Comparative Analysis of Results of Assessing the Central Federal District's Regions' Economic Development by Using Linear and Non-Linear Models

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Abstract

This paper provides a comparative analysis of results of estimating specific efficacy that characterizes the amount of gross regional product depending on expenditure on technological innovations, the fixed capital and the average annual number of persons employed. This approach includes comparison of actual data with normative values calculated for three types of models: linear, logarithmic, and power multiplicative. The particular performance indicator is determined as a relation between actual and normative values for the Russian Federation's Central District's regions. The investigation's information base was statistical data on the regions of the RF CFD for 2007–2016. The issues under discussion are differences in results of evaluation obtained in using the first, second and third type models.

Keywords

Indicators, gross domestic product by region, modeling

JEL code

C10, P25, R11, R15

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INTRODUCTION

The economy of the region is characterized by a number of indicators, one of which is the regions' gross domestic product (GDP).

However, the comparative analysis of the regions' GDP in terms of absolute values is not quite correct since each subject, its condition and functioning are determined by a number of specific factors and distinctive features. Consequently, it is necessary to use such indicators that would be immune to measuring and standardization while taking into account the specific properties of the objects (regions) under study. Such indicators can be formed using an econometric approach that enables construction of the models on whose basis it is possible to establish a link between the results of functioning of a complex system (an individual region in this case) and the factors of its condition and impact.

The latter of those determine the impact of the control agencies used for obtaining the desired result. The choice of model specifications for describing the object of research is generally subjective. However, this subjectivity rests on a number of criteria, including the previous investigation results for similar objects, the essential nature of factors, included in the model, its adequacy and accuracy, the applicability, etc. On the one hand, the variety of models and tools for analyzing complex systems gives the researcher an array of options, while on the other it creates a problem of substantiating the choice of relevant approaches to evaluating its condition and operability. This is what determines the relevance of the investigation being carried out. The object of research is the Central Federal District (CFD) of the Russian Federation (RF). The subject of research is the influence of condition and impact factors on the regions' GDP.

The aim of the investigation is to conduct a comparative analysis of results of evaluating the regions' economy with the aid of the method being developed and using linear and non-linear models.

1 SURVEY AND LITERATURE REVIEW

The investigations related to evaluation of the state and functioning of complex systems, including regions, are based on three approaches.

The first of them presupposes the use of generally accepted indicators of evaluating the state and functioning of complex systems. These indicators are shown in various statistical databases.[7] Thus it becomes possible to observe the general dynamics and the current state of the objects of investigation and forecast the behavior of trend models.

The second approach is based on building mathematical models that connect the results of the system's functioning with operability factors. In most cases they represent an extension of classical models of an economic balance and economic growth and that of cyclic models.

The economic balance models are associated with works of such researchers as Keynes (1936) who proposed the "cost – production" model and Leontief (1925) who developed the intersectional balance model. Those models were further developed by the optimum resource distribution models (Kantorovich, 1939), a model of general economic equilibrium (Neumann, 1945, 1946), asymmetric information market models (Akerlof et al., 2001) and others.

The economic growth models are dealt with in the works of Harrod (1939), Solow (1956), Domar (1946) and others.

At the basis of cyclic models are the works of Kondratiev and Oparin (1928), Schumpeter (1935), Freeman (1979), Kleinkneht (1987) and others.

Modern mathematical models for describing an economy or part thereof can be divided into two classes that include models which are a system of equations consisting of derivative functions with

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[7] Federal State Statistics Service of the Russian Federation (ROSSTAT) [online]. <*http://www.gks.ru*>. Eurostat [online]. <*http://www. ec.europa.eu/eurostat/data/database*>.

a different set of restrictions and a system of differential equations describing changes in the economy's main indicators in compliance with its condition.

At their basis are the well known function classes of Cobb et Douglas (1928), Solow (1956), Leontief (1941), production function with constant elasticity of substitution (CES-function), multifactor production function with constant elasticity of substitution of Mihalevsky (CESM-function) and others. For more details see Kleyner (2016). The problem of substantiating their choice for describing the objects and subjects remains open to date.

The World-3 model (Meadows et al., 1972) that describes not only economic but also social trends at the global level can be referred to the second class.

The third approach is associated with the econometric modeling, in which case the functional type of model and the composition of its factors are determined using correlation and regression analysis. In many cases, the analysis of state and functioning of the object under study, including a region's economy and its components, involves linear models (Dreyer and Schmid, 2017; Sayaria et al., 2018) since they are the simplest; quadratic models (Charfeddineand and Mrabet, 2017); logarithm models (Lin and Benjamin, 2018); translog model from the Cobb–Douglas production function (Zhenhua and Guangsheng, 2016); and power multiplicative models, including those with allowance for an innovative component (Makarov et al., 2016). Over 800 works were published in 2018 on building econometric models for different objects of investigation,[8] which in some way or other use the above mentioned models.

The models built serve as a basis for developing forecasts and evaluating the level of development of objects (regions) under study. Used for this purpose are particular and general (integral) evaluation indicators. The procedures of building integral evaluation indicators are quite diverse. Worth noting, among them, are the procedures of calculating mean characteristics by using weight coefficients (Tretyakova and Osipova, 2016); formation of an indicator as the chief component (Aivazian, 2003); size reduction by multiple dimension scaling (Tolstova, 2006); data envelopment analysis (Charnes et al., 1978) and others.

The proposed investigation is based on previous works of the authors involved in building models for evaluating the region's level of development (Zhukov et al., 2016a, 2016b; Zhukov, 2018a, 2018b), which include formation of particular and general evaluation indicators with the use of the econometric approach. This article focuses on analysis of models used for evaluating a region's economy, including its particular efficiency indicators.

2 DATA AND METHODS

The methodology of evaluating a regions' economy, used in this work, presupposes calculating particular efficiency indicators determined as a ratio between actual and normative results of functioning of objects under study.

The particular performance indicator can be calculated using the formula:

$$\xi_{k,i}(t) = \frac{y_{k,i}{}^{0}(t)}{y_{k,i}{}^{)0}(t)}, \tag{1}$$

where $y_{k,i}{}^{0}(t)$, $y_{k,i}{}^{)0}(t)$ – normalized actual and normative values of indicator, k – region index, i – indicator index.

Normalization is carried out using the formulas:

$$y_{k,i}{}^{0}(t) = \frac{y_{k,i}{}^{*} - \min\{y_{k,i}{}^{*}, y_{k,i}{}^{)*}\}}{\max\{y_{k,i}{}^{*}, y_{k,i}{}^{)*}\} - \min\{y_{k,i}{}^{*}, y_{k,i}{}^{)*}\}}, \tag{2}$$

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[8] The data are obtained based on the relevant request to the international scientometric database [online]. [cit. 8.11.2018]. *<http:// www.sciencedirect.com>*.

$$y_{k,i}^{)0}(t) = \frac{y_{k,i}^{)*} - \min\{y_{k,i}^{*}, y_{k,i}^{)*}\}}{\max\{y_{k,i}^{*}, y_{k,i}^{)*}\} - \min\{y_{k,i}^{*}, y_{k,i}^{)*}\}}, \tag{3}$$

here $y_{k,i}^{*}$, $y_{k,i}^{)*}$ – standardized actual and normative values determined using the formulas:

$$y_{k,i}^{*}(t) = \frac{y_{k,i} - M(y_i(t))}{\sigma(y_i(t))}, \tag{4}$$

$$y_{k,i}^{)*}(t) = \frac{y_{k,i}^{)} - M(y_i^{)}(t))}{\sigma(y_i^{)}(t))}, \tag{5}$$

where $M(y_i(t))$, $M(y_i^{)}(t))$, $\sigma(y_i(t))$, $\sigma(y_i^{)}(t))$ – respectively, mean and standard deviation.

The special feature of this approach is that the indicators, thus formed, enables to avoid the problems of comparing the objects of investigation in using different units of measurement and data change ranges. Under such circumstances the indicator allows for a simple interpretation. If its value is larger than a one, the functioning of the system can be considered satisfactory.

The normative value is a regressive model connecting the system's functioning results with the factors of its state and an impact on it.

We shall use a linear, logarithmic and power multiplication form of the model:

$$y_{k,i}^{)*} = \sum_{j=1}^{n} C_{i,j} \cdot x_j^{*} + \sum_{s=1}^{s} D_{i,s} \cdot z_s^{*}, \tag{6}$$

$$\ln(y_i^{)*}) = \sum_{j=1}^{n} C_{i,j} \cdot \ln(x_j^{*}) + \sum_{s=1}^{s} D_{i,s} \cdot \ln(z_s^{*}), \tag{7}$$

$$y_i^{)*} = \prod_{j=1}^{n} x_j^{*C_{i,j}} \cdot \prod_{s=1}^{s} z_s^{*D_{i,j}}, \tag{8}$$

where n is the number of state factors, s is the number of impact factors, $C_{i,j}$, $D_{i,s}$, are corresponding weight coefficients between i productive (result of functioning of system) and j and s standardized factors of x_j^{*} state and z_s^{*} impact. State factors are a set of essential properties the system possesses at a given moment in time. Impact factors are a set of controlled properties leading to changes in the system's functioning results. The subjects of management can change impact factors. Substitution of actual values x_j^{*} and z_s^{*} in (1) for k region can produce an individual norm. The models described by Formulas (7) and (8) are identical.

Chosen as objects of investigation were regions of the Russian Federation's Central District (17 regions less Moscow).

The 2007–2016 reports of the Russian Federation's State Statistics Service (ROSSTAT) became an information basis of the investigation.[9]

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[7] Federal State Statistics Service of the Russian Federation (ROSSTAT) [online]. [cit. 20.11.2018]. <*http://www.gks.ru/wps/wcm/connect/rosstat_main/rosstat/ru/statistics/publications/catalog/doc_113862350615*>.

Chosen as an efficiency indicator was the particular assessment indicator characterizing the regions' GDP. Also involved in the process were the state and impact factors.

The least square method (backward selection) was used to select included variables. The absolute indicator, presented in terms of value, was adjusted according to purchasing power parity (PPP) in US dollars for comparing with the international level (see Table 1).

Table 1 Ruble exchange rate at purchasing power parity (PPP) in US dollars

Year	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Rub/USD	13.98	14.34	14.02	15.82	17.35	18.46	20.48	21.26	23.98	23.70

Source: ROSSTAT, own construction

3 RESULTS AND DISCUSSION

Model specification was presented by the following formulas:

$$y_{10.1/lin}^{)*} = \underset{(0.044)}{0.602 \cdot x_{1,1}^{*}} + \underset{(0.042)}{0.337 \cdot x_{1,2}^{*}} + \underset{(0.014)}{0.076 \cdot z_{22.17}^{*}},\ (R^2 = 0.986,\ v = 166), \qquad (9)$$

$$y_{10.1/log}^{)*} = \underset{(0.049)}{0.565 \cdot \ln(x_{1,1}^{*})} + \underset{(0.054)}{0.347 \cdot \ln(x_{1,2}^{*})} + \underset{(0.030)}{0.097 \cdot \ln(z_{22.17}^{*})},\ (R^2 = 0.932,\ v = 166), \qquad (10)$$

$$y_{10.1/mul}^{)*} = \underset{(0.049)}{x_{1,1}^{*0.565}} \cdot \underset{(0.054)}{x_{1,2}^{*0.347}} \cdot \underset{(0.030)}{z_{22.17}^{*0.097}},\ (R^2 = 0.984,\ v = 166), \qquad (11)$$

where $y_{10\ 1/lin}^{)*}$, $y_{10.1/log}^{)*}$, $y_{10\ 1/mul}^{)*}$ are the model linear, logarithm and multiplicative GDP by region in terms of PPP in US dollars, respectively, $x_{1,1}^{*}$ is the fixed capital, $x_{1,2}^{*}$ is the average annual number of persons employed, $z_{22.17}^{*}$ is the expenditure on technological innovation, () are standard errors, R^2 is determination coefficient, v are degrees of freedom.

For these models, the determination coefficient is statistically significant at 1% level. F-test was used for assessment. For assessing the model coefficients t-test was used. All coefficients are statistically significant at 5% level. The highest coefficient of determination is characteristic for the entire linear model.

Table 2 Results of tests to verify the models' adequacy

Model/test	t	TP	DW	R/S	SRC
Linear	1.58E-04	122	1.929	6.531	1.060, 1.432, 1.245
Logarithm	1.66E-03	120	1.944	5.323	0.553, 0.019, 0.683
Multiplicative	2.89E-01	128	1.752	7.326	0.553, 0.019, 0.683
Table values	1.974	101	$d_L = 1.61$ $d_U = 1.73$	$R/S_L = 5.112$ $R/S_U = 8.560$	1.974
Result	adequate	adequate	adequate	adequate	adequate

Note: Valid at 5% level.
Source: Author's calculations

Five (5) tests with residuals were carried out for checking the model quality:

- the correspondence of the equality 0 of the mean (Student statistic (t-test));
- random character (the test on turning points (TP-test));
- presence (absence) of autocorrelation (the Durbin–Watson statistic (DW-test));
- correspondence of normal probability distribution (R/S –criterion (R/S-test));
- checking for heteroscedasticity (Spearman rank correlation (SRC-test)).

Results of the tests are shown in Table 2. Table 1 shows that all models are adequate. For interval evaluation of model values the following formula was used:

$$y_{10.1}^{)} = y_{10.1}^{)} \pm t_{1-\alpha,v=n-p-1} \cdot s_y \cdot (1 + [XZ]_0^{\,T} \cdot ([XZ]^T \cdot [XZ])^{-1} \cdot [XZ]_0)^{1/2}, \tag{12}$$

where s_y is standard error, $y_i^{)}$ is calculated value by Formulas (9) to (11), $t_{\alpha,n-p-1}$ is the coefficient characterizing the confidence level (determined by the Student distribution table), α is value (at 5% level), n is the number of observations, p is the number of model parameters; $[XZ]$ is matrix of state and impact factors, $[XZ]_0$ is vector of specified values.

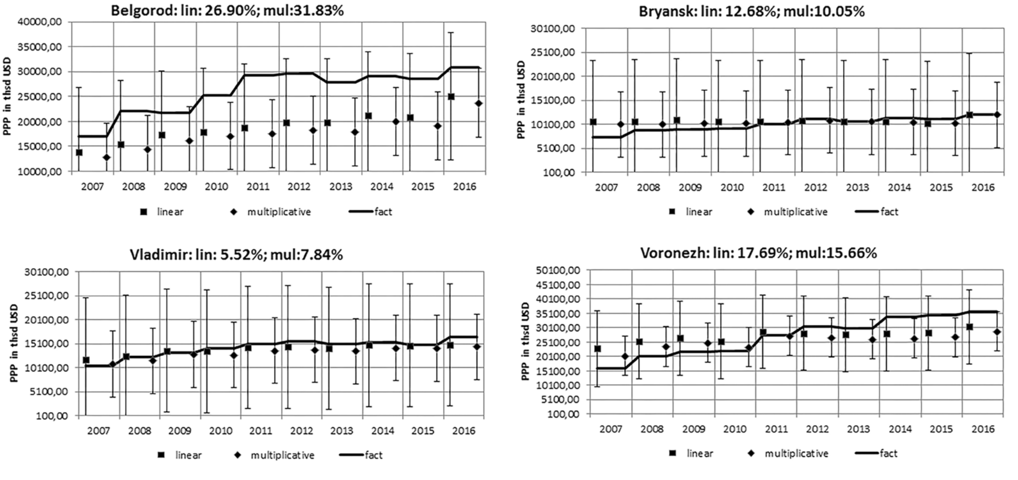
Dynamics of relevant indicators is shown in Figures 1 through 4.

Given that in transition to absolute values the models (7) and (8) are identical to ($y_{10\,1/mul}^{\;)*}$ = exp ($y_{10.1/log}^{\;)*}$)), only linear and multiplicative models are supplied in these Figures. Digits in the title (lin and mul) characterize the average relative error:

$$E_{rel} = \sum_{i=1}^{n} \left| \frac{y_i - y_i^{)}}{y_i} \right| \cdot 100\%. \tag{13}$$

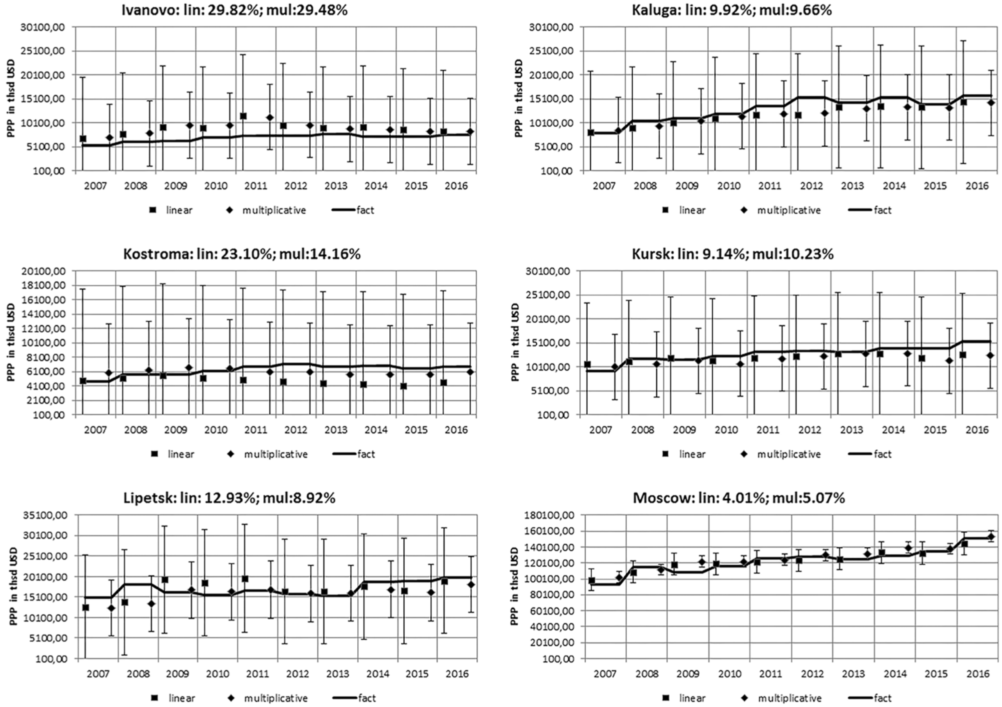
Figures 1 through 4 show that for some regions, for instance, Belgorod, Bryansk, Vladimir, Voronezh, Kursk, Lipetsk, Tver, Tula and Yaroslavl, the values calculated using the linear model are higher than

Figure 1 Dynamics of actual and calculated values of GDP by regions (Belgorod to Voronezh regions)



Note: Linear, multiplicative are calculated values by linear and multiplicative models, respectively, fact is actual (empirical) data, lin and mul values are average relative error.
Source: Author's calculations

Figure 2 Dynamics of actual and calculated values of the GDP by regions (Ivanovo to Moscow regions)



Note: Linear, multiplicative are calculated values by linear and multiplicative models, respectively, fact is actual (empirical) data, lin and mul values are average relative error.
Source: Author's calculations

those calculated using the multiplicative model. For Kostroma, Moscow and Orel regions the values calculated using the linear model are lower than values, calculated using the multiplicative model. For Ivanovo, Kaluga, Ryazan, Smolensk and Tambov regions this difference varies between positive and negative.

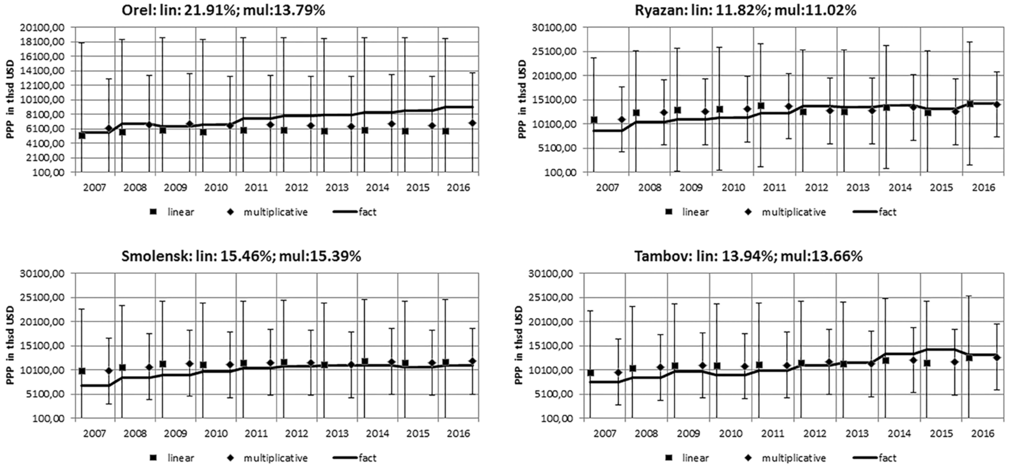
It means that at this stage the issue of choosing a model for forming the normative remains open.

For regions of the Central Federal District the particular assessment indicator is calculated using Formula (1). The results are shown in Figures 5 through 6.

Figures 5 and 6 show that the quantitative assessment of different models produces different results. However, the linear model gives an upper estimate of the particular assessment indicators for Orel region, the logarithm model gives upper estimate for Smolensk and Tver regions, while the multiplicative model gives an upper estimate for Belgorod region only. The estimates vary for remaining regions. The dynamics of these values is similar for linear, logarithm, and multiplicative models. To determine the causes that impact these results, it is necessary to additionally study the curvatures obtained in building particular assessment indicators.

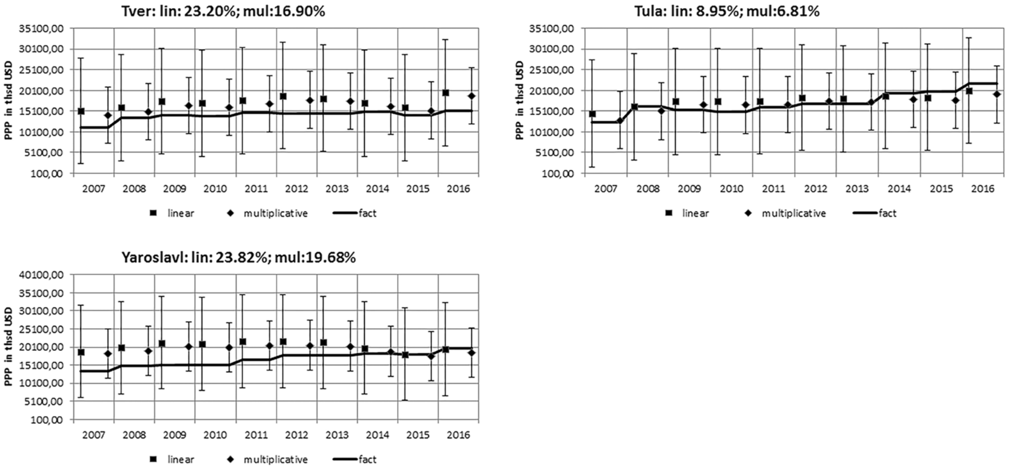
For this purpose, the isoquants are built while fixing one of the variables, for example, the expense on technological innovation ($z_{22.17}^{*}$), presenting them in three versions: absolute units of measurement, standardized and a normalized version.

Figure 3 Dynamics of actual and calculated values of GDP by regions (Orel to Tambov regions)



Note: Linear, multiplicative are calculated values by linear and multiplicative models, respectively, fact is actual (empirical) data, lin and mul values are average relative error.
Source: Author's calculations

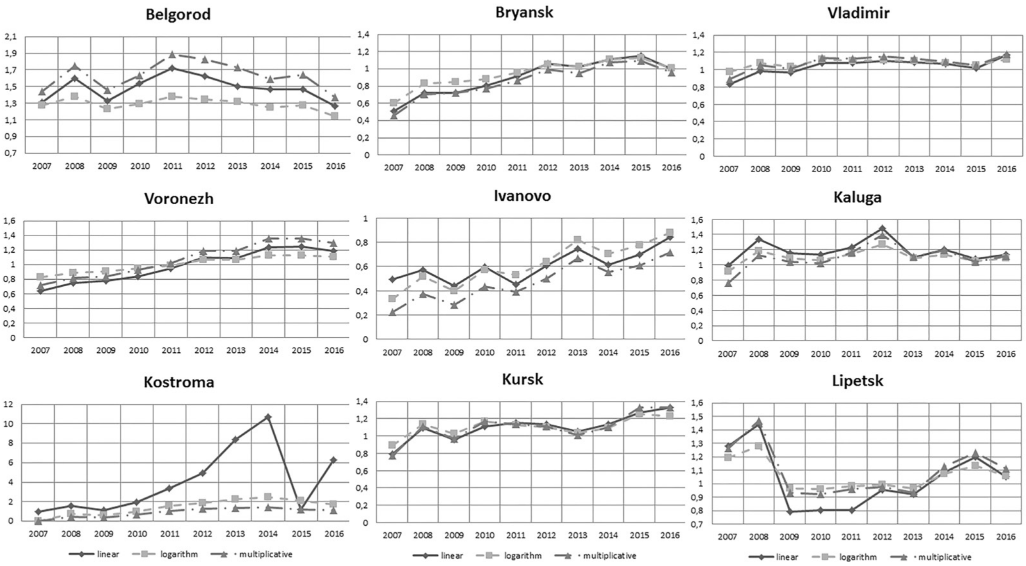
Figure 4 Dynamics of actual and calculated values of GDP by regions (Tver to Tula and Yaroslavl regions)



Note: Linear, multiplicative are calculated values by linear and multiplicative models, respectively, fact is actual (empirical) data, lin and mul values are average relative error.
Source: Author's calculations

The results obtained in absolute units of measurement for various GDP values by regions in US$ ($20, $50 and $80 million) are shown in Figure 7. The fixed expense on technological innovation is $11.9 million, which corresponds to a standardized value equal to a one.

Figure 5 Dynamics of particular performance indicator for regions (Belgorod to Lipetsk regions)



Belgorod

Bryansk

Vladimir

Voronezh

Ivanovo

Kaluga

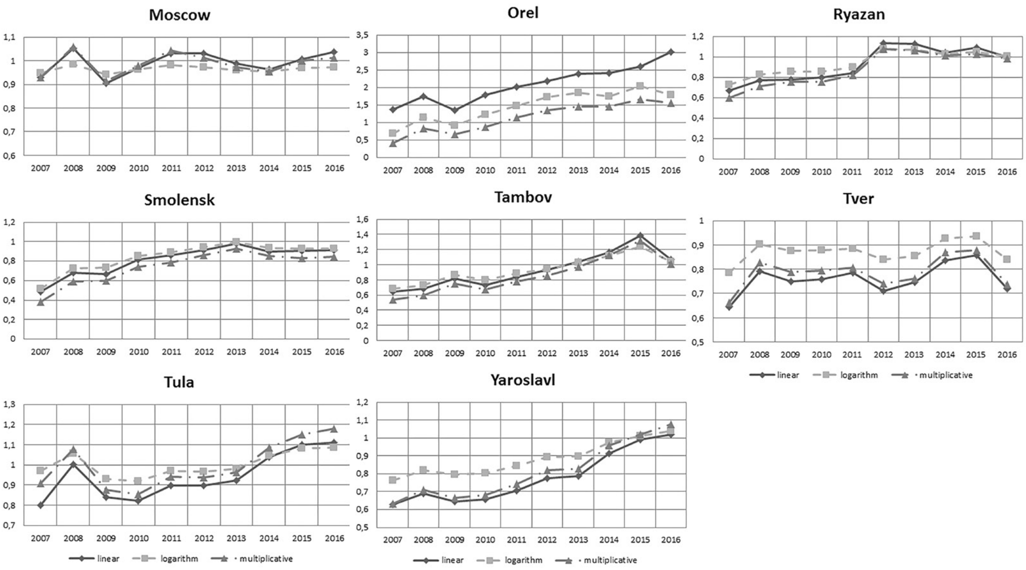
Kostroma

Kursk

Lipetsk

Source: Author's calculations

Figure 6 Dynamics of particular performance indicator for regions (Moscow to Yaroslavl regions)



Moscow

Orel

Ryazan

Smolensk

Tambov

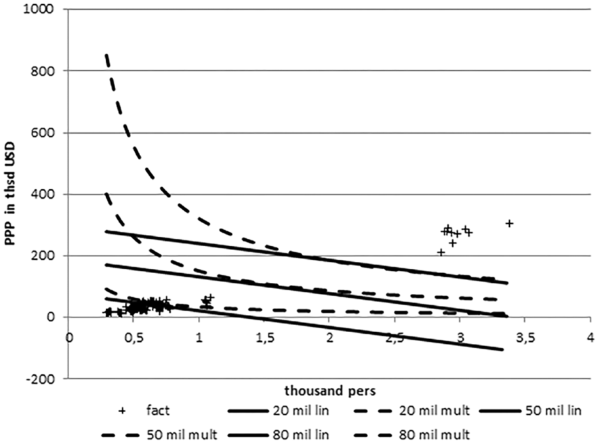
Tver

Tula

Yaroslavl

Source: Author's calculations

Figure 7 Regions' GDP isoquants in terms of PPP in US dollars



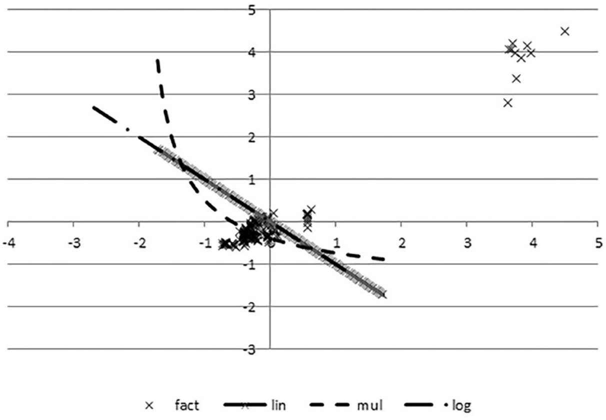
Note: Mil lin, mil mul are isoquants which are calculated by linear and multiplicative models for $20, $50 and $80 million regions' GDP respectively; fact is actual (empirical) data, x-axis is an average annual number of persons employed (thousands persons), y-axis is the fixed capital (PPP in thousand US dollars).
Source: Author's calculations

Figure 7 shows only linear and multiplicative models since in converting the logarithmic model to absolute values, the logarithmic and multiplicative model forms coincide.

In increasing the expenditure on technological innovation to the maximum actual value, equal to $5.6 million, the level lines for the linear and multiplicative models did not overlap.

If the data are presented in a standardized format, by using Formulas (4) and (5), then the change of GDP absolute values by regions and the expense on technological innovations of the values inclination and shift for linear and multiplicative models will not be observed (Figure 8).

Figure 8 Standardized regions' GDP isoquants



Note: Lin, mul, log are calculated values by linear and multiplicative models, respectively; fact is actual (empirical) data, x-axis is the standardized average annual number of persons employed, y-axis is the standardized fixed capital.
Source: Author's calculations

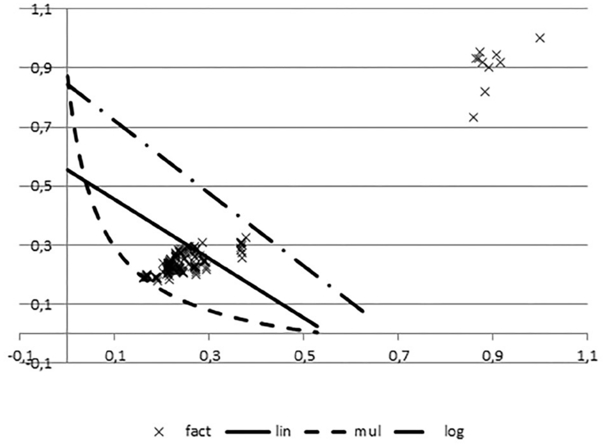
Figure 8 shows that values calculated using the logarithmic model lie on the linear model but are shifted to the left in relation to the linear model. This is explained by the fact that the initial data were processed for logarithms. Also, it can be seen that at one and the same set value of the regions' GDP and the expenditure on technological innovations, the norms calculated using linear and logarithmic models allow for larger values of $x_{1,1}^*$ and $x_{1,2}^*$ in an area formed by overlapping of relevant isoquants. In this case the area contains most of the empirical data for the CFD regions.

Following normalization according to Formulas (2) and (3) the logarithmic level line came out above others. This is easily seen in Figure 9. The change in the regions' GDP and expenditure on technological innovations does not change the level lines formed by the set values of $x_{1,2}^0$ and $x_{1,2}^0$.

To convert the variables $x_{1,1}^*$ and $x_{1,2}^*$ to a normalized format ($x_{1,1}^0$ and $x_{1,2}^0$), formulas similar to Formulas (2) and (3) were used.

Figure 9 shows that the logarithmic model allows for higher values of $x_{1,1}^0$ and $x_{1,2}^0$, while the multiplicative one does for lower values of $x_{1,1}^0$ and $x_{1,2}^0$. It means that in forming the norms, at all other conditions being equal, the logarithmic model will yield the lowest norms for assessing the regions' GDP, the linear model will produce mean values, while the multiplicative model will yield the highest or the most rigorous (strictest) norms.

Figure 9 Normalized regions' GDP isoquants



Note: Lin, mul, log are calculated values by linear and multiplicative models, respectively; fact is actual (empirical) data, x-axis is the normalized average annual number of persons employed, y-axis is the normalized fixed capital at full accounting value at the end of the year.
Source: Author's calculations

It is known that the norms are established by control agencies. Therefore, at the current stage the choice of a model can be determined depending on the importance of developing the regions' economy compared to social development and securing their ecological safety, that are the agencies' priority functions. Among other things, these priorities must be tied up with the concept of sustainable development.

Another aspect that can be a basis for choosing the described models is the qualitative nature of influence exerted by capital, labor, and innovations. It is obvious that the production included in the regions' GDP, needs both labor and production assets, while the use of innovations heightens the production efficiency and determines competitiveness in the modern rapidly developing world. It is expressly the existence of these three components that are the essence of the generalized form of the Cobb-Douglas function presented by Formula (8) for 3 variables. The logarithmic model, as Figures 7 to 9 show, allows for formation of lower norms for the regions, which may decrease the competitiveness level. The linear model, while giving a mean evaluation of the norm, allows the production process without the use of either labor or capital or innovations, which at the current level of development is not possible on the regions' scale.

Consequently, the choice of the model can be determined using three mutually supportive criteria: 1) model quality and adequacy; 2) controls' priorities (the higher the normative, the more important is the indicator under study and the higher the competitiveness); and 3) qualitative (semantic) content of the model.

Table 3 assesses the models chosen for investigation.

Table 3 Results of the estimation of models by criteria

Model/Criteria	Quality and adequacy	Priority	Substantive content
Linear	+++	++	–
Logarithm	+	+	–
Multiplicative	++	+++	+

Source: Own construction

Table 3 shows that all models are good quality and adequate. In this case, the most qualitative is the linear model and in the simplest case it is possible to do with a linear model in the formation of the normative for of GDP by region. However, in order to satisfy all criteria (taking into account the substantive content of the models and the importance of GRP for the regional economy) it is necessary to use the multiplicative model. The multiplicative model significantly all requirements; it is qualitative and adequate and has the highest priority. Therefore, the authors the authors used this model to form particular performance indicator.

The results of assessing the GDP by region; the multiplicative model is presented in Table 4.

Table 4 Particular performance indicator

Region/Year	2012	2013	2014	2015	2016
Belgorod	1.822	1.733	1.593	1.642	1.375
Bryansk	0.993	0.953	1.072	1.091	0.956
Vladimir	1.150	1.129	1.097	1.053	1.182
Voronezh	1.191	1.186	1.357	1.356	1.297
Ivanovo	0.499	0.668	0.553	0.609	0.716
Kaluga	1.399	1.104	1.192	1.040	1.119
Kostroma	1.259	1.326	1.445	1.195	1.132
Kursk	1.109	1.013	1.105	1.329	1.330
Lipetsk	0.974	0.930	1.126	1.232	1.110

Table 4 (continuation)

Region/Year	2012	2013	2014	2015	2016
Moscow	1.012	0.972	0.953	1.001	1.011
Orel	1.360	1.454	1.452	1.670	1.564
Ryazan	1.074	1.065	1.010	1.028	0.981
Smolensk	0.86	0.924	0.852	0.833	0.846
Tambov	0.856	0.974	1.127	1.317	1.010
Tver	0.739	0.761	0.869	0.88	0.737
Tula	0.938	0.964	1.086	1.152	1.179
Yaroslavl	0.818	0.828	0.957	1.020	1.078

Source: Author's calculations

In 2016, most of the particular performance indicators exceeding a one, excepting Bryansk, Ivanovo, Ryazan, Smolensk, and Tver regions. For these regions, the GDP does not reach the norm, and this is a negative result. For the rest of the regions, their functioning can be considered satisfactory. However, compared to 2015, in most of the regions the indicator values dropped. In order to analyze the phenomena that affected the result, it is essential to study the extent and intensity of using the labor and capital as well as innovations, which calls for a more detailed study at the level of a concrete region.

CONCLUSION

This article deals with a comparative analysis of results of appraising Russia's Central Regions' GDP depending on their chosen model based on the author's approach, by means of which it is possible to formulate normal standards for each, taking into consideration its specifics. This makes it different from other approaches used for formulating complex systems' functioning results evaluation indicators.

In order to create normative standards the following models were chosen: linear, logarithmic, and multiplicative, which included factors characterizing labor, capital and innovations.

For substantiating the choice of the model, checks were conducted for the models' quality and adequacy. All the models proved to be adequate and of proper quality.

Following calculation of the particular performance indicator it turned out that the use of different models brings about different results. This called for the study on the isoquant's shape determined by the chosen models. The analysis showed that the highest value of evaluation indicators results from the logarithmic model, middle from the linear model and the lowest (more rigorous requirement for the regions' GDP) from the multiplicative model. At this stage the choice of the model can be determined taking into account the priorities of development of a particular region.

The next stage was evaluation of the models' semantic purport.

Thus, the developed three-level algorithm of substantiating the choice of a model for establishing the norms made it possible to adopt the multiplicative model as a basis.

The investigation results can be used for determining the causes of reduction in the regions' expected GDP values as well as for taking well-grounded managerial decisions.

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