Payout Phase of Defined Contribution Systems: the Case of Slovakia

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Received 8.3.2023 (revision received 30.6.2023), **Accepted** (reviewed) 4.7.2023, **Published** 15.12.2023

Abstract

The paper aims to assess various aspects concerning the payment phase of the old-age pension scheme, the so-called second pillar of the pension system in Slovakia. However, the conclusions may also be useful for other pension systems. Using the Lee-Carter model and standard actuarial methods, we conclude that the second pillar is advantageous for the high-income groups or in case of high performance of pension funds. We also address the issue of deferring the purchase of a lifetime annuity. Deferral can be beneficial when the yield of the pension fund exceeds a certain threshold value. This threshold usually raises with increasing age. We argue that the temporary pension is a disadvantageous product and its recent cancellation is correct. The main contribution of the paper subsists in a three-state model of long-term care insurance, using which we calculate corresponding replacement rates. Combined with a lifetime annuity, long-term care insurance can be beneficial.

Keywords	***DOI***	***JEL code***
Pension system in Slovakia, Svensson yield curve, Lee-Carter model, lifetime annuity, deferred annuity, long-term care insurance	*https://doi.org/10.54694/stat.2023.12*	*G22, H75, I13, J11, J26*

INTRODUCTION

Since January 2005, pensions in Slovakia are operated by a three-pillar system: the compulsory, Pay-As-You-Go (PAYG) first pillar, the second pillar in the form of old-age pension saving, and the third pillar as a voluntary supplementary pension saving. Only the first pillar is compulsory and future pensioners can redirect part of their contributions to the saving (second) pillar. In this case, the first pillar pension is reduced accordingly. This reduced pension can be supplemented by benefits from savings of the second pillar. Savings in the second pillar and the subsequent payment phase are governed by the Act of the National Council of the Slovak Republic No. 43/2004 Coll. on the old-age pension scheme, as amended ("Act 43/2004 Coll.").

Several publications addressed the question of whether participation in the second pillar is beneficial. The answer is not uniform and depends on a specific wage profile. For low-income groups, currently

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valid Act 43/2004 Coll. only offers the possibility of paying benefits in the form of a lifetime annuity. According to Melicherčík et al. (2015), lifetime annuity benefits may not cover (even in the case of zero fees of insurance companies) the reduction of the first pillar pension. For higher-income savers, the probability of covering the reduction is higher due to the partial solidarity of the first pillar. In Gubalová et al. (2022), a calculation of the Global Pension Index for Slovakia is presented. The authors point to the problem that very low pensions for low-income groups negatively affect the value of the pension index of Slovakia. This is a serious issue, which is mainly caused by the inappropriate setting of saving strategies in the funded pillars of the Slovak pension system and the ways of receiving pension benefits from the second and third pillars. In Špirková et al. (2021), authors offer a critical view of pension savings in the Slovak pension system. Within several case studies, they analyze risk factors that may affect the amount of future pension benefits and discuss the importance of choosing a suitable payout product.

However, currently valid Act 43/2004 Coll. provides more possibilities for using pension savings. If the sum of pension benefits paid from other sources reaches a minimum reference amount (currently an average old-age pension), the pensioner may apply for a programmed withdrawal or a temporary pension. The programmed withdrawal also includes the possibility to withdraw the entire saved amount at once. This greatly expands the possibilities of using pension savings.

Recently, the first offers of life insurance companies to pay lifetime benefits from the second pillar have appeared. The level of benefits was low mainly due to the risk of longevity and low interest rates. On the other hand, many authors (see e.g., Milevsky, 1998; Šebo and Šebová, 2016) argued that the immediate purchase of a lifetime annuity is not an optimal use of pension savings. The Slovak government, therefore, came up with the amendment to Act 43/2004 Coll., where the payment phase is implemented in the form of a specific programmed withdrawal and a deferred purchase of a lifetime annuity. Temporary pensions and all other forms of programmed withdrawal are cancelled. One of the contributions of this paper is a discussion, of whether such a change is beneficial for pensioners.

Pension benefits should be set to meet the needs of pensioners. Due to health problems, long-term care is often necessary during retirement. The extent of care depends on the specific disability and can also be full-time care. Pensioners typically do not have the necessary resources to finance it. The main contribution of this paper is a model focused on using pension savings to purchase long-term care insurance. We have calculated the resulting replacement rates that can be achieved using realistic savings levels.

To summarize, the Slovak pension system after the reform in 2005 combines several principles. The PAYG first pillar is supplemented by a defined-contribution second pillar. The payment phase of the second pillar offers several options. Therefore, even though the article is based on the reality of the pension system in Slovakia, several issues that are also interesting for other pension schemes are discussed. The aim of the paper is to thoroughly evaluate the payment phase of the second pillar of the pension system in Slovakia. We will also discuss the appropriateness of the latest legislative changes in Act 43/2004. Based on the above, we set the following hypotheses:

H1: The benefits from the second pillar are not sufficient to compensate the corresponding reduction of the first pillar.

H2: Immediate purchase of a lifetime annuity is not an optimal use of pension savings.

H3: Temporary pension is an unsuitable pension product.

H4: Long-term care insurance is a reasonable pension product that could suitably expand the current options in the payout phase of the second pillar.

When discussing H1-H4, the Lee-Carter stochastic demographic mortality model (Lee and Carter, 1992) and the Svensson model (Svensson, 1994) are used. To convert savings into replacement rates, we use the approach of Špirková et al. (2019). Regarding deferred annuities, we follow the results of Milevsky (1998). However, we extend these calculations by considering the more complicated structure of the fees associated with annuities and the Svensson yield curve for discounting. Our three-state model is used for calculations related to long-term care insurance.

The paper is organized as follows. In the first Section, we assess the level of pension benefits when buying lifetime annuities. In the second Section, we first deal with programmed withdrawal and temporary pensions. Then, we discuss in detail the deferral of the purchase of lifetime annuities and the convenience of the latest changes in Act 43/2004 Coll. regarding the payment phase of the second pillar. The third Section deals with long-term care insurance. In the final part we conclude.

1 EXPECTED LEVEL OF SAVINGS AND LIFETIME ANNUITIES

The mandatory part of the pension system in Slovakia has two pillars: the public, compulsory, non-funded (Pay-As-You-Go) first pillar, and the private, fully funded second pillar. The contribution rate (for the old-age pensions) is currently set at 18% for the first pillar (in the case a pensioner decides to stay only in the public scheme only) or 12.5% for the first pillar and 5.5% for the second pillar (in the case a pensioner decides to participate in both pillars) with a future planned increase to 6%.

The adequacy of pension savings can be assessed in several ways. In Kilianová et al. (2006) authors introduced a retirement-years indicator (D_T). It was calculated as the ratio of the sum S_T saved at the time of retirement T and the last yearly wage W_T before retirement: $D_T = \frac{S_T}{W_T}$. This indicator can be easily recalculated to the replacement rate (the ratio of the first pension to W_T, cf. Melicherčík et al., 2015).

1.1 Survival and mortality probabilities

In order to recalculate the savings to the replacement rate, we have used an approach of Špirková et al. (2019). Let us assume that all ages are expressed in years. Pricing of annuity products has been based on the relevant survival probabilities ${}_tp_x$ representing the probability that an individual at age x years survives at least until the age of $x + t$ years, where $t \in \mathbb{N}$. Denote by q_x the probability that an individual being at age x years dies before age $x + 1$ years. Then ${}_tp_x = \prod_{h=0}^{t-1}(1 - q_{x+h})$. In our practical calculations we have used three sets of mortality rates (for $x \in \{62, 63, \ldots, \omega\}$, where $\omega = 105$ years denotes the maximum age):

- $q_x^{(S)}$ representing the static mortality rates from the Mortality Tables of the SO SR (SO SR, 2022) for the total Slovak population in the year 2018,
- $q_x^{(H)}$ denoting the predicted mortality rates using the Lee-Carter longevity model for the future period from 2020 to 2063,
- $q_x^{(L)}$ representing the estimated mortality rates from the lower bound of the 99% prediction interval for the aforementioned Lee-Carter predictions; in our calculations we have applied them as components of the pessimistic (in terms of insurance) longevity model (see also Špirková et al., 2019).

For this mortality rates applies: $q_x^{(S)} \geq q_x^{(H)} \geq q_x^{(L)}$, $\forall x \in \{62, 63, \ldots, 105\}$.

The Lee-Carter stochastic demographic mortality model is a statistical model that is widely used in the fields of demography, actuarial science, and old-age pension modelling to analyse and forecast mortality rates. The model was developed in 1992 by Ronald D. Lee and Lawrence R. Carter (Lee and Carter, 1992) and has since become one of the most popular and well-regarded mortality models. The Lee-Carter model is based on the idea that mortality rates can be expressed as a function of age and a time trend, which captures the overall pattern of mortality improvement over time. Specifically, the model assumes that the logarithm of the age-specific mortality rates follows a linear trend with age, with the slope of the trend varying over time according to a stochastic process. In our case, we have estimated the parameters of the Lee-Carter model based on data from the Human Mortality Database (HMD, 2022) for the total Slovak population from 1990 to 2019 using the 'demography' package (Hyndman, 2023) of the statistical software R (R Core Team, 2022). Subsequently, we have predicted the mortality rates for the future period of 2020–2063 for persons aged 62 to 105 years using the R package 'forecast' (Hyndman and Khandakar, 2008; Hyndman et al., 2023). The Lee-Carter longevity model, i.e., vector of mortality rates $q_x^{(H)}$ for $x \in \{62, 63, \ldots, 105\}$, has been constructed by selecting the diagonal elements of the matrix of predicted mortality rates. The pessimistic longevity model has been built similarly, but when constructing the vector $q_x^{(L)}$, $x \in \{62, 63, \ldots, 105\}$, we have used the diagonal elements of the matrix containing the lower bound values of 99% prediction intervals.

1.2 Svensson yield curve

In accordance with ECB (2022b), we have used the Svensson yield curve as a functional form for the spot interest rates depending on corresponding maturities. The Svensson yield curve is given by (Svensson, 1994):

$$R(t) = \beta_0 + \beta_1 \frac{1-\exp\left(-\frac{t}{\tau_1}\right)}{\frac{t}{\tau_1}} + \beta_2 \left[\frac{1-\exp\left(-\frac{t}{\tau_1}\right)}{\frac{t}{\tau_1}} - \exp\left(-\frac{t}{\tau_1}\right)\right] + \beta_3 \left[\frac{1-\exp\left(-\frac{t}{\tau_2}\right)}{\frac{t}{\tau_2}} - \exp\left(-\frac{t}{\tau_2}\right)\right], \tag{1}$$

where $R(t)$ is a yield from a bond investment with continuous compounding, t is a time to maturity, $t \in (0, T_{\max}]$, $T_{\max}$ is the maximum time to maturity, τ_1, τ_2, β_0, β_1, β_2, β_3 are parameters of the Svensson yield curve. The discounting factor corresponding to the maturity t is then given by $P(t) = e^{-R(t)t}$. The interpretation of the parameters is available e.g., in Aljinović et al. (2012): β_0 is the long-term asymptotic value of $R(t)$, β_1 is the spread between the long-term and short-term rates, i.e., $\beta_0 + \beta_1$ is the short-term rate (the rate corresponding to zero maturity). The parameters τ_1, τ_2, β_2, β_3 specify the positions, magnitudes and directions of two humps corresponding to the Svensson curve.

1.3 Pension annuity product

Consider a person with a retirement age x years having saved the amount S_T. The basic equivalence equation represents the expected present values of all cash-flows related to the yearly annuity payment P_x:

$$S_T = P_x a_x (1 + \beta) + P_x \alpha. \tag{2}$$

On the left-hand side stands the accumulated sum S_T representing a premium of the product. The value $P_x a_x$ is the expected present value of the whole life yearly paid annuity-immediate P_x, where:

$$a_x = \sum_{t=1}^{\omega-x} {}_tp_x \, P(t)$$

denotes the expected present value of a whole life 1 monetary unit (m. u.), paid at the end of each year under the condition that the person is alive and ω is the maximum age (regarding used life tables $\omega = 105$ years). According to the current version of Act 43/2004 Coll., the monthly benefits P_x are paid in the first seven years of the retirement period regardless of whether the beneficiary is alive. Therefore, we set ${}_tp_x = 1$ for $t = 1, 2, \ldots, 7$. Finally, α and β represent fees charged to the first and following annuity payments. Denote by

$$\tilde{a}_x = a_x(1 + \beta) + \alpha,$$

the value of the whole life 1 m. u. including fees. Dividing both sides of (2) by W_T and making some minor adjustments one has

$$\mathrm{RE}_x = \frac{P_x}{W_T} = \frac{D_T}{\tilde{a}_x}, \tag{3}$$

where RE_x is the replacement rate (the ratio of the yearly pension to the last yearly salary before retirement).

Table 1 contains replacement rates for different levels of savings and retirement ages calculated using the Svensson ECB all bonds curve (the parameters have been estimated in ECB (2022a) as of 9 June 2022) and fees $\alpha = 50\%$, $\beta = 8\%$. We have applied probabilities of death $q_x^{(H)}$ calculated using the Lee-Carter longevity model (see Section 1.1). In Table 2 the values of $\tilde{a}_x$ for various ages together with corresponding conditional life expectancies (LE_x) are presented. One can observe that the $\tilde{a}_{62}$ is slightly less than LE_{62}, which is a consequence of the appreciation of savings using interest rates. However, due to the 7-year warranty and transaction costs (especially, in the case of shorter LE_x, the charged α plays a more significant role), with increasing age, the price of $\tilde{a}_x$ becomes significantly higher than the expected lifetime LE_x. It is worth noting, that Act 43/2004 Coll. has been recently amended and the payout phase will (from 1 January 2024) have a different form. This will be in detail discussed in Section 2.

Table 1 Replacement rates of lifetime annuity payments for various levels of savings and initial ages of the pensioner (x in years) using mortality rates $q_x^{(H)}$ with 7-year guarantee applied

D_T / age x	62	65	70	75	80
1.5	0.0782	0.0830	0.0932	0.1063	0.1210
2.0	0.1043	0.1108	0.1243	0.1417	0.1613
2.5	0.1303	0.1385	0.1553	0.1772	0.2017
3.0	0.1564	0.1661	0.1864	0.2126	0.2420
3.5	0.1825	0.1938	0.2175	0.2480	0.2823
4.0	0.2085	0.2215	0.2485	0.2835	0.3226

Source: Own construction

Table 2 Values of $\tilde{a}_x$ for different ages x (in years) using mortality rates $q_x^{(H)}$ with 7-year guarantee applied together with corresponding conditional life expectancies at age x (LE_x in years) in the total Slovak population in 2019 according to HMD (2022)

x	62	65	70	75	80
$\tilde{a}_x$	19.18	18.06	16.09	14.11	12.40
LE_x	19.97	17.78	14.28	11.09	8.21

Source: Own construction according to HMD (2022)

To illustrate the calculated levels of replacement rates, let us consider a person contributing to the second pillar at 6% of the gross wage. Following recent legislative changes, the total contribution rate used for the reduction of the first pillar pension was increased from 18% to 22.75%.[3] Therefore, this saver will receive 16.75/22.75 of the pension from the first pillar designed for 50% replacement rate.[4] As a result, the required compensation from the savings of the second pillar is roughly $13.2\% \approx \frac{6}{22.75} \times 50\%$. Using the results from Table 1 one can conclude, that achieving such a replacement rate requires a level of savings of approximately 2.5 of yearly salaries. In Melicherčík et al. (2015) authors reported a realistic level

[3] The first pillar pension is reduced by the part δ / 22.75 for the period of participation in the second pillar, where δ is the contribution rate (in percentage) to the second pillar. The original reduction ratio was δ / 18.

[4] According to the amendment to Act of the National Council of the Slovak Republic No. 461/2003, the newly granted pensions will not be increased by the entire increase of wages (only by 95% of the increase of wages). The replacement rate of 50% will therefore be reduced in the future.

of savings D_T in the second pillar after 40 years of saving between 1.5 and 4 yearly salaries. Compensation for the reduction of the first pillar by savings from the second pillar is therefore questionable.

However, this calculation only applies to certain income groups. The amount of the starting pension is linear with respect to a quantity called the Average Personal Wage Point (APWP). This quantity is the average of the ratios of personal wages and average wages in the national economy for a working career.[5] To calculate the pension, however, the APWP is adjusted according to Table 3. The adjusted APWP is denoted by APWP*. The above calculation is valid for APWP ∈ [1, 1.25). For other values of APWP, the required compensation of the first pillar benefits can be estimated as:

$$\frac{6.00}{22.75} \times \frac{\mathrm{APWP}^*}{\mathrm{APWP}} \times 50\%. \tag{4}$$

Table 3 APWP and adjusted APWP (APWP*)

APWP	APWP*
APWP < 1	APWP + (1 – APWP) × 0.2
APWP ∈ [1, 1.25]	APWP
APWP > 1.25	min(1.25 + (APWP – 1.25) × 0.68 ,3)

Source: Own construction

The values of the required compensations according to (4) for selected values of the APWP are presented in Table 4. One can observe that the second pillar is beneficial for high-income groups, for which the required compensation rates are lower.

Table 4 Required compensations (in %) of the first pillar benefits

APWP	0.50	0.75	1.00	1.25	1.50	2.00	3.00	5.00	7.00
APWP*	0.60	0.80	1.00	1.25	1.42	1.76	2.44	3.00	3.00
Compensation	15.82	14.07	13.19	13.19	12.48	11.60	10.73	7.91	5.65

Source: Own construction

2 OTHER LEGAL POSSIBILITIES OF USING PENSION SAVINGS

2.1 Temporary pension and programmed withdrawal

According to current legislation, a saver is entitled to a programmed withdrawal or a temporary pension if the sum of pension benefits paid from other sources reaches a minimum reference amount (currently an average old-age pension). A temporary pension is an insurance product paid by insurance companies. The length of the contract can be 5, 7 or 10 years. Unlike the second pillar lifetime pension, the temporary pension does not include the insurer's obligation to pay 7 years of pension benefits. At the event of the beneficiary's death, the payment of benefits shall cease. In addition, a temporary pension does not insure longevity. The product can be valued using Formula (2) while omitting the condition $_tp_x = 1$ for $t = 1, 2, \ldots, 7$. To highlight the disadvantage of the temporary pension, we have calculated the results using fees $\beta = 8\%$ (the same as in the case of a lifetime annuity) and $\alpha = 0\%$ (50% for the lifetime annuity). Values $\tilde{a}_x$ used for the calculation of the replacement rates for temporary pensions can be found in Table 5. One can observe that the values $\tilde{a}_x$ are close to the lengths of the pensions. Therefore, it is questionable

[5] For details, see Act of the National Council of the Slovak Republic No. 461/2003, as amended (Articles 62 and 63).

(especially for lower ages and pension lengths) whether there is a big difference between the temporary pension and the withdrawal of the full amount (which is a legal form of the programmed withdrawal) with gradual spending without any institutional assistance. A motivational difference can be observed only for higher ages or lengths of temporary pensions. On the other hand, in this case, the risk of death before the end of the planned period is substantial. To conclude, a temporary pension is probably not a good alternative for using pension savings. The main reasons are low interest rates and low probabilities of death. The cancellation of this option in the current amendment to the law is therefore correct.

Table 5 Values of $\tilde{a}_x$ used for calculation of the replacement rates for temporary pensions using mortality rates $q_x^{(H)}$ for various ages of the pensioners (x in years) and durations of pensions

Age x / duration (in years)	5	7	10
62	4.9741	6.7022	9.0094
65	4.9392	6.6379	8.8837
70	4.8617	6.4923	8.5946
75	4.7361	6.2501	8.0914
80	4.4725	5.7547	7.1639

Source: Own construction

In contrast, the programmed withdrawal is not an insurance product. When using this method of payment, the savings remain in the pension management company (PMC), with which the pensioner concludes a retirement benefit plan. Under this agreement, the PMC will pay a pension from a personal account under pre-agreed terms. The beneficiary determines the monthly amount and the length of the retirement benefits. At the event of death, the remaining funds are subject to inheritance. It is worth noting that by using the programmed withdrawal, one can avoid the annuity fees. An interesting set of dynamic and static strategies of programmed withdrawal can be found in Šebo and Šebová (2016). The authors also calculated the expected value of the bequest corresponding to the respective strategies. Most of the strategies presented avoided ruin and can be considered a more effective alternative compared to lifetime annuities. However, according to the latest legislative changes, the programmed withdrawal is subject to income tax, which partially disadvantages this form of using pension savings.

2.2 Deferred purchase of an annuity

The legislation does not require the purchase of a lifetime annuity from the second pillar savings even in the case of receiving a pension from the first pillar. There is no reason to rush to buy an annuity when the pensioner's income is sufficient. Such a situation occurs, e.g., when the beneficiary is working after retirement age. In such a case, it may be advantageous to defer the purchase of an annuity, or not to buy an annuity at all, and to use the savings later for a more reasonable purpose or to leave them as a bequest. Paragraph 46i of Act 43/2004 Coll. gives a possibility to apply for the return on investment payment if the saver has reached the retirement age, while not being the recipient of the retirement pension or early retirement pension by programmed withdrawal. Note that the return on investment is not a retirement pension. Therefore, savings remain the property of the saver. It is worth noting that in some years the return on investment can be negative and therefore zero benefits may be paid.

A rational question is whether it is worth delaying the purchase of a lifetime annuity. In IFP (2022) authors discussed a combination of a programmed withdrawal and buying the lifetime annuity. They considered the programmed withdrawal with monthly payments one would receive when buying an annuity at retirement. While monthly benefits were paid, the remaining savings were invested. After 10 years, the lifetime annuity was purchased from the rest of the savings. The authors reported, that the resulting monthly lifetime benefit was with probability 93% higher than that resulting from the lifetime annuity purchased at retirement. A thorough analysis of strategies using deferred lifetime annuities considering stochastic interest rates and mortality rates can be found in Milevsky (1998).

Assume that a retiree at age x years has an amount $\tilde{a}_x$ to purchase a unit lifetime annuity. However, the purchase of an annuity may be deferred by one year and the funds $\tilde{a}_x$ may be invested instead. Assuming an annual return of r, such a strategy is advantageous if:

$$\tilde{a}_x(1 + r) \geq \tilde{a}_{x+1} + 1. \tag{5}$$

From (5), a minimum yield r_x^* can be derived to make this strategy beneficial:

$$r_x^* = \frac{\tilde{a}_{x+1}}{\tilde{a}_x} + \frac{1}{\tilde{a}_x} - 1. \tag{6}$$

Compared to the Milevsky (1998) approach, we consider in addition the transaction cost α associated with the first payment and different interest rates for different maturities according to the Svensson curve. For a deeper insight into Formula (6), let's write $\tilde{a}_x$ in a simpler form

$$\tilde{a}_x = (1 + \beta)\sum_{t=1}^{\omega-x} \frac{{}_tp_x}{(1 + y)^t} + \alpha,$$

where y is a yield to maturity. Supposing that the yield y is the same for x and $x + 1$ and using a natural actuarial identity:

$${}_tp_{x+1} = \frac{{}_{t+1}p_x}{{}_1p_x},$$

Formula (6) takes the form

$$r_x^* = \frac{1 + y}{{}_1p_x} - 1 - \frac{\beta}{\tilde{a}_x} - \frac{\alpha}{\tilde{a}_x}\left(\frac{1 + y}{{}_1p_x} - 1\right). \tag{7}$$

The first term on the right-hand side of (7) is increasing with respect to x. On the other hand, the terms with fees α and β are decreasing with respect to x. Thus, for zero fees, the minimum yield r_x^* increases with age and sooner or later the purchase of the lifetime annuity is advantageous. On the other hand, high fees can increase the optimal purchase age.

The minimum values of r_x^* calculated according to Formula (6) for selected ages are in Table 6. The results e.g., show that if the deferral of the lifetime annuity is to be beneficial for 10 years, a yield of around 4% is required. The calculations do not assume a 7-years payment guarantee as in Section 1.3.

Table 6 Minimum values of r_x^* for various ages (in years) using mortality rates $q_x^{(H)}$

x	r_x^*	x	r_x^*	x	r_x^*	x	r_x^*
62	0.0333	72	0.0399	82	0.0649	92	0.1452
64	0.0344	74	0.0421	84	0.0775	94	0.1751
66	0.0354	76	0.0449	86	0.0922	96	0.2188
68	0.0366	78	0.0490	88	0.1076	98	0.2805
70	0.0380	80	0.0553	90	0.1243	100	0.3621

Source: Own construction

Such a guarantee affects too much the price of the annuity and does not make sense for higher ages. Therefore, we omit it throughout this section. Results presented in Table 6 confirm that the minimum yield r_x^* is increasing with x.

The current amendment to Act 43/2004 Coll. assumes that it is advantageous to postpone the purchase of the lifetime annuity.[6] According to this change, the saved funds will be divided into two equal parts at the start of the payout phase. Define D_x as half of the median life expectancy common to men and women of the saver age. Half of the savings are then paid evenly on monthly basis during the period D_x (this is a specific form of programmed withdrawal, which is not taxed). The other half of the savings (invested in the meantime) will be used to buy a life annuity after the D_x period. Denote by M the amount saved at the retirement age. Suppose that one half of the savings were invested with a yield y. To continue with at least the same payments after period D_x, one has the inequality:

$$\frac{M}{2}\frac{(1+y)^{D_x}}{\tilde{a}_{x+D_x}} \geq \frac{M}{2D_x},$$

hence:

$$\tilde{a}_{x+D_x} \leq D_x(1+y)^{D_x}. \tag{8}$$

The values of D_x, $\tilde{a}_{x+D_x}$, and threshold yields for ages over 60 years are shown in Table 7. One can observe that e.g. for the beginning of using funds from the second pillar at the age of 64 years, a yield of 2.83% is required to continue with the same benefits after the period D_x. In recent financial market conditions, this is realistic, but not certain. Increasing y^* with increasing age x is consistent with the results of Table 6. Moreover, a programmed withdrawal during the D_x period will prevent loss of potential inheritance at the event of death. Finally, let's recall the fact that fixed transaction costs α have a greater impact on annuity prices at older ages. To summarize, the combination of a programmed withdrawal with the subsequent purchase of a lifetime annuity provides an interesting solution, balancing the reduction of the risk of losing funds at the event of earlier death with the potential problem of a reduction in benefits after switching to a lifetime annuity.

Table 7 The estimated values of D_x (in years) using the Mortality Tables of the SO SR (SO SR, 2022) for the total Slovak population in the year 2019, values of $\tilde{a}_{x+D_x}$ and threshold yields y^* for ages over 60 years, using mortality rates $q_x^{(H)}$ and no guarantee applied

Age x (in years)	60	62	64	66	68	70
D_x	11.25	10.40	9.57	8.75	7.97	7.19
$\tilde{a}_{x+D_x}$	13.59	13.06	12.50	11.93	11.34	10.74
y^*	0.01696	0.02211	0.02831	0.03610	0.04523	0.05739

Source: Own construction

3 LONG-TERM CARE INSURANCE

At present, the need for a long-term care in the case of dependency is a common problem. Most pensioners do not have the means to cover the associated costs. In this section, we present our model of long-term care insurance as well as the replacement rates it could provide. We have supposed that a person can be in one of the following three states:

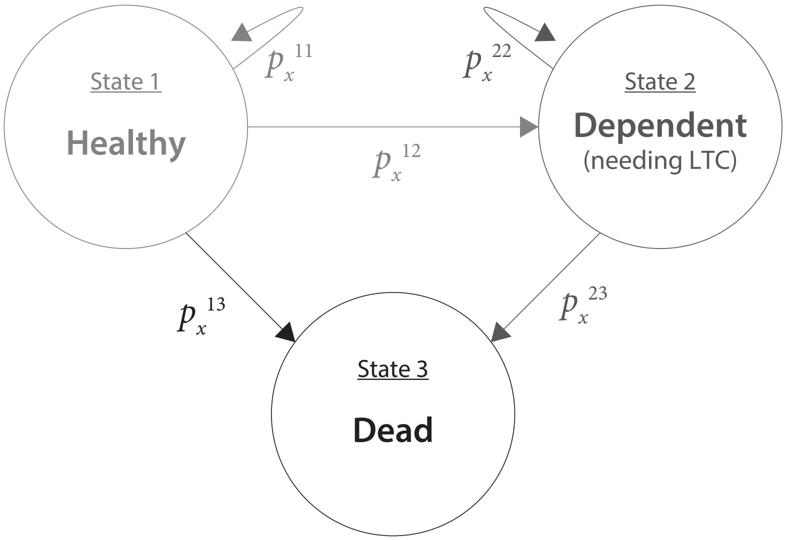
1. Healthy,

[6] It is in accord with already mentioned results of IFP (2022).

2. Dependent (needing a long-term care),
3. Dead.

Furthermore, we have assumed that a healthy person can become dependent or die and a dependent person cannot become healthy. A graphical representation of the model is shown in Figure 1.

Figure 1 Graphical representation of the long-term care (LTC) insurance model



Source: Own construction

Table 8 Shares requiring long-term care (LTC) in the population of Belgium (2007)

Age cohort (in years)	% of LTC	Age cohort (in years)	% of LTC
50–54	4.7465	70–74	15.1312
55–59	5.8612	75–79	20.5790
60–64	8.3343	80–84	36.6185
65–69	8.5914	85+	72.2630

Source: Willemé (2010)

Since realistic data of number of persons requiring long-term care in Slovakia are not available, we have used the data from Belgium (Willemé, 2010). The shares requiring long-term care in Belgium are in Table 8. In the first step, we have interpolated these data using weighted averages, with weights corresponding to population sizes at ages 65, 66, …, 105 years in the total Slovak population; see Mortality Tables of the SO SR for 2018 (SO SR, 2022). In the second step, we have fitted the interpolated values by a polynomial-exponential function. The resulting estimates of shares γ_x requiring long-term care for various age cohorts x are available in Table 9.

Let us denote the probabilities of remaining in the corresponding states, respectively transitions between the states as follows:

- p_x^{ii} – the probability that an individual being at age x years in state $i \in \{1, 2\}$ remains in this state at least to age $x + 1$ years,

- ${}_{m}p_x^{ii}$ – the probability that an individual being at age x years in state $i \in \{1, 2\}$ remains in this state at least to age $x + m$ years, where $m \in \mathbb{N}$ is a multiple of year,
- p_x^{ij} – the probability that an individual being at age x years in state $i \in \{1, 2\}$ transits to state $j \in \{2, 3\}$, $j > i$ before age $x + 1$ years.

Table 9 Shares requiring long-term care (γ_x) for various ages x (in years) – our estimates

x	γ_x	x	γ_x	x	γ_x	x	γ_x
62	0.0833	73	0.1489	84	0.5005	95	0.8968
63	0.0840	74	0.1691	85	0.5549	96	0.9288
64	0.0846	75	0.1913	86	0.6049	97	0.9440
65	0.0852	76	0.2151	87	0.6477	98	0.9627
66	0.0859	77	0.2401	88	0.6890	99	0.9757
67	0.0865	78	0.2661	89	0.7239	100	0.9821
68	0.0888	79	0.2958	90	0.7623	101	0.9848
69	0.0944	80	0.3400	91	0.7911	102	0.9869
70	0.1034	81	0.3775	92	0.8204	103	0.9892
71	0.1157	82	0.4122	93	0.8492	104	0.9920
72	0.1310	83	0.4500	94	0.8693	105	0.9920

Source: Own construction

The following equations apply to the mentioned probabilities:

$$p_x^{11} + p_x^{12} + p_x^{13} = 1, \tag{9}$$

$$p_x^{22} + p_x^{23} = 1. \tag{10}$$

By shifting individuals of age x years and balancing the number of dependent ones we have:

$$(1 - \gamma_x)p_x^{12} - \gamma_x p_x^{23} = \gamma_{x+1}(1 - \bar{q}_x) - \gamma_x, \tag{11}$$

$$\bar{q}_x = (1 - \gamma_x)p_x^{13} + \gamma_x p_x^{23}. \tag{12}$$

Formulas (9)–(12) are not sufficient to determine all the necessary probabilities p_x^{ij}. Missing equations can be replaced by defining the relationship between the mortality probabilities p_x^{13} and p_x^{23}. One has more options for this definition, e.g.

1. $p_x^{13} = p_x^{23} = q_x^{(H)}$, the case of mortality rates using Lee-Carter longevity model,
2. $p_x^{13} = p_x^{23} = q_x^{(L)}$, the case of pessimistic longevity (in terms of insurance),
3. $p_x^{13} = q_x^{(L)}$ with additional equation $q_x^{(S)} = (1 - \gamma_x)\, p_x^{13} + \gamma_x p_x^{23}$ setting the whole-population mortality rate at the higher level; in this case the dependents (persons dependent on the long-term care) have lower life expectancy comparing to the healthy population,
4. $p_x^{13} = q_x^{(L)}$ and $p_x^{23} = \kappa\, \dfrac{q_x^{(S)} - (1 - \gamma_x)p_x^{13}}{\gamma_x}$, where κ is a constant factor such that the dependents have the half life expectancy comparing to the healthy population,

5. $p_x^{13} = q_x^{(S)}$ and $p_x^{23} = \lambda \dfrac{q_x^{(S)} - (1 - \gamma_x)p_x^{13}}{\gamma_x} = \lambda p_x^{13}$, where $\lambda > 1$ is a constant factor such that the dependents have the half life expectancy comparing to the healthy population.

Combining Formulas (9)–(12) with any of these options one can determine all the necessary probabilities p_x^{ij}, $i, j \in \{1, 2, 3\}$, $i \leq j$.

Denote by Z_x a random variable representing the time (in years) in which a healthy person of age x years switches to state 2 (long-term care dependency). It is obvious, that $\Pr(Z_x = 1) = p_x^{12}$. The probabilities for further times $k > 1$, where $k \in \mathbb{N}$, can be calculated as

$$\Pr(Z_x = k) = p_x^{11} \times p_{x+1}^{11} \times \ldots \times p_{x+k-2}^{11} \times p_{x+k-1}^{12}. \tag{13}$$

Suppose that a person of age x years becomes dependent after m years. The expected present value of long-term care lifetime benefits L_x is $L_x \times {}_{m|}a_x^L$, where:

$$_{m|}a_x^L = P(m + 1) + \textstyle\sum_{t=m+1}^{\omega-x-1} P(t + 1) \times {}_{t-m}p_{x+m}^{22}. \tag{14}$$

The probabilities ${}_kp_y^{22}$ that the person of age y years remains in state 2 at least next $k \in \mathbb{N}$ years are calculated as follows:

$$_kp_y^{22} = p_y^{22} \times p_{y+1}^{22} \times p_{y+2}^{22} \times \ldots \times p_{y+k-1}^{22}. \tag{15}$$

The expected present value of unit long-term care benefits is then:

$$\mathrm{E}\left[{}_{Z_x|}a_x^L\right] = \textstyle\sum_{m=1}^{\omega-x-1} {}_{m|}a_x^L \times \Pr(Z_x = m). \tag{16}$$

Denote by $\tilde{a}_x{}^L$ the value of unit long-term benefits including fees:

$$\tilde{a}_x^L = \mathrm{E}\left[{}_{Z_x|}a_x^L\right](1 + \beta) + \alpha.$$

The replacement rate of long-term care benefits RL_x can be then calculated as:

$$\mathrm{RL}_x = \frac{L_x}{W_T} = \frac{D_T}{\tilde{a}_x{}^L}. \tag{17}$$

We have calculated replacement rates for three options of the relationship between the mortality probabilities p_x^{13} and p_x^{23} (see above). In our calculations, we have considered the fees $\alpha = 10\%$ and $\beta = 9\%$. Compared to the case of the lifetime annuity, the fee α is lower (50% for the lifetime annuity). This reflects the fact that the expected lifetime (EL) in state 2 is typically lower than the life expectancy when calculating a lifetime annuity and a high fee corresponding to the first pension could significantly lower the benefits. The decrease in the fee α is compensated with a higher fee β ($\beta = 8\%$ for the lifetime annuity). In our calculations, we have considered the level of savings $D_T = 3$ yearly salaries.

Option 2, where $p_x^{13} = p_x^{23} = q_x{}^{(L)}$, can be used to estimate the upper limit of the long-term care (LTC) insurance price. The replacement rates of the LTC benefits and the expected value of 1 m. u. lifetime benefits (paid in the case of necessary LTC) are presented in Table 10 (columns RR and $\mathrm{E}\left[{}_{Z_x|}a_x^L\right]$, respectively). Compared to the replacement rates for the lifetime annuities (Table 1) one can observe significantly higher values. For example, for the age of 62 years, the replacement rate for LTC benefits is more than 6 times higher than in the case of lifetime annuity. This value is influenced by two factors. As the age increases, the probability of transition to state 2 increases, causing the insurance price to rise.

On the other hand, life expectancy decreases with increasing age, which has the opposite effect on the insurance price. According to values $\mathrm{E}\left[{}_{Z_x|}a_x^L\right]$ in Table 10 for lower ages, the first factor prevails, for higher ages the decisive factor is the decrease of the life expectancy. In the last two columns of Table 10, we present the expected value of 1 m. u. lifetime benefits in the case of LTC benefits paid from the age according to the first column and expected lifetime in state 2 for a person with the age according to the first column (${}_{0|}a_x^L$ and EL in 2 respectively). Compared to $\mathrm{E}\left[{}_{Z_x|}a_x^L\right]$, the ${}_{0|}a_x^L$ values are significantly higher, which is related to the uncertainty of a healthy person's transition to state 2.

Option 5, in which applies $p_x^{13} = q_x^{(S)}$ and $p_x^{23} = \lambda p_x^{13}$, is appropriate to estimate the lower limit of the LTC insurance price. The corresponding values are in Table 11. Compared to Option 2, one can observe significantly higher replacement rates and lower expected lifetimes in years in state 2.

Option 3, where $p_x^{13} = q_x^{(L)}$ with additional equation $q_x^{(S)} = (1 - \gamma_x)\, p_x^{13} + \gamma_x p_x^{23}$, respects the fact, that people needing LTC have lower life expectancy comparing to healthy ones (see e.g., Murtaugh et al., 2001). Moreover, the mortality rate for the whole population is the realistic value $q_x^{(S)}$. The results corresponding to Option 3 are presented in Table 12. For example, the replacement rate corresponding to the age of 62 years is about 9 times higher compared to lifetime annuities (Table 1). The life expectancy of healthy people (it can be seen in the last column of Table 10) is significantly higher than that of people in the need of LTC (see the last column of Table 12).

Table 10 Replacement rates (RR) of long-term care benefits (D_T = 3) and expected lifetimes (EL) in years in the state 2 for different ages and mortality settings according to Option 2

Age *x* (in years)	RR	$\mathrm{E}\left({}_{Z_x|}a_x^L\right)$	${}_{0|}a_x^L$	EL in 2
62	0.9740	2.7340	17.5089	24.17
65	0.8696	3.0734	16.4735	22.03
70	0.7466	3.5947	14.5665	18.46
75	0.7567	3.5453	12.4312	14.91
80	0.8484	3.1523	10.1182	11.47

Source: Own construction

One can observe, that results in Tables 10–12 vary significantly, which shows a big impact of the p_x^{13} and p_x^{23} choices. The precise adjustment would require relevant data on the mortality of the dependents. Since we do not yet have these data, we present only three options, two of which represent the lower and upper limits of the insurance price.

Table 11 Replacement rates (RR) of long-term care benefits (D_T = 3) and expected lifetimes (EL) in years in the state 2 for different ages and mortality settings according to Option 5

Age *x* (in years)	RR	$\mathrm{E}\left({}_{Z_x|}a_x^L\right)$	${}_{0|}a_x^L$	EL in 2
62	2.3034	1.1031	8.8124	9.75
65	2.1393	1.1948	7.9958	8.64
70	1.9261	1.3372	6.6674	6.90
75	2.0513	1.2500	5.3240	5.25
80	2.4398	1.0363	4.0677	3.80

Source: Own construction

Table 12 Replacement rates (RR) of long-term care benefits (D_T = 3) and expected lifetimes (EL) in years in the state 2 for different ages and mortality settings according to Option 3

Age *x* (in years)	RR	$E\left({}_{Z_x\|}a_x^L\right)$	${}_{0\|}a_x^L$	EL in 2
62	1.4036	1.8691	10.0721	11.63
65	1.2726	2.0710	8.6030	9.66
70	1.1362	2.3307	7.4501	8.08
75	1.1460	2.3100	6.5696	6.89
80	1.2852	2.0498	5.6600	5.71

Source: Own construction

When deciding to purchase LTC insurance, it is useful to know the probability that it will be used sometime in the future. The probability p_x^U that a retiree at age x years will receive LTC benefits can be calculated as:

$$p_x^U = \sum_{m=1}^{\omega-x-1} \Pr(Z_x = m).$$

The probabilities p_x^U for selected ages can be found in Table 13. One can observe probabilities around 50% of receiving the LTC benefits. It is worth noting, that the probabilities of real using the insurance must be assessed together with the corresponding replacement rate, which is times higher compared to a classic lifetime pension.

Table 13 Probabilities that a retiree at age *x* will receive LTC benefits according to Options 1–5

Age *x* (in years) / Option	Option 1	Option 2	Option 3	Option 4	Option 5
62	0.4183	0.4829	0.5621	0.5816	0.4589
65	0.4345	0.5003	0.5806	0.5912	0.4666
70	0.4621	0.5278	0.6049	0.6181	0.4850
75	0.4653	0.5305	0.6049	0.6248	0.4774
80	0.4491	0.5125	0.5782	0.5990	0.4493

Source: Own construction

Buying the LTC insurance means giving up a large amount in favour of potential benefits. As in the case of a lifetime annuity, one may consider deferring the purchase of LTC insurance. Assume that a retiree at age x years has an amount $\tilde{a}_x^L$ to purchase a potential unit LTC benefit. If the individual decides to postpone the purchase for one year, the amount $\tilde{a}_x^L$ is invested with return r and three possibilities can arise:

- With probability p_x^{11} the retiree remains in a healthy state. After one year the retiree can buy the LTC insurance for the new price $\tilde{a}_{x+1}^L$.
- With probability p_x^{12} the retiree switches to the dependent state. After one year it is impossible to buy the LTC insurance and the retiree will lose the benefits with value ${}_{0|}a_{x+1}^L$.
- With probability p_x^{13} the retiree dies. In such a case, the amount of $\tilde{a}_x^L(1 + r)$ remains as an inheritance.

The expected profit from the one-year deferral of the purchase of the LTC insurance can then be calculated as:

$$\text{Edprof}_x = p_x^{11}\left(\tilde{a}_x^L(1 + r) - a_{x+1}^L\right) + p_x^{12}\left(\tilde{a}_x^L(1 + r) - {}_{0|}a_{x+1}^L\right) + p_x^{13}\tilde{a}_x^L(1 + r).$$

For Edprof$_x$ to be positive, the yield must be greater than the threshold value:

$$r_x^{L*} = \frac{p_x^{11}\tilde{a}_{x+1}^L + p_x^{12}\,{}_{0|}a_{x+1}^L}{\tilde{a}_x^L} - 1.$$

The minimum values of r_x^{L*} for selected ages are presented in Table 14. The threshold yields r_x^{L*} are positive for ages above 80 years. On the other hand, according to Tables 10–12, the values of ${}_{0|}a_{x+1}^L$ are significantly higher compared to $\tilde{a}_x^L$ and the pensioner faces the risk of losing the LTC benefits when he/she decides to defer the buying the LTC insurance. One can observe an increase in threshold yields with respect to age (except for minor anomalies at ages 62 and 65 years). This is a consequence of increasing (with respect to age) p_x^{12} probabilities and the resulting risk of falling into the state of dependence.

Table 14 The minimum values of yearly returns in case of one-year deferral of the LTC insurance purchase for selected ages according to Options 1–5

Age (in years) / Option	Option 1	Option 2	Option 3	Option 4	Option 5
62	–0.0522	–0.0508	–0.0417	–0.0408	–0.0577
65	–0.0556	–0.0530	–0.0378	–0.0367	–0.0642
70	–0.0256	–0.0256	–0.0184	–0.0179	–0.0395
75	–0.0098	–0.0109	–0.0049	–0.0036	–0.0203
80	0.0129	0.0097	0.0170	0.0232	0.0134

Source: Own construction

It is probably unrealistic to renounce all savings in favour of purchasing the LTC insurance. The authors in Murtaugh et al. (2001) presented an interesting idea of purchasing a combined product of a lifetime annuity and LTC insurance. They argued that a cohort buying such a combination had a higher mortality rate than a cohort buying a lifetime annuity only. Therefore, setting the mortality rates according to Option 3 is appropriate for calculating the price of the combined product. In Table 15, lifetime annuity replacement rates calculated with mortality $q_x^{(S)}$ are displayed. Compared to Table 1, where replacement rates are calculated with mortality rates $q_x^{(H)}$, the values from Table 15 are higher. The saved amount D_T (in our calculations we use $D_T = 3$ and age $x = 62$ years) can be divided between the purchase of the lifetime annuity and the LTC insurance. For example, a 1:1 split of the saved amount offers a replacement rate for the lifetime annuity of 8.37% and an additional 140.36% / 2 = 70.18% if needed the LTC. Other options can be calculated using the linearity of replacement rates with respect to D_T.

Table 15 Replacement rates of whole-life annuity payments for various levels of savings and current initial ages of the pensioner using mortality rates $q_x^{(S)}$ with 7-year guarantee applied

D_T / age x (in years)	62	65	70	75	80
1.5	0.0837	0.0899	0.1027	0.1185	0.1365
2.0	0.1116	0.1199	0.1370	0.1581	0.1820
2.5	0.1396	0.1499	0.1712	0.1976	0.2275
3.0	0.1675	0.1799	0.2054	0.2370	0.2730
3.5	0.1954	0.2099	0.2397	0.2766	0.3185
4.0	0.2233	0.2399	0.2739	0.3161	0.3640

Source: Own construction

CONCLUSIONS

The introduction of the second pension pillar created a variety of possible types of pension benefits in Slovakia. We discussed several alternatives for using the second pillar savings.

When recalculating savings from the second pillar to the replacement rate, we have used an approach from Špirková et al. (2019). Lifetime annuities may have a problem to compensate the shortening of the first pillar. We found that the validity of hypothesis H1 depends on the level of income and the development of financial markets. For low-income cohorts, compensating for the reduction in the first pillar would require unrealistically high returns on assets. For high-income cohorts, participation in the second pillar is highly likely to be beneficial.

In the analysis of deferred annuities, we modified the approach of Milevsky (1998), considering a more general fee structure and discounting using the Svensson yield curve. The later purchase of an annuity can be an attractive solution for savers who do not need a pension benefit from the second pillar immediately after retirement. However, the advantage of such a strategy requires an adequate yield for invested savings. In accordance with Milevsky (1998), the threshold yield typically increases with the saver's age. This also applies to the new way of paying out savings from the second pillar, which is a combination of a programmed withdrawal and the later purchase of a lifetime annuity. On the other hand, however, unpaid money is still the property of the pensioner. Thus, the validity of hypothesis H2 depends on age and the level of market returns. In general, we can recommend deferring the purchase of an annuity only at the early stages of retirement.

An interesting possibility of using savings is the investment return withdrawal. This option can also be combined with a later purchase of the annuity. Its advantage is that unpaid savings still belong to the property of the saver. The drawback is that in some years zero benefits might be paid.

An early purchase of a temporary pension seems disadvantageous. There is little difference between a temporary pension benefit and gradually spending the saved amount. More promising benefits can only be obtained for higher ages or lengths of temporary pensions. On the other hand, this carries a high risk of death before the contract expiration. Thus, our calculations presented in Section 2.1 prove the validity of hypothesis H3.

The main contribution of the paper is the valuation of long-term care insurance. We used a three-state model, where it was necessary to differentiate the mortality of healthy people and those in need of long-term care. Since we did not have exact data, we formulated five options of relationships between the mentioned mortality rates. Options with the lowest and the highest mortality were used for the upper and the lower estimate of the insurance price, respectively. We consider Option 3 from Section 3 as a realistic setting. In this setup, the population in general has high mortality probabilities $q_x^{(S)}$, while the healthy have low mortality rates $q_x^{(L)}$. The resulting pension benefits can guarantee a reasonable level of long-term care. Also in the case of the LTC insurance, an increase in the threshold returns for investment can be observed for the advantageous deferral of the purchase. Deferred purchase of the LTC insurance is suitable for lower ages. Later, the risk of falling into a state of dependence and losing potential benefits increases.

The LTC insurance product implies renouncing of savings in favour of purchasing insurance. A tempting alternative is to purchase a combined product consisting of the life annuity and the LTC insurance. For a cohort that opts for this choice, the cost of the lifetime annuity may be lower (Murtaugh et al., 2001). This is consistent with the Option 3 mortality setting. Along with (Murtaugh et al., 2001), we believe that the combined product is realistic and that hypothesis H4 about the meaningfulness of the LTC insurance is valid.

In the next research, it would be useful to obtain more accurate data on the number of people in the need of long-term care in Slovakia. Based on them, the proportions of dependents γ_x could be refined. An interesting separate research task is to investigate the relationship between the mortality rates of healthy p_x^{13} and dependents p_x^{23}. For this, it is also necessary to obtain relevant data.

ACKNOWLEDGEMENTS

This work was supported by the Slovak Research and Development Agency under the project APVV-20-0311 and Scientific Grant Agency of the Ministry of Education, Science, Research and Sport of the Slovak Republic under the project VEGA 1/0760/22.

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