimated for physical flows. The resulting estimates are then applied on the regional input-output tables and are used to construct a standard Leontief interregional model. To analyse the difference between the two models, we use a graphical representation. Furthermore, we assess the percentage deviation of the average Leontief multiplier in the regional submatrices. We proved that, although the interregional output flows appear different and the relative structure of Leontief matrix is different, the resulting impacts on the regions do not fundamentally differ.

Keywords	***JEL code***
Regional Input-Output tables, Input-Output analysis, entropy theory, gravity method	*C67, R13, E21*

INTRODUCTION

Regional Input-Output tables provide the detailed information about regional economy. We have published regional Input-Output Tables for the Czech Republic (2011, 2013) and the methodology, see Sixta and Vltavská (2016). Even if these tables include significant amount of data, their linkages and arrangements into interregional model describing also the product flows between the regions, multiply the usefulness for the users. The crucial point lies in the methods for the arrangements these regional matrices into a one Sigle matrix. Both the entropy method and the gravity method were devised for uses different from those for interregional flow estimation. Given their number of uses, these two methods for regional flow estimation can be considered the main methods for interregional output flow estimation. The gravity method is based on Newton's law of gravitation, where the force of attraction between two objects is proportional to the product of their masses and inversely proportional to the square of the distance between them (all multiplied by the gravitational constant). The application for estimating output flows between cities from the 18th century (Banzhaf, 2000; Kurz and Salvadori, 2000a and 2000b) can be considered

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the first economic use of the gravity method. In the input-output (I-O) analysis, this method was not applied until the 1970s by Leontief and Strout (1963) and Theil (1967), independently of each other. Since then, the gravity method has been used in the I-O analysis and data estimation for flows between regions and/or states themselves in many applications. For example, it has been used to construct multinational I-O tables where data sources are not sufficient – such as in the FIGARO project (Rueda-Cantuche and Rémond Tiedrez, 2016) or world Input-Output tables (Foster and Stehrer, 2010). In the decades following its introduction, the gravity method methodology acquired different forms of gravity equation estimation, especially with regard to the added data sources and the information contained therein (Anderson and van Wincoop, 2003). The main problem of gravity method estimation is that it is necessary to know the output flows in order to make it possible to calibrate the gravity method, which, however, directly thwarts its use. For this reason, some authors choose unit parameter values of the gravity method (e.g. Kieslichová, 2016). However, as shown by other authors, this leads to the degradation of this method to calibration from the inverse distance method (Šafr and Sixta, 2017). The solution of this estimate appears to be the use of alternative data sources available. Some authors use data flows between regions (or states) for other countries to for calibrate the parameters (or their estimation), and then apply these coefficients to the state being examined. This can be used, because the dynamics of relationships are determined by the parameters of variables, but the setting of flow levels is determined by the gravitational constant that can be established exogenously, based on the total level of flows in the economy (or between states). For these reasons, we have decided to use an alternative approach (Šafr and Sixta, 2017), estimating the parameters from the physical output flows in the economy, which will bring the real parameters closer to the territory of the Czech Republic that we examined.

The entropy theory was developed by Edwin T. Jaynes (1922–1998), who first used this method for studying the information theory in statistical mechanics (Jaynes, 1957). The theory is based on Bayesian statistics. However, compared to Bayes theorem, which is used to calculate probability, entropy maximization leads to the "assignment" of the probability of a priori distribution (Jaynes, 1988). The technique has been applied both in technical sciences and economics. In Input-Output analysis, A. G. Wilson can be considered the pioneer of this theory in estimating interregional flows (Wilson, 1970). The application of these methods has been subsequently dealt with by many authors. As a comprehensive view, Sargento's work can be mentioned, dealing with both the numerical optimization of this problem and the possible general solution of the optimization problem (Sargento, 2009). In general, there are two main approaches based on entropy. The principle of the first one is to maximize entropy under the conditions of meeting the sum of exports and imports between regions. In fact, it is the maximum possible distribution (decay) of output flows in the flow matrix, assuming the sum of columns and rows (export and import consistency between regions). The second option of applying this method (used in this article) is to minimize the import distance, assuming the sum of exports and imports and the exogenously determined entropy rate of the entries in the output flow matrix.

1 METHODOLOGY

1.1 Interregional I-O model

This article uses a standard interregional Input-Output model. Based on its construction, this model is considered to be a model "with full information" (Oosterhaven and Hewings, 2014) or Isard's standard model (Isard, 1951). In the view of the I-O model, this model is a model that considers full mutual interconnection of regions, facilitating also to analyse the retrospective impacts on individual regions (type III model according to Lenzen et al., 2004). The core of the I-O analysis is I-O tables (IOTs) and, for regional application, it is regional Input-Output tables (RIOTs). Regional Input-Output tables show the economy structure from the perspective of products and aggregates in regional categorization. This table can be illustrated by the following simplified form – see Figure 1.

Figure 1 Simplified interregional Input-Output table

			Region						Hous. Cons.			Gov. cons.			i_i^{pr}			export	product
	Region	Region	1		2		3		1	2	3	1	2	3	1	2	3		-
		Products	1	2	1	2	1	2	-	-	-	-	-	-	-	-	-		-
Region	1	1	z_{11}^{11}	z_{12}^{11}	z_{11}^{12}	z_{12}^{12}	z_{11}^{13}	z_{12}^{13}	c_1^{11}	c_1^{12}	c_1^{13}	g_1^{11}	g_1^{12}	g_1^{13}	i_1^{11}	i_1^{12}	i_1^{13}	e_1^1	x_1^1
		2	z_{21}^{11}	z_{22}^{11}	z_{21}^{12}	z_{22}^{12}	z_{21}^{13}	z_{22}^{13}	c_2^{11}	c_2^{12}	c_2^{13}	g_2^{11}	g_2^{12}	g_2^{13}	i_2^{11}	i_2^{12}	i_2^{13}	e_2^1	x_2^1
	2	1	z_{11}^{21}	z_{12}^{21}	z_{11}^{22}	z_{12}^{22}	z_{11}^{23}	z_{12}^{23}	c_1^{21}	c_1^{22}	c_1^{23}	g_1^{21}	g_1^{22}	g_1^{23}	i_1^{21}	i_1^{22}	i_1^{23}	e_1^2	x_1^2
		2	z_{21}^{21}	z_{22}^{21}	z_{21}^{22}	z_{22}^{22}	z_{21}^{23}	z_{22}^{23}	c_2^{21}	c_2^{22}	c_2^{23}	g_2^{21}	g_2^{22}	g_2^{23}	i_2^{21}	i_2^{22}	i_2^{23}	e_2^2	x_2^2
	3	1	z_{11}^{31}	z_{12}^{31}	z_{11}^{32}	z_{12}^{32}	z_{11}^{33}	z_{12}^{33}	c_1^{31}	c_1^{32}	c_1^{33}	g_1^{31}	g_1^{32}	g_1^{33}	i_1^{31}	i_1^{32}	i_1^{33}	e_1^3	x_1^3
		2	z_{21}^{31}	z_{22}^{31}	z_{21}^{32}	z_{22}^{32}	z_{21}^{33}	z_{22}^{33}	c_2^{31}	c_2^{32}	c_2^{33}	g_2^{31}	g_2^{32}	g_2^{33}	i_2^{31}	i_2^{32}	i_2^{33}	e_2^3	x_2^3
			l_1^1	l_2^1	l_1^2	l_2^2	l_1^3	l_2^3											
	Value	added	n_1^1	n_2^1	n_1^2	n_2^2	n_1^3	n_2^3											
		Import	m_1^1	m_2^1	m_1^2	m_2^2	m_1^3	m_2^3											
		x_i^p	x_1^1	x_2^1	x_1^2	x_2^2	x_1^3	x_2^3											

Source: Author's work based on Miller and Blair (2009)

Generally, input-output tables are made in two variant – in the industries classification (NACE) or in product classification (CPA). Input-output model can be than interpreted in both – as industries or as products. The interpretation depends on the source data. Where variable z_{ij}^{pr} represents the intermediate use flow of product i from region p to region r to produce good j. c_i^{pr} represents the output flow to household consumption, and it is product i from region p to household consumption in region r. g_i^{pr} represents to the output flow to government consumption, and it is product i from region p to consumption in region r. By analogy, i_i^{pr} represents the output flow to investment, and it is product i from region p to region r. Exports are represented by e_i^p, and it is exports from region p in product i. Variables l_j^r and n_j^r represent the variables of gross value added (GVA) for product j, produced in connection with product creation in region r (e.g. wages, profits ...etc). The last variable is the imports of the product (abroad) m_j^r – from region r (product j). The total number of region is P which is same as R. The number of products (or industries) is n (and it is i as well as j same). The total output can then be represented as x_i^p. The following relationships apply to this table from the use perspective:

$$\Sigma_{r=1}^{R}(\Sigma_{j=1}^{n}(z_{ij}^{pr}) + c_i^{pr} + g_i^{pr} + i_i^{pr}) + e_i^p = x_i^p \ , \tag{1}$$

and from the resource perspective:

$$\sum_{p=1}^{P}\sum_{i=1}^{n}(z_{ij}^{pr}) + l_j^r + g_j^r + m_j^r = x_j^r \ , \tag{2}$$

total use can be represented as:

$$\sum_{r=1}^{R} c_i^{pr} + \sum_{r=1}^{R} g_i^{pr} + \sum_{r=1}^{R} i_i^{pr} + e_i^p = f_i^p \ . \tag{3}$$

The Input-Output analysis is based on the matrix describing the ratio between intermediate use inputs and outputs of individual industries; the total matrix (composed of regional submatrices) can be represented as matrix A^T composed of submatrices:

$$\mathbf{A}^{\mathrm{T}} = \begin{bmatrix} \mathbf{A}^{11} & \mathbf{A}^{1r} & \mathbf{A}^{1R} \\ \mathbf{A}^{p1} & \mathbf{A}^{pr} & \mathbf{A}^{pR} \\ \mathbf{A}^{P1} & \mathbf{A}^{Pr} & \mathbf{A}^{PR} \end{bmatrix}, \text{ where } \mathbf{A}^{pr} = (a_{ij}^{rp})_{nn}, \tag{4}$$

for r, p = 1, 2......P, (P=R), and, therefore, the size of A^{T} is $(Rn) \times (Rn)$.

The individual entries of matrices A^{pr} can then be defined as:

$$\text{matrix: } a_{ij}^{pr} = \frac{z_{ij}^{pr}}{x_j^r}. \tag{5}$$

Technical coefficients represent the parameters of Leontief production function. This function is known as the fixed proportions production function. This production function represents extreme case of production function without any elasticity of substitution of inputs.

It can be shown (e.g. Šafr, 2016a) that regional coefficients must be a disaggregation of national coefficients ($\mathbf{A}^{\mathbf{N}}$ – national technical coefficients, $\mathbf{x}^{\mathbf{N}}$ – national output vector):

$$\left[\sum_{p=1}^{P}\sum_{r=1}^{R}\mathbf{A}^{pr}\, diag\,(\mathbf{x}^{r})\right] diag\,((\mathbf{x}^{\mathrm{N}})^{-1}) = \mathbf{A}^{\mathrm{N}}. \tag{6}$$

Then, both in the classic I-O model and in the interregional I-O model, the following is true:

$$(\mathbf{I} - \mathbf{A}^{\mathrm{T}})^{-1}\mathbf{f}^{\mathrm{T}} = \mathbf{x}^{\mathrm{T}}, \tag{7}$$

where the $(\mathbf{I} - \mathbf{A}^{T})^{-1}$ is Leontief inverse, known as **L**. This directly shows the overall impact of total end use on total output in the economy. The elements of matrix **L** is interpreted as a derivative of total product by final use – the chance of total product caused by chance of final use. In the case of the three regions in question, this equation can be broken down into individual output vectors and submatrices of technical coefficients and vectors of use:

$$\begin{aligned} \mathbf{f}^{1} &= +\,(\mathbf{I} - \mathbf{A}^{11})\mathbf{x}^{1} - \mathbf{A}^{12}\mathbf{x}^{2} - \mathbf{A}^{13}\mathbf{x}^{3}, \\ \mathbf{f}^{2} &= -\,\mathbf{A}^{21}\mathbf{x}^{1} + (\mathbf{I} - \mathbf{A}^{22})\mathbf{x}^{2} - \mathbf{A}^{23}\mathbf{x}^{3}, \\ \mathbf{f}^{3} &= -\,\mathbf{A}^{31}\mathbf{x}^{1} - \mathbf{A}^{32}\mathbf{x}^{2} + (\mathbf{I} - \mathbf{A}^{33})\mathbf{x}^{3}. \end{aligned} \tag{8}$$

By solving this system of regional I-O equations, the solution for each region separately can then be gained, i.e. the following equation can be gained::

$$\mathbf{x}^{1} = \mathbf{J}^{-1}\mathbf{f}^{1} - \mathbf{J}^{-1}\mathbf{O}\mathbf{f}^{2} - \mathbf{J}^{-1}\mathbf{G}\mathbf{f}^{3}, \tag{9}$$

where element J^{-1} shows the impact effect of multiplying the increase in the end use of the first region on the total output of the first region, $\mathrm{J}^{-1}\mathrm{O}$ shows the effect of increasing the end use of the second region on the total output of the first region, and $\mathrm{J}^{-1}\mathrm{G}$ represents the effect of increasing the end use in the third region on the output of the first region. It can be said that these submatrices are submatrices of the Leontief total matrix. Their values are as follows:

$$\mathbf{J} = [\mathbf{I} - \mathbf{A}^{11} - \mathbf{A}^{12}\mathbf{E} - \mathbf{A}^{13}\,(\mathbf{I} - \mathbf{A}^{33})^{-1}\,\mathbf{A}^{31} - \mathbf{A}^{13}\,(\mathbf{I} - \mathbf{A}^{33})^{-1}\,\mathbf{A}^{32}\mathbf{E}], \tag{10}$$

where we simplified a part of the calculation using submatrix **E**:

$$\mathbf{E} = [\mathbf{R}^{-1}\mathbf{A}^{32}\,(\mathbf{I} - \mathbf{A}^{33})^{-1}\,\mathbf{A}^{31} + \mathbf{R}^{-1}\mathbf{A}^{21}]\ , \tag{11}$$

whose simplification removes matrix **R**:

$$\mathbf{R} = [\mathbf{I} - \mathbf{A}^{22} - \mathbf{A}^{23}\,(\mathbf{I} - \mathbf{A}^{33})^{-1}\mathbf{A}^{32}]\ . \tag{12}$$

Matrix **O** can be calculated as follows:

$$\mathbf{O} = [\mathbf{A}^{12}\mathbf{R}^{-1} + \mathbf{A}^{13}\,(\mathbf{I} - \mathbf{A}^{33})^{-1}\mathbf{A}^{32}\mathbf{R}^{-1}]\ , \tag{13}$$

and matrix **G** as follows:

$$\mathbf{G} = [\mathbf{A}^{12}\mathbf{R}^{-1}\mathbf{A}^{23}\,(\mathbf{I} - \mathbf{A}^{33})^{-1} + \mathbf{A}^{13}\,(\mathbf{I} - \mathbf{A}^{33})^{-1} + \mathbf{A}^{13}\,(\mathbf{I} - \mathbf{A}^{33})^{-1}\,\mathbf{A}^{32}\mathbf{R}^{-1}\mathbf{A}^{23}\,(\mathbf{I} - \mathbf{A}^{33})^{-1}]\ . \tag{14}$$

This is how we expressed the impact of total use in individual regions on the total increase in the output in the first region. Increase for the output of the second region and the third region can be expressed analogously. Its reasoning makes it an analogous equation as presented by Oosterhaven and Hewings (2014) or Miller and Blair (2009),[3] but for three regions, not two, in this case. This change is particularly important when one region is analysed in the context of all other regions and foreign countries – which can be simplified into 3 regions of the interregional I-O model.

The resultant equation (Formula 9) provided same results as the total Leontief inverse. The advantage of this approach lies in analytical use – due that these equations (Formulas 9–14) allows to analyse the channel of the change of total use. For example, it allows you to separate secondary effects of the chance of total use in region 1 caused by chance of final use in region 1 from other regions – which is not possible to take from total Leontief inverse matrix or from the submatrices of this matrix.

1.2 Interregional flows

Estimation of interregional output flows is based on the assumption that total exports and imports are known in individual regions. This can then be represented separately for each product as a flow matrix for product i as follows:

$$\boldsymbol{U}_i = (u_i^{p,r})_{P\times P}\ . \tag{15}$$

However, these are total output flows, i.e. not only to intermediate use, but also to end use, hence:

$$u_i^{pr} = \sum_{j=1}^{n} z_{ij}^{pr} + f_i^{pr}\ . \tag{16}$$

And, in the retrospective reconstruction, it will be necessary to estimate z_{ij}^{pr}, f_i^{pr} retrospectively, which can be conducted from the import table, assuming that international imports have the same structure of use and intermediate use as interregional imports to individual regions.

1.2.1 Entropy approach

This method is based on the assumption that the values of output flows between regions represent a microstate. Each unit of this output flow represents an individual movement (state). The total volume

[3] Miller and Blair (2009) describe this equation only from the perspective of the increase in the output of the industry, abstracting from elements **JO** and **JG**.

of flows in the matrix representation is the macro state of the system (identical to matrix $\mathbf{u}_i$). By using combinatorics, the total number of combinations of individual movements of output flows can be determined. If we assume that we know the matrix of output flows ($\mathbf{u}_i$), the total possible number of microstate combinations can be represented as function wfor interregional flows:

$$w(\boldsymbol{u}_i) = \frac{\sum_{p=1}^{P} \sum_{r=1}^{R} u_i^{pr}}{\prod_{p=1}^{P} \prod_{r=1}^{R} u_i^{pr}!} . \tag{17}$$

Entropy maximization as defined by Jaynes (1957) consists in the maximization of $w_p(\mathbf{u}_i)$, which expresses the number of possible combinations of microstates. Batten (1982) also points to other possible definitions and solutions of the maximization entropy equation (or, rather, uncertainty) of output flows that can be used for maximization. Stirling's approximation can lead us to the model defined by Batten and Boyce (1986),[4] where we minimize the import distance of individual output flows, assuming a predetermined entropy rate:

$$\begin{aligned}
&\min_{u_i^{pr}} f_i = \textstyle\sum_{p=1}^{P} \sum_{r=1}^{R} \delta_i^{pr} u_i^{pr} , \\
&-\sum_{p=1}^{P}\sum_{r=1}^{R} (u_i^{pr} \text{ In } u_i^{pr} - u_i^{pr}) \geq \phi_i \;,^{5} \\
&v_i^p = \sum_{r=1}^{R} u_i^{pr} , \\
&d_i^r = \sum_{p=1}^{P} u_i^{pr} , \\
&u_i^{pr} \geq 0 \text{ for } p \neq r , \\
&u_i^{pr} = 0 \text{ for } p = r ,
\end{aligned} \tag{18}$$

where δ_i^{pr} is the distance between region p and region r. Parameter ϕ_i is the rate of exogenously determined entropy for product i. v_i^p is the known total regional exports of product i from region p. d_i^r is the known regional total imports of product i to region r. The first equation in the limitation shows us an approximate entropy rate that must be greater than or equal to the predetermined entropy rate (ϕ_i). The parameter of entropy (which is in boundaries) is key variable which affect the final distribution of flows in economy. If this boundary is omitted, result of minimization is same as minimal distance. The maximum of this parameter (ϕ_i) is proportional distribution (calculated by the way of unconditional probability). The true size of this parameter is generally unknown. One way how to calculate it consists in using transport tables in different classification/or aggregation. These tables should be rescaled to the same size as is the estimated tables.

The only unsolved problem in estimating interregional output flows remains the problem of how to determine the entropy rate for individual products. In my case, we started from the structure of output flows in physical representation. Thus, this data shows the volume of exports and imports based on individual NST product classifications in natural representation – tonnes, kilograms, etc.

1.2.2 Gravity approach

As we mentioned in the introduction above, the gravity method is based on Newton's law of gravitation. In the case of I-O tables, it is assumed that the export/import rate (force) of two regions (objects) is directly

[4] The detailed procedure and other different model variants are shown by Batten (1982) and Sargento (2009).

[5] If the flow value is zero, then the expression (u_i^{pr} In $u_i^{pr} - u_i^{pr}$) is considered to be zero.

proportional to the product of the total output of the regions in question and indirectly proportional to their distance. The standard gravity model can then be presented as follows:

$$u_i^{pr} = G_i \, \frac{(x_i^p)^{\alpha_i} \, (x_i^r)^{\beta_i}}{(\delta^{pr})^{\omega_i}} \, , \tag{19}$$

where G_i is the total export/import level of product i in the economy between regions α_i and β_i then represent the elasticity (of the importing region and the exporting region) for product i. As with the entropy approach, variable δ^{pr} is the distance between region p and region r, and constant ω_i is the degree of distance decay between regions. In the case of international trade, this equation is supplemented by other variables that affect how much the countries in question cooperate with each other. However, this cannot be applied to the case of one country with a single fiscal and monetary framework. The estimation of individual parameters can be gained either by calibration or, using regression, by logarithmizing to the following expression:

$$\log(u_i^{pr}) = \log(G_i) + \alpha_i \, \log(x_i^p) + \beta_i \, \log(x_i^r) - \omega_i \, \log(\delta^{pr}) \, . \tag{20}$$

As mentioned in the introduction above, in order to use the gravity model, it is necessary to know the model parameters that must be estimated from the output flows between regions. With regard to the availability of this data, we use the output flows between regions in natural representation. However, this data may be used under the following conditions:

1. It is assumed that the individual CPA and NST classifications have homogeneous outputs.
2. It is possible to approximate the CPA by means of the NST classification.
3. The values of products do not vary in individual regions and product flows.
4. These are net output flows (not quasi-transit).

Although these are strong conditions, we assume that conditions 1–3 have been met. However, the problem is condition 4, which is not met in our data as it is published, containing all flows in the economy (and thus also to stock). Another reason for adjusting the gravity equation can be found in the fact that we know the total volumes of exports and imports to individual regions. For this reason, we used the adjusted gravity form of the equation (for more information, see Šafr and Sixta, 2017):

$$u_i^{pr} = G_i \, \frac{(x_i^p)^{\alpha_i} \, (x_i^r)^{\beta_i}}{(\delta^{pr})^{\omega_i}} \, \Big(\frac{l^{\psi_i^p}}{l^{\xi_i^r}}\Big) \, , \tag{21}$$

where variables $l_p^{\psi j} l_r^{\xi j}$ represent stock inventories in individual regions, ξ_i^r represents the effect of stock inventories in demanding region, ψ_i^p represents the effect of stock of inventories in supplying region. Unfortunately, this data is not directly available and, for this reason, we approximated stock using the number of workers in warehouses as in the original article by Šafr and Sixta (2017).

The most used way how to obtain these parameters (α_i, β_i, ω^{pr}, ψ_i^p, ξ_i^r) is to estimate them by regression method (Shepherd, 2013). Due to fact that the flows between regions have to be estimated, the approximation is generally used – the parameters are estimated on the basis of different data sources such as another states or different classification (our case).

2 DATA

The main data source for the estimation of interregional flows is regional I-O tables prepared at the Department of Economic Statistics of the University of Economics (KEST, 2017), as well as the national accounts of the Czech Statistical Office (2017). Imports and exports are estimated by the model based on structure of Use. Regional estimates are made separately from international ones (Vltavská and Sixta, 2017). Another important source is the employment data provided by Trexima (2017) on the number of workers in warehouses, as well as data on exports and imports between regions in physical representation, and in a different classification from the Ministry of Transport (MD ČR, 2017). Parameters ϕ_i was estimated from NTS classification.

With regard to the NST and CPA classification mismatch, we used the following approximation of parameter estimates – see Table 1.

Table 1 NST proxy structures pro CPA products

NST	CPA
NST **01**	CZ-CPA **01–03**
NST **02**	CZ-CPA **05–06**
NST **03**	CZ-CPA **07–09, 41–42**
NST **04**	CZ-CPA **10–12**
NST **05**	CZ-CPA **13–15**
NST **06**	CZ-CPA **16–18, 58–63**
NST **07**	CZ-CPA **19**
NST **08**	CZ-CPA **20–22**
NST **09**	CZ-CPA **23**
NST **10**	CZ-CPA **24–25**
NST **11**	CZ-CPA **26–28**
NST **12**	CZ-CPA **29–30, 45–47**
NST **13**	CZ-CPA **31–33**
NST **14**	CZ-CPA **36–39**
NST **15**	CZ-CPA **49–53**
NST **18**	CZ-CPA **64–99**

Source: Author´s work

3 RESULTS

We applied the above two methods to data to gain interregional flow structures. Subsequently, we retrospectively estimated flows to intermediate use between the regions to individual products. Considering the magnitude of these results (1 148 × 1 148 matrix), we aggregated these structures, presenting them in the diagrams describing the volume of exports and imports between regions – see Figures 2 and 3.

The same figure can be shown for Gravity approach.

The results show that the entropy method achieves more extreme values compared to the gravity method. The entropy rate in Entropy method is about 3% higher than in model without entropy in constraints. The entropy rate in Entropy Method is then much closer to Newton model than to minimal distance approach. This is due to the minimization of import distances, which is limited by the degree of entropy of matrix entries. We can see the strongest relationship has Prague region in both method. This is caused by the constrain which comes from RIOTs. These two matrices can be compared by criteria. The WAD criterion is usually used to assess the difference between two intermediate use matrices[6] in the I-O analysis:

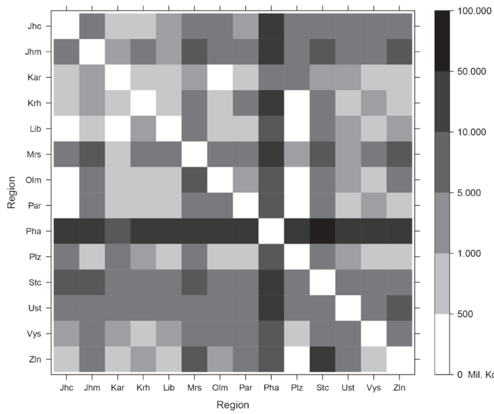
$$Err^{\mathrm{WAD}} = \frac{\sum_{j=1}^{n}\sum_{i=1}^{n} m_{ij}\left| m_{ij} - q_{ij}\right|}{\sum_{j=1}^{n}\sum_{i=1}^{n}\left(m_{ij} + q_{ij}\right)} \times 100\,, \qquad (22)$$

or MAPE:

$$Err^{\mathrm{MAPE}} = \frac{1}{n^2}\sum\nolimits_{j=1}^{n}\sum\nolimits_{i=1}^{n}\frac{\left| m_{ij} - q_{ij}\right|}{\left| m_{ij}\right|} \times 100\,. \quad (23)$$

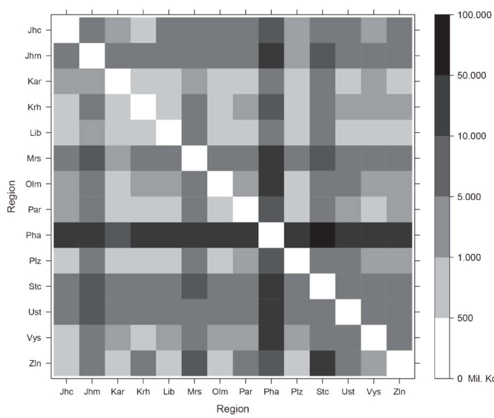
The problem with these criteria (as well as others, such as WAPE, SWAD, etc.) is that they require a reference matrix through which the force of the entry change is weighed. However, these two estimates are completely independent of each other, and no matrix is the starting point here. This then leads to the situation that the intermediate use matrix entries can be zero in one case and non-zero in the second case; however, the aforementioned criteria can lead to distorted results. For this reason, we constructed the MWAD criterion for this application:

Figure 2 Entropy approach: exports and imports between regions



Source: Author's work

Figure 3 Gravity approach: exports and imports between regions



Source: Author's work

$$Err^{\mathrm{MWAD}} = \frac{\sum_{j=1}^{n}\sum_{i=1}^{n}\dfrac{m_{ij} + q_{ij}}{2}\left| m_{ij} - q_{ij}\right|}{\sum_{j=1}^{n}\sum_{i=1}^{n}\left(m_{ij}\right)} \times 100\,. \qquad (24)$$

They show that the difference between them is 0.223 and, according to RMSE (* 100), it is 0.428. Thus, it appears that compared to other methods (as published in Šafr and Sixta, 2017), these two methods do not lead to extremely different results. Table 2 shows the MWAD criterion structure for differences between regions (taking products) in the economy.

[6] We consider two matrices: **M** and **G** with elements: m_{ij}, resp. g_{ij}.

Table 2 MWAD criterion between Entropy and Gravity approach

REGION	JHC	JHM	KAR	KRH	LIB	MRS	OLM
MWAD	0.35	1.27	0.05	0.24	0.10	1.50	0.13
REGION	PAR	PHA	PLZ	STC	UST	VYS	ZLN
MWAD	0.16	4.67	0.26	4.68	0.29	0.51	0.78

Source: Author´s work

What is interesting about these results is the fact that the results for the Prague Region and the Central Bohemian Region show the most significant differences. On the contrary, the Karlovy Vary Region and the Liberec Region show the most significant similarities.

The biggest difference can be seen at the level of individual products. For the case of CZ-CPA 1 ("Products of agriculture, hunting and related services"), we have calculated the flow matrices by both methods. These matrices are summarized in Tables 3 and 4. The readers can see that the Entropy approach is providing more extreme results and lot of relationships estimated as zero. These results will provide totally different structure of multiplication in interregional Input-Output models.

Table 3 CZ-CPA 1: Imports and Exports between regions (Entropy approach)

	Jhc	Jhm	Kar	Krh	Lib	Mrs	Olm	Par	Pha	Plz	Stc	Ust	Vys	Zln
Jhc	0	53	3	1	11	70	1	0	2811	0	168	387	41	95
Jhm	2	0	0	0	0	337	21	0	1348	0	5	9	23	1146
Kar	0	0	0	0	4	13	0	0	0	0	2	9	1	4
Krh	4	4	0	0	27	84	10	0	1905	0	58	133	2	1
Lib	0	0	0	0	0	12	1	0	51	0	19	24	0	1
Mrs	0	12	0	1	2	0	27	0	3	0	7	0	0	169
Olm	0	35	0	1	7	208	0	0	421	0	10	8	0	94
Par	0	43	25	27	137	832	24	0	615	0	71	271	37	14
Pha	0	2	1	0	0	6	0	0	0	0	13	22	0	1
Plz	44	14	261	3	63	56	4	0	1587	0	126	194	19	41
Stc	2	6	20	2	52	13	1	0	2443	0	0	117	3	3
Ust	1	2	15	0	1	12	0	0	93	0	5	0	1	2
Vys	51	160	100	6	116	864	45	0	2457	0	239	457	0	296
Zln	0	2	0	0	0	8	1	0	47	0	0	0	0	0

Source: Author´s work

Table 4 CZ-CPA 1: Imports and Exports between regions (Newton approach)

	Jhc	Jhm	Kar	Krh	Lib	Mrs	Olm	Par	Pha	Plz	Stc	Ust	Vys	Zln
Jhc	0	60	67	7	66	401	22	0	2185	0	129	259	26	296
Jhm	16	0	54	6	53	324	18	0	1764	0	104	209	21	239
Kar	0	1	0	0	1	5	0	0	25	0	1	3	0	3
Krh	12	37	41	0	40	245	14	0	1333	0	79	158	16	180
Lib	1	2	2	0	0	13	1	0	69	0	4	8	1	9
Mrs	1	4	5	1	5	0	2	0	152	0	9	18	2	21
Olm	4	14	15	2	15	91	0	0	496	0	29	59	6	67
Par	13	40	44	5	43	263	15	0	1432	0	85	170	17	194
Pha	1	2	2	0	2	14	1	0	0	0	4	9	1	10
Plz	13	42	46	5	46	276	15	0	1503	0	89	178	18	203
Stc	15	45	50	5	49	299	17	0	1637	0	0	193	19	220
Ust	1	2	3	0	3	16	1	0	89	0	5	0	1	12
Vys	27	84	94	10	93	561	31	0	3057	0	181	363	0	414
Zln	0	1	1	0	1	7	0	0	38	0	2	5	0	0

Source: Author´s work

3.1 Analytical impact

Interregional flow estimation is based on total export and import volumes between regions. For this reason, the total shocks for the economy calculated using the interregional model are the same, but the *shock structures are different*. Therefore, we focused on Formula (9) and decomposition of Leontief matrix (matrix **L**). Subsequently, we expressed the average value of the multiplication in the region in question due to the increase in the end use in the examined region and compared these results between the volumes of the method. Table 5 summarizes the submatrices of the Leontief Inverse matrix (**L**) of this calculation.

Table 5 Percentage difference of submatrices of interregional Leontief matrix

	Jhc	Jhm	Kar	Krh	Lib	Mrs	Olm	Par	Pha	Plz	Stc	Ust	Vys	Zln
Jhc	100%	88%	68%	88%	67%	60%	106%	64%	120%	154%	89%	128%	173%	98%
Jhm	65%	100%	55%	74%	70%	105%	139%	133%	92%	182%	77%	59%	141%	191%
Kar	72%	69%	100%	75%	99%	121%	53%	71%	104%	253%	61%	241%	56%	69%
Krh	54%	67%	82%	100%	85%	143%	49%	181%	108%	49%	107%	62%	136%	35%

Table 5 (continuation)

	Jhc	Jhm	Kar	Krh	Lib	Mrs	Olm	Par	Pha	Plz	Stc	Ust	Vys	Zln
Lib	59%	41%	53%	202%	100%	93%	57%	84%	112%	54%	106%	197%	50%	154%
Mrs	97%	142%	78%	134%	99%	100%	202%	94%	80%	50%	116%	77%	46%	175%
Olm	31%	184%	60%	34%	58%	257%	100%	119%	58%	47%	50%	83%	52%	296%
Par	39%	96%	92%	124%	91%	168%	216%	100%	89%	44%	100%	63%	129%	64%
Pha	108%	95%	94%	105%	102%	95%	93%	99%	100%	89%	107%	110%	104%	89%
Plz	223%	42%	711%	151%	55%	80%	50%	82%	113%	100%	81%	68%	39%	52%
Stc	127%	107%	77%	52%	135%	88%	57%	98%	108%	145%	100%	92%	109%	54%
Ust	68%	43%	137%	128%	138%	61%	79%	84%	127%	119%	83%	100%	99%	100%
Vys	124%	200%	96%	95%	95%	98%	132%	134%	86%	60%	95%	72%	100%	104%
Zln	54%	285%	93%	49%	43%	138%	257%	105%	78%	80%	86%	56%	41%	100%

Source: Author´s work

Thus, if we interpret the relationship between the South Moravian Region and the South Bohemian Region (second row, first column), i.e. 65%, this value states that the entropy-based calculation approach will cause that the average increase in the output of the average product of the South Bohemian Region is 65% lower than in the calculation using the gravity model. From the row perspective (i.e. multiplication for the region in the row caused by the increase in end use in the region in the column), the Karlovy Vary Region (average 128%) and the Moravian-Silesian Region (114%) are the most overvalued compared to the gravity model, with the South Bohemian Region (average 87%) and Liberec Region (88%) being the most undervalued.

These results appear to be crucial, although the absolute values according to the graphs do not seem to be very varied. For this reason, we further focused on the decomposition of Leontief multiplication in the relative structure. For each region, we calculated the multiplier structure of the average Leontief multiplier for the region in question, and then we entered this data for all regions in the web diagram. The following set of 14 images illustrates the results of this calculation.

Figure 4 Structure of average multiplication out of the region (without Prague)

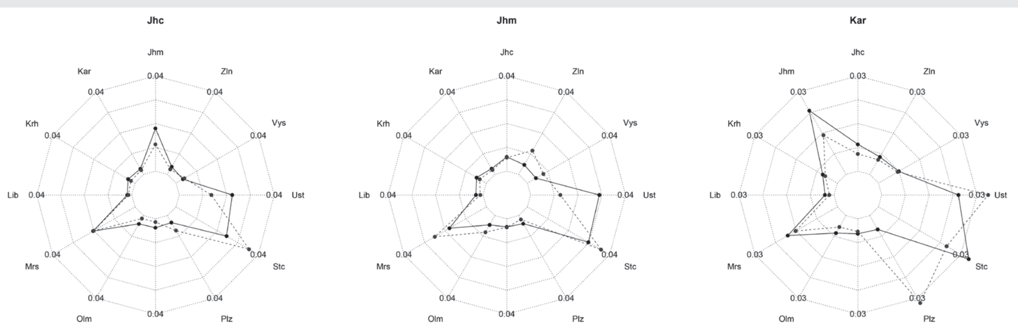
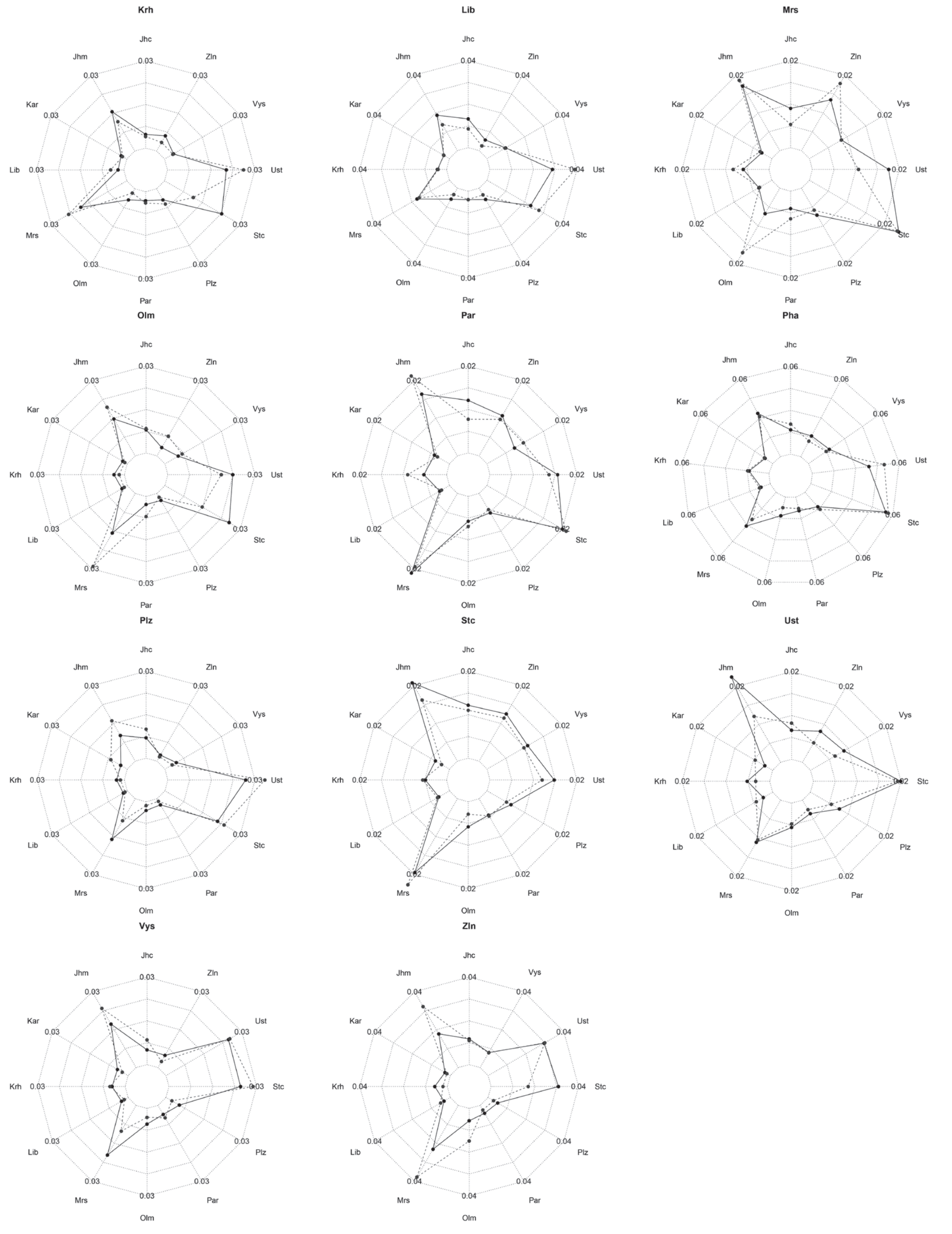


Figure 4 (continuation)



Note: The hatched line represents the entropy approach, and the black line represents the gravity approach.
Source: Author's work

The results clearly show that the considerable (up to 700%) difference is not significant in size, with very small shares in the Leontief total multiplier. This means that, from the size perspective, these are not the major differences in multiplication. The only major differences can be found in the South Bohemian Region, Karlovy Vary Region and Prague Region.

CONCLUSION

Input-output analysis is a powerful tool for modelling of a wide range of effects. Such models are often used on the level of total economy but they can be used on the level of regional economy, as well. The availability of regional input-output tables for the Czech Republic (KEST, 2017) allowed us to focus on interregional model. It means that it emphases the importance of the method used for modelling of interregional flows (in the case of Czech Republic for year 2013). Gravity method is commonly used but we show that entropy can be use as well. The selection of the correct method may be fundamental.

The results showed that despite the fact that the results in absolute values graphically do not show fundamental differences between regions, differences can be measured across individual regions from the perspective of multiplication process of the particular product in interregional model. With regard to the construction of the model, impacts on the entire economy are the same, but their structure differs significantly (retrospective multiplication to other regions). At first glance, the relative share of the Leontief matrices revealed fundamental differences, but the web diagram showed that this difference is not as fundamental in terms of the volume of multiplication between these methods. Therefore, it can be assumed that, if these two methods are used, the results of the impacts on individual regions will be very similar, except for the South Bohemian Region, Karlovy Vary Region and Prague Region, as mentioned above.

The results described in the paper can be used for further modelling and/or impact assessment analysis for the Czech Republic or they can serve as a guidance for those who are trying to construct interregional model for different country. We proved that interregional model can be constructed for smaller country, as well. We illustrate that the elements of Leontief matrix can be dependent on the method selected and can influence the forecasts of gross value added and employment across both regions and products. This has to be considered when conducting similar research and therefore we recommend to use at least two mentioned approaches.

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APPENDIX

Table A1 Estimates of parameters (Estimates and their 95% bounds)

Coefficient	Lower bound	Estimate	Upper bound
α_i	0.065	0.068	0.071
β_i	–0.013	–0.010	–0.007
ω^{pr}	1.057	1.068	1.079
ψ_i^p	0.595	0.602	0.609
ξ_i^r	–0.758	–0.749	–0.740

Source: Author's work

Table A2 Czech regions – Abbreviations and their full names

Abbreviation	Full name
Jhc	South Bohemian Region
Jhm	South Moravian Region
Kar	Karlovy Vary Region
Krh	Hradec Králové Region
Lib	Liberec Region
Mrs	Moravian-Silesian Region
Olm	Olomouc Region
Par	Pardubice Region
Pha	Prague
Plz	Plzeň Region
Stc	Central Bohemian
Ust	Ústí nad Labem Region
Vys	Vysočina Region
Zln	Zlín Region