Multicriteria Evaluation of Randomized Response Techniques for Population Mean

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Received 24.7.2023, **Accepted** (reviewed) 3.8.2023, **Published** 15.12.2023

Abstract

Increasing demand in surveys focused on quantitative characteristics (population mean among others) of controversial issues like corruption, tax evasion, drugs consumption of sensitive variables like spending on drugs or illegal sources of income which lead to lively research, are of randomized response techniques for quantitative variables. Therefore, we propose complex multicriteria evaluation methodology for randomized response techniques for population mean. Based on extensive review in the literature, following ranges of criteria were proposed: statistical properties of estimator, implementation and parameter choice, respondent burden and credibility and confidentiality protection of respondents' data. Finally, we evaluate in this setting standard techniques using scramble variables and recently proposed techniques of dichotomous question.

Keywords	***DOI***	***JEL code***
Randomized response techniques, scramble variable, multicriteria evaluation, survey sampling, Horvitz-Thompson estimator, population mean	*https://doi.org/10.54694/stat.2023.32*	*C83, C10*

INTRODUCTION

The field randomized response techniques for quantitative variables have been experiencing rapid development both in theory and practice (e.g. Christofides and Chaudhuri, 2013; or Chaudhuri et al., 2016; doctoral thesis of Cobo Rodrigues, 2018) since the first proposal fifty years ago (Eriksson, 1973). There are three main approaches in this field:

- Methods using scramble variables (Eriksson, 1973; Eichorn and Hayre, 1983), where instead of true values respondent provides linearly transformed values of sensitive variables depending on results of a random experiment.

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- Methods using scramble variables with use of auxiliary variable known for the whole population strongly correlated to surveyed sensitive variable (Diana and Perri, 2013; Cobo Rodriguez, 2018).
- Methods using dichotomous response (Antoch et al., 2022), where respondent provides only dichotomous response (“Yes/No”) instead of any numerical value related to value of surveyed sensitive variable.

Recently, research has also been focused on comparison of techniques to find optimal techniques both in theoretical criteria and practical performance of competing estimators. For example, Azeem and Ali (2023) study vast number of estimators using scramble variables. To our best knowledge, all studies only focus on statistical criteria like unbiasedness of estimators, their variance and performance of asymptotic confidence interval. There is a need for methodology incorporating also relevant issues like design of randomization of responses (choice of scramble variables), comfort and credibility for respondents, risk of disclosure of respondent data.

Main purpose of the paper is proposing such methodology and evaluate standard techniques using scramble variables and techniques using dichotomous response. Techniques using auxiliary variable are excluded at this first stage to keep the clarity of our evaluation. Moreover, the application of methods using auxiliary variable is not feasible in the most of the real life populations. First, there is no such auxiliary variables nature of studied population (convicts, members of small community). The second, more important objection is, that if auxiliary variable is strongly correlated with the sensitive variable, the auxiliary variable is also sensitive. Therefore, such auxiliary variable would be also available in very poor quality.

The rest of the paper is organized as follows. In Section 1, basic notions of estimation of population mean of a finite population by randomized response techniques and notation is introduced. Section 2 reviews evaluated randomized response techniques, both standard ones using scramble variables and recently proposed ones with use of dichotomous response. In Section 3, ranges of quality criteria and evaluation methodology are proposed, and methods reviewed in the previous section are evaluated. The main findings and conclusions of the paper are summarized in the last section.

1 BASIC NOTIONS FOR ESTIMATION OF POPULATION MEAN

The purpose of survey sampling is to estimate characteristics of a finite population $U = \{1, 2, \ldots, N\}$ of N unambiguously identified objects. For a quantitative variable Y the most common objective is to estimate its population total $t_Y = \sum_{i \in U} Y_i$ or population mean $\overline{t}_Y = t_Y/N$. To achieve that, a random sample s of fixed sample size n is selected with probability $p(s)$. Using probabilities π_i, ($\pi_i = \sum_{s \ni i} p(s)$) of selection of i^{th} unit of the population U, population mean is then estimated by unbiased Horvitz-Thompson estimator

$$\overline{t}_{Y,HT} = \frac{1}{N} \sum_{i \in s} \frac{Y_i}{\pi_i}. \tag{1}$$

For statistical properties of estimators and theorethical proofs consult Horvitz and Thompson (1952) and Section 2.8 in Tillé (2006) for details.

Because the surveyed variable is sensitive, respondents frequently refuse to answer or provide fabricated answers. Therefore, survey statisticians try to obtain at least randomized variable Z correlated to variable of interest Y. At the second stage, randomization of responses is always carried out independently for each unit selected in sample s on the sampling procedure $p(s)$. Randomized response Z is further transformed to random variable R, which follows standard model of randomized responses proposed by Arnab (1994):

$$E_q(R_i) = Y_i,\ Var_q(R_i) = \phi_i \text{ for all } i \in U,\ Cov_q(R_i, R_j) = 0, \text{ if } i \neq j\ i, j \in U, \tag{2}$$

where E_q, Var_q and Cov_q denote mean, variance and covariance with respect to probability distribution $q(r|s)$ of randomization of response of a selected sample s. Finally, population mean is estimated by unbiased Horvitz-Thompson estimator using transformed randomized responses R_i instead of values of sensitive variable Y_i

$$\bar{t}_Y^R = \frac{1}{N} \Sigma_{i \in s} \frac{R_i}{\pi_i}, \tag{3}$$

where upper subscript R denotes the used randomized response technique.

2 EVALUATED METHODS

This section presents both standard methods using scramble variable for population mean without use of auxiliary variable and methods using dichotomous response by Antoch et al. (2022). Only these methods are evaluated. In total, five methods are evaluated in the paper.

2.1 Standard methods using scramble variables

Idea of scramble variables comes from the seminal paper of Eriksson (1973). Ten years later, Eichhorn and Hayre (1983) generalized this concept and laid theoretical foundation of scramble variable. Idea of scramble variables in setting of Erikson (1973) is as follows. Respondent generates value of random scramble variable S, unknown to an interviewer. Then respondent provides transformed response $Z = SY$. The distribution of the S must be chosen to mask the sensitive variable Y. This setup was generalized by Arcos et al. (2015), who defined a model describing all cases for methods using scramble variables including general formula for population mean estimator of Horvitz-Thompson type and its variance. This model is widely used both in theoretical research for comparison of methods and applications.

2.1.1 Method of Eriksson (1973)

Eriksson (1973) proposed that each respondent randomly selects a card from a package of L cards with numbers $b_1, b_2, \ldots, b_L$. The value in card selected is unknown to an interviewer. The i^{th} respondent provides transformed value $b_i y_i$ instead of original value y_i. Randomized response of i^{th} respondent is then defined as

$$Z_{i,E} = y_i S_1, \tag{4}$$

where S_1 is a scramble variable with non-zero mean μ_1 and positive variance σ_1^2.

Transformed randomized response is then given as

$$R_{i,E} = \frac{Z_{i,A}}{\mu_1}. \tag{5}$$

Unbiased population mean estimator of Horvitz-Thompson type is then

$$\bar{t}_{Y,E}^{HT,R} = \frac{1}{N} \sum_{i \in s} \frac{R_{i,E}}{\pi_i}. \tag{6}$$

2.1.2 Method of Chaudhuri (1987)

Chaudhuri (1987) modified method of Eriksson (1973) as follows. Each respondent randomly selects one card from two packages. The first package consists of L cards with numbers $b_1, b_2, \ldots, b_L$; the second one consists of K cards with numbers $c_1, c_2, \ldots, c_K$. Both selected cards are unknown to an interviewer. The ith respondent provides transformed value $b_i y_i + c_i$ instead of original value y_i. Randomized response of ith respondent is then defined as

$$Z_{i,CH} = y_i\, S_1 + S_2\,, \tag{7}$$

where S_2 S_1 are scramble variables with non-zero means μ_1, μ_2 and positive variances $\sigma_1^{\,2}$, $\sigma_2^{\,2}$.
Transformed randomized response is then given as

$$R_{i,CH} = \frac{Z_{i,CH} - \mu_2}{\mu_1}\,. \tag{8}$$

Unbiased population mean estimator of Horvitz-Thompson type is then

$$\bar{t}_{Y,CH}^{\,HT,R} = \frac{1}{N}\sum_{i\in s}\frac{R_{i,CH}}{\pi_i}\,. \tag{9}$$

2.1.3 Method of Bar-Lev et al. (2004)

Bar-Lev et al. (2004) modified proposal of Eriksson (1973) in following way. With fixed probability p unknown both to respondent and interviewer, each respondent reports its true value of sensitive variable y_i. With probability $1 - p$ each respondent randomly selects a card from a package of L cards with numbers $b_1, b_2, \ldots, b_L$. Then, the ith respondent provides transformed value $b_i\, y_i$ instead of original value y_i. Randomized response of ith respondent is then defined as

$$Z_{i,B} = \begin{cases} y_1 & \text{with probability } p\,, \\ y_i S_1 & \text{with probability } 1 - p\,, \end{cases} \tag{10}$$

where S_1 is a scramble variable with non-zero mean μ_1 and positive variance $\sigma_1^{\,2}$, and p fixed probability $(0 < p < 1)$, that respondent provides the true value of sensitive variable y_i.

If $p + (1 - p)\mu_1 \neq 0$, transformed randomized response is then given as

$$R_{i,B} = \frac{Z_{i,B}}{p + (1 - p)\mu_1}\,. \tag{11}$$

Unbiased population mean estimator of Horvitz-Thompson type is then

$$\bar{t}_{Y,B}^{\,HT,R} = \frac{1}{N}\sum_{i\in s}\frac{R_{i,B}}{\pi_i}\,. \tag{12}$$

The main concern is usually respondent privacy. Low probabilities like $p = 0.1$ or $p = 0.2$ are chosen, because respondent privacy is the main concern of randomized response techniques.

2.2 Methods using dichotomous responses

Methods using scramble variables have several drawbacks. The first drawback is missing practical guidelines for designing scramble variable. In the literature it is recommended to rely on the survey statistician experience (Chaudhuri, 1987). The second drawback is, that the calculations can be too demanding for respondents, and they can lead to severe errors or refusal. The third drawback is that this method can be less trustworthy for respondents, because they can feel that interviewer can guess somewhat the sensitive value. Moreover, if the interviewer knows the values of scramble variable, he can calculate the true value of sensitive variable.

To resolve these issues, Antoch et al. (2022) proposed completely different approach. They assume that the surveyed sensitive variable Y is both non-negative and bounded from above, i.e., $0 < m \leq Y \leq M$. They assume the both bounds m, M of the variable Y are known. Each respondent generates, independently of the others, a pseudorandom number U from the uniform distribution on interval (m, M), while the interviewer does not know this value. The respondent then answers a simple question: "Is the value of Y greater than U?". For example: "Is your monthly income greater than U?"

Note, that even if an interviewer knows the value of random number U, he cannot guess the true value of U accurately (unless $Y = U = M$). Therefore, they proposed more accurate estimator using the values of random numbers U. Unbiased variance estimators using plug-in technique of Arnab (1994) and Arnab (1995) were derived by Vozár (2023). However, the variance estimators for both methods require knowledge of random numbers U.

2.2.1 Original method of Antoch et al. (2022)

Randomized response of i^{th} respondent follows alternative distribution with parameter $\frac{y_i - M}{M - m}$

$$Z_{i,(m,M)} = \begin{cases} 1 \text{ with probability } \frac{y_i - M}{M - m}, \text{ if } U_i < y_i . \\ 0 \text{ with probability } 1 - \frac{y_i - M}{M - m}, \text{ if } U_i \geq y_i . \end{cases} \tag{13}$$

Transformed randomized response is then given as

$$R_{i,(m,M)} = m + (M - m)\, Z_{i,(m,M)} . \tag{14}$$

Unbiased population mean estimator of Horvitz-Thompson type is then

$$\bar{t}^{\,HT,R}_{Y,(m,M)} = \frac{1}{N} \sum_{i \in s} \frac{R_{i,(m,M)}}{\pi_i} . \tag{15}$$

2.2.2 Method of Antoch et al. (2022) using values of random numbers

Randomized response of i^{th} respondent incorporates information on random number in the following manner

$$Z_{i,\alpha,(m,M)} = \begin{cases} 1 - \alpha + 2\alpha \frac{U_i - m}{M - m} \text{ with probability } \frac{y_i - m}{M - m}, \text{ if } U_i < y_i , \\ -\alpha + 2\alpha \frac{U_i - m}{M - m} \text{ with probability } 1 - \frac{y_i - M}{M - m}, \text{ if } U_i \geq y_i , \end{cases} \tag{16}$$

where α is a tuning parameter. Its value is a priori set by the interviewer, is fixed and unknown to the respondent. Antoch et al. (2022) derived its optimal value minimizing variance for case of sample with constant selection probabilities π_i.

Transformed randomized response is then given as

$$R_{i,\alpha,(m,M)} = (M - m)\, Z_{i,\,\alpha,(m,M)} + m \tag{17}$$

Unbiased population mean estimator of Horvitz-Thompson type is then

$$\bar{t}^{HT,R}_{Y,\alpha,(m,M)} = \frac{1}{N}\sum_{i\in s}\frac{R_{i,\alpha,(m,M)}}{\pi_i}. \tag{18}$$

3 EVALUATIONS OF METHODS

In the first subsection, range of evaluation criteria randomized response techniques for population mean relevant for design of real-life survey are discussed and multicriteria evaluation methodology is proposed. In the second part, methods presented in Section 2 are evaluated. As explained before, only methods without use of auxiliary variables are evaluated.

3.1 Quality criteria of randomized response techniques

When choosing a technique and the values of its parameters for a given sample survey, in addition to purely statistical considerations (unbiasedness and consistency of estimates), also other criteria must also be considered (see Chaudhuri and Mukerjee, 1988; Chaudhuri and Christofides, 2013; the paper of Blair et al., 2015; the doctoral thesis of Cobo Rodriguez, 2018).

The first range of quality criteria for estimates based on the randomized response technique represents statistical criteria, which are a necessary condition for all newly proposed estimates and statistical methods in general. Namely, they are unbiasedness (at least asymptotic one), consistency of estimators, and consistent estimates of variances allowing the construction of asymptotic confidence intervals. The next three ranges of estimation quality criteria are already specific only to the techniques of randomized response.

The remaining ranges of the quality of estimates are related to the randomized response technique used. We sum up these issues in these three ranges of criteria:

- technical implementation of a random experiment of randomized response,
- the optimal choice of the parameters of the randomized response technique and the existence of theoretical results for the choice of these parameters, when it is always necessary to balance between the accuracy of the estimates and the protection of respondents' privacy ("accuracy-privacy trade-off", Chaudhuri and Mukerjee, 1988),
- clarity and credibility of the randomized response technique for the respondent.

The implementation of a randomized response is important for the practical feasibility of the survey, and more non-statistical aspects must be considered, such as education, cultural habits, the relationship, and trust of the investigated population towards the organizers and users of this survey (Chaudhuri and Christofides, 2013; Blair et al. 2015). Most applications of randomized response for a quantitative variable have focused on a discrete random variable taking on a relatively small number of values. Therefore, the generation of the masking variable was realized in the past by simple tools such as drawing from a deck of cards, fate, and wheel of fortune. For practically continuous sensitive variables acquiring many values, these methods are impractical, so it is necessary to use random number generators. These procedures are used more and more often due to the use of web, tablets, or mobile applications when polling.

When applying the randomized response technique, the choice of the parameters of the chosen method is crucial, because there is an inverse relationship between the accuracy of the estimates on the one hand and the risk of a sufficiently accurate estimate or even the disclosure of the respondent's sensitive data on the other ("accuracy-privacy trade-off", Chaudhuri and Mukerjee, 1988). It is observed (Blair et al., 2015; Warner, 1965) that respondents perceive these risks of revealing their confidential data sensitively. For qualitative variables (specifically, the relative frequency of occurrence of a given character), theoretical results and practical guidelines were obtained by Blair et al. (2015). For quantitative variables, we do not know such results, many authors (e.g. Chaudhuri, 1987) refer to the experience of the statistician designing the given statistical survey.

Critical to the success of the survey is clarity and user comfort for the respondent. Several studies (see discussion Blair et al., 2015) show that even relatively simple methods such as Warner's mirror question method are incomprehensible to a non-negligible part of respondents (e.g. 20% or more). Also, complicated techniques or techniques requiring complex arithmetic operations (e.g., Chaudhuri's method, 1987) may lead to refusal to answer, gross errors, or cause respondents to mistrust that the interviewer is trying to trick them into revealing a sensitive answer.

The criteria for the evaluation of methods of randomized response are summarized in Table 1, including the weights for the evaluation of individual techniques of randomized response in the next part of the chapter. The same weighting was chosen for all four criteria ranges, each criterion also has the same weight within each criteria range. Each criterion is evaluated on a four-point scale ranging from 0 to 3 with the following meaning:

- 0: the method completely violates the given criterion (completely unsatisfactory),
- 1: the method partially fulfills the given criterion, but insufficiently (unsatisfactory),
- 2: the method fulfills the given criterion to a sufficient extent (satisfactory),
- 3: the method meets the given criterion in full (excellent, optimal value).

We assigned equal weight to each range of criteria. Also, each criterion has the same weight within the given range of criteria. This is because we consider all ranges of criteria and criteria to be equally important.

Table 1 Evaluation criteria of randomized response techniques for population mean

Range	Criterion	Weight
1. Statistical properties of estimator	1.1 Unbiasedness and consistency of estimator	1/12
	1.2 Unbiasedness and consistency of variance estimator	1/12
	1.3 Functional asymptotic confidence intervals	1/12
2. Implementation and parameter choice	2.1 Difficulty of technical implementation of the method	1/8
	2.2 Rules for the optimal choice of method parameters	1/8
3. Respondent burden and credibility	3.1 Difficulty and clarity of the method for the respondent	1/8
	3.2 Credibility of the method for the respondent	1/8
4. Confidentiality protection of respondents' data	4.1 The degree of risk of disclosure of sensitive information	1/8
	4.2 Leakage of sensitive information when knowing the result of a randomized response	1/8

Source: Own construction

3.2 Evaluation of randomized response techniques for population mean

In the multicriteria evaluation, we compare standard methods for estimating the population mean without using an auxiliary variable with newly proposed estimates using a dichotomous response. A total of five estimates using the respective methods are evaluated:

1) estimator $\bar{t}_{Y,E}^{HT,R}$ using Eriksson technique (1973),
2) estimator $\bar{t}_{Y,CH}^{HT,R}$ using Chaudhuri technique (1987),
3) estimator $\bar{t}_{Y,B}^{HT,R}$ using technique of Bar-Lev et al. (2004),
4) estimator $\bar{t}_{Y,(m,M)}^{HT,R}$ using dichotomous response (Antoch et al., 2022),
5) estimator $\bar{t}_{Y,\alpha,(m,M)}^{HT,R}$ using dichotomous response with knowledge random number (Antoch et al., 2022).

In terms of the statistical properties of the estimation, the methods are completely comparable. The estimators have all the desired statistical properties (Eriksson, 1973; Chaudhuri, 1987; Bar-Lev et al., 2004; Antoch et al., 2022), the respective Horvitz-Thompson-type estimators are unbiased and consistent. Variance estimates of using the Arnab (1994) technique (see Vozár, 2023 for estimators using dichotomous response) are also unbiased and consistent. Therefore, all methods fully meet criteria 1.1 and 1.2 and they are rated with four points. The results of simulation studies (Vozár, 2023) show that the variance estimates are strongly influenced either by outliers of transformed weighted responses (Eriksson technique, 1973; Chaudhuri, 1987; Bar-Lev et al., 2004) or by negative values in Horvitz-Thompson estimates and variance estimates for methods using dichotomous responses (Antoch et al., 2022). All methods are therefore evaluated with two points in criterion 1.3.

The evaluation of the methods, including its justification, according to the second range of criteria dealing with the implementation and choice of method parameters is summarized in Table 2. The evaluation of the methods, including its justification, according to the third range of criteria dealing with the respondent's burden and credibility is summarized in Table 3.

Table 2 Evaluation of implementation and parameter choice

Method	Criterion	Evaluation rationale	Score
$\bar{t}_{Y,E}^{HT,R}$	2.1 Difficulty of technical implementation of the method	Fairly undemanding, standard techniques (fate, cards, random number generator)	3
	2.2 Rules for the optimal choice of method parameters	Arbitrary, relying on the experience of the statistician designing the survey	1
$\bar{t}_{Y,CH}^{HT,R}$	2.1 Difficulty of technical implementation of the method	Fairly undemanding, standard techniques (fate, cards, random number generator)	3
	2.2 Rules for the optimal choice of method parameters	Arbitrary, relying on the experience of the statistician designing the survey	1
$\bar{t}_{Y,B}^{HT,R}$	2.1 Difficulty of technical implementation of the method	Fairly undemanding, standard techniques (fate, cards, random number generator)	3
	2.2 Rules for the optimal choice of method parameters	Arbitrary, relying on the experience of the statistician designing the survey	1
$\bar{t}_{Y,(m,M)}^{HT,R}$	2.1 Difficulty of technical implementation of the method	Fairly undemanding, standard techniques (fate, cards, random number generator)	3
	2.2 Rules for the optimal choice of method parameters	Rules of thumb for choice of an interval for generating random numbers (using a priori information about the range and quantiles of the sensitive variable)	2
$\bar{t}_{Y,\alpha,(m,M)}^{HT,R}$	2.1 Difficulty of technical implementation of the method	Fairly undemanding, standard techniques (fate, cards, random number generator)	3
	2.2 Rules for the optimal choice of method parameters	Rules of thumb for choice of an interval for generating random numbers (using a priori information about the range and quantiles of the sensitive variable)	2

Source: Own construction

Table 3 Evaluation of respondent burden and credibility

Method	Criterion	Evaluation rationale	Score
$\bar{t}_{Y,E}^{HT,R}$	3.1 Difficulty and clarity of the method for the respondent	The method is quite understandable, risk of numerical error	2
	3.2 Credibility of the method for the respondent	The method can be seen as tricky, because the interviewer knows the masking variables and recalculates the sensitive value	1
$\bar{t}_{Y,CH}^{HT,R}$	3.1 Difficulty and clarity of the method for the respondent	The method may be perceived as less comprehensible due to the two card moves and more complex arithmetic operations, higher risk of numerical error	1
	3.2 Credibility of the method for the respondent	The method can be seen as tricky, because the interviewer knows the masking variables and recalculates the sensitive value	1

Table 3			(continuation)
Method	**Criterion**	**Evaluation rationale**	**Score**
$\overline{t}_{Y,B}^{HT,R}$	3.1 Difficulty and clarity of the method for the respondent	The method is quite understandable, risk of numerical error	2
	3.2 Credibility of the method for the respondent	The method can be seen as tricky, because the interviewer knows the masking variables and recalculates the sensitive value. Reliability is lower because the respondent may be asked to provide the true value of a sensitive variable	1
$\overline{t}_{Y,(m,M)}^{HT,R}$	3.1 Difficulty and clarity of the method for the respondent	Easy method[1)], "Yes/No" answer to a simple question	3
	3.2 Credibility of the method for the respondent	The method is relatively credible1), it depends on how the respondent perceives the risk associated with knowing the value of the random number in the question	2
$\overline{t}_{Y,\alpha,(m,M)}^{HT,R}$	3.1 Difficulty and clarity of the method for the respondent	Easy method[1)], "Yes/No" answer to a simple question	3
	3.2 Credibility of the method for the respondent	The method is relatively credible,[1)] it depends on how the respondent perceives the risk associated with knowing the value of the random number in the question	2

Note: [1)] The difficulty and credibility of the method depends on the technical way of implementing random number generation.
Source: Own construction

The evaluation of the methods, including its justification, according to the fourth range of criteria dealing with the protection of the confidentiality of the respondent's data is summarized in Table 4.

Table 4 Evaluation of confidentiality protection of respondents' data

Method	Criterion	Evaluation rationale	Score
$\overline{t}_{Y,E}^{HT,R}$	4.1 The degree of risk of disclosure of sensitive information	Fairly good	2
	4.2 Leakage of sensitive information when knowing the result of a randomized response	One hundred percent, given the knowledge of the masking variable	0
$\overline{t}_{Y,CH}^{HT,R}$	4.1 The degree of risk of disclosure of sensitive information	Fairly good	2
	4.2 Leakage of sensitive information when knowing the result of a randomized response	One hundred percent, given the knowledge of the masking variable	0
$\overline{t}_{Y,B}^{HT,R}$	4.1 The degree of risk of disclosure of sensitive information	Fairly good	2
	4.2 Leakage of sensitive information when knowing the result of a randomized response	One hundred percent, given the knowledge of the masking variable	0
$\overline{t}_{Y,(m,M)}^{HT,R}$	4.1 The degree of risk of disclosure of sensitive information	Good, except for extremely low or high values	2
	4.2 Leakage of sensitive information when knowing the result of a randomized response	The risk is very low except for very low or high values, leakage of the exact value only if $y_i = M$	2
$\overline{t}_{Y,\alpha,(m,M)}^{HT,R}$	4.1 The degree of risk of disclosure of sensitive information	Good, except for extremely low or high values	2
	4.2 Leakage of sensitive information when knowing the result of a randomized response	The risk is very low except for very low or high values, leakage of the exact value only if $y_i = M$	2

Source: Own construction

The evaluation of the methods in the individual ranges of quality criteria for the individual randomized response techniques, including their order, is summarized in Table 5. From the point of view of statistical criteria (unbiasedness and consistency of estimates), all compared techniques are equivalent.

The evaluation results showed that the newly proposed methods compared to the standard methods bring improvements in the areas of respondent burden, method credibility, and respondent confidentiality protection. Moreover, for methods using dichotomous response there are rules for choice of an interval for random numbers based on very rough a priori information about the interval of values of the sensitive variable. Since knowledge of random numbers is needed to estimate the variance to construct asymptotic confidence intervals, we are inclined to use the more precise estimate $\overline{t}^{HT,R}_{Y,\alpha,(m,M)}$ in practice. Therefore, methods using dichotomous response can be implemented as dynamic questionnaires in computer or web assisted surveys. The survey tool would use its own random number generator and the generated random number *U* would be used to create question "Is your monthly income greater than *U*?". Value of random number *U* is stored to compute estimate $\overline{t}^{HT,R}_{Y,\alpha,(m,M)}$ and its variance estimator. This approach would require very careful training of intervievers and drafting survey information for respondents'.

Table 5 Evaluation of randomized response technique for population mean

Criterion	Scoring (weighted) and method ranking				
	$\overline{t}^{HT,R}_{Y,E}$	$\overline{t}^{HT,R}_{Y,CH}$	$\overline{t}^{HT,R}_{Y,B}$	$\overline{t}^{HT,R}_{Y,(m,M)}$	$\overline{t}^{HT,R}_{Y,\alpha,(m,M)}$
Cumulative point rating (total)	1.853 **(3.-4.)**	1.728 (5.)	1.853 (3.-4.)	2.383 (1.-2.)	2.383 (1.-2.)
1. Statistical properties of estimator	0.633 (1.-5.)	0.633 (1.-5.)	0.633 (1.-5.)	0.633 (1.-5.)	0.633 (1.-5.)
1.1 Unbiasedness and consistency of estimator	0.250 (1.-5.)	0.250 (1.-5.)	0.250 (1.-5.)	0.250 (1.-5.)	0.2501 (1.-5.)
1.2 Unbiasedness and consistency of variance estimator	0.250 (1.-5.)	0.250 (1.-5.)	0.250 (1.-5.)	0.250 (1.-5.)	0.250 (1.-5.)
1.3 Functional asymptotic confidence intervals	0.133 (1.-5.)	0.133 (1.-5.)	0.133 (1.-5.)	0.133 (1.-5.)	0.133 (1.-5.)
2. Implementation and parameter choice	0.500 (3.-5.)	0.500 (3.-5.)	0.500 (3.-5.)	0.625 (1.-2.)	0.625 (1.-2.)
2.1 Difficulty of technical implementation of the method	0.375 (1.-5.)	0.375 (1.-5.)	0.375 (1.-5.)	0.375 (1.-5.)	0.375 (1.-5.)
2.2 Rules for the optimal choice of method parameters	0.125 (3.-5.)	0.125 (3.-5.)	0.125 (3.-5.)	0.250 (1.-2.)	0.250 (1.-2.)
3. Respondent burden and credibility	0.375 (3.-4.)	0. 250 (5.)	0.375 (3.-4.)	0.625 (1.-2.)	0.625 (1.-2.)
3.1 Difficulty and clarity of the method for the respondent	0.250 (3.-4.)	0.125 (4.-5.)	0.250 (3.-4.)	0.375 (1.-2.)	0.375 (1.-2.)
3.2 Credibility of the method for the respondent	0.125 (3.-5.)	0.125 (3.-5.)	0.125 (3.-5.)	0.250 (1.-2.)	0.250 (1.-2.)
4. Confidentiality protection of respondents' data	0.375 (3.-5.)	0.375 (3.-5.)	0.375 (3.-5.)	0.500 (1.-2.)	0.500 (1.-2.)
4.1 The degree of risk of disclosure of sensitive information	0.250 (1.-5.)	0.250 (1.-5.)	0.250 (1.-5.)	0.250 (1.-5.)	0.250 (1.-5.)
4.2 Leakage of sensitive information when knowing the result of a randomized response	0.125 (3.-5.)	0.125 (3.-5.)	0.125 (3.-5.)	0.250 (1.-2.)	0.250 (1.-2.)

Source: Own construction

CONCLUSION

We proposed methodology of multicriteria evaluation of randomized response techniques for population mean, which incorporate all, both statistical and non-statistical issues relevant for implementation in real-life survey using randomized response. Based on extensive review in the literature, we identified four ranges of criteria relevant to randomized response techniques: statistical properties of estimator, implementation and parameter choice, respondent burden and credibility and confidentiality protection of respondents' data. This methodology was applied to compare standard techniques based on scramble variables and recently proposed techniques using dichotomous response. The techniques using dichotomous response are superior, because of its ease of implementation, higher comfort of respondents (answer "Yes/No" instead of calculations) and better protection of sensitive data. Even if interviewer knows the results of randomized response, he cannot guess the true sensitive values. Because both past and recent studies (Azeem and Ali, 2023 among others) focus mostly on statistical properties of the estimator, we proposed for the future research methodology and set of model populations with different shapes of distribution to compare estimators more objectively. This methodology must study the influence of the following factors affecting the performance of the studied techniques: shape, location, and variability of studied sensitive variable, population and sample size, choice of parameters (i.e. scramble variables). The assessment of extended scope of methods from this paper using the set of these model populations is the topic of our paper under preparation.

There are several areas for the future research. The first area is to assess performance methods and variance estimators (covering all methods in Azeem and Ali, 2023) depending on the size of sample, population and the shape of sensitive variables. The second one is to apply these techniques for the real data – wage data of the Czech and Slovak Republic. The paper under preparation focuses also on important application in official statistics – estimation of year-on-year growth of mean of sensitive variable. Applying the simulation study with the use of distributions of sensitive variables with different shape, we also resolved parameter choice of interval of random number and tuning parameter α. The use of robust techniques for estimates applying dichotomous variables, construction of variance estimators using of robust methods and bootstrap techniques represent open problems.

ACKNOWLEDGMENT

This work was supported by Institutional Support to Long-Term Conceptual Development of Research Organization, the Faculty of Informatics and Statistics of the Prague University of Economics and Business, Prague. The authors are grateful to the unknown reviewers for their valuable comments that considerably improved the contents of this paper.

References

ANTOCH, J., MOLA, F., VOZÁR, O. (2022). New Randomized Response Technique for Estimating the Population Total of a Quantitative Variable [online]. *Statistika: Statistics and Economy Journal*, 102(2): 205–227. <https://doi.org/10.54694/stat.2022.11>.

ARCOS, A., RUEDA, M., SINGH, S. (2015). A generalized approach to randomized response to quantitative data. *Quantity and Quality*, 49: 1239–1256.

ARNAB, R. (1994). Nonnegative variance estimation in randomized response surveys [online]. *Communication in Statistics – Theory and Methods*, 23(6): 1743–1752. <https://doi.org/abs/10.1080/03610929408831351>.

ARNAB, R. (1995). Optimal estimation of a finite population total under randomized response surveys. *Statistics*, 27: 175–180.

AZEEM, M., ALI S. (2023) A neutral comparative analysis of additive, multiplicative, and mixed quantitative randomized response models [online]. *PLoS ONE*. 18(4): e0284995. <https://doi.org/10.1371/journal.pone.0284995>.

BAR-LEV, S., K., BOBOVITCH, E., B, BOUKAI (2004). A note on randomized response models for quantitative data [online]. *Metrika*, 60: 255–260. <https://doi.org/10.1007/s001840300308>.

BLAIR, G., KOSUKE, I., ZHOU, Y. (2015). Design and analysis of randomized response technique. *Journal of American Statistical Association*, 101: 1304–1319.

CHAUDHURI, A. (1987). Randomized response surveys of a finite population: A unified approach with quantitative data [online]. *Journal of Statistical Planning and Inference*, 15: 157–165. <https://doi.org/10.1016/0378-3758(86)90094-7>.

CHAUDHURI, A., MUKERJEE, R. (1988). *Randomized Response, Theory and Techniques.* New York: Marcel Dekker.

CHAUDHURI, A., CHRISTOFIDES, T. (2013). *Indirect Questioning in Sample Surveys.* Heidelberg: Springer. ISBN 978-3642-36275-0

CHAUDHURI, A., CHRISTOFIDES, T., RAO, C. (2016). *Handbook of Statistics 34. Data Gathering, Analysis and Protection of Privacy Through Randomized Response Techniques.* Amsterdam: Elsevier. ISBN 978-0444-63570-9
COBO RODRIGUEZ, B. (2018). *Using auxiliary information in indirect questioning techniques.* Doctoral thesis, University of Granada.
DIANA, G., PERRI, P. F. (2013). Scrambled response models based on auxiliary variables. In: TORELLI, N., PESARI, F., BAR-HEN, A. (eds.) *Advances in Theoretical and Applied Statistics*, Berlin: Springer-Verlag, 281–291.
EICHHORN, B., HAYRE, L. S. (1983). Scramble randomized response methods for obtaining sensitive data. *Journal of Statistical Planning and Inference*, 7: 307–316.
ERIKSSON, S. (1973). A new model for randomized response [online]. *International Statistical Review*, 41: 101–113. <https://doi.org/10.2307/1402791>.
HORVITZ, D., THOMPSON, D. (1952). A generalization of sampling without replacement from a finite universe [online]. *Journal of American Statistical Association*, 47: 663–685. <https://doi.org/10.1080/01621459.1952.10483446>.
TILLÉ, Y. (2006). *Sampling Algorithms.* New York: Springer. ISBN 978-0387-30814-2
VOZÁR, O. (2023). Unbiased variance estimator of the randomised response techniques for population mean [online]. *Statistika: Statistics and Economy Journal*, 103(1): 113–120. <https://doi.org/10.54694/stat.2022.38>.
WARNER, S. (1965). Randomized response: A survey technique for eliminating evasive answer bias [online]. *Journal of American Statistical Association*, 60: 63–69. <https://doi.org/10.2307/2283137>.