Equivalence of Fault Trees and Stochastic Petri Nets in Reliability Modeling

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Abstract

Modeling of reliability of the complex systems (machines, large networks, human body) is an important area of recent research. There are two main approaches applied: i) fault trees, ii) Petri nets. For the probabilistic study of a system is vital to know its minimal cut/minimal path sets. Both for fault trees and Petri Nets it is an NP-hard problem. Liu and Chiou (1997) described the equivalence of both representations for a given system. Furthermore, they found a top-down matrix algorithm to find critical cuts and minimal paths of the Petri net of the system. They claim without proof that their algorithm is more efficient than the ones for fault trees. We present both representations of a system. The algorithm is illustrated on a simple example of a three-masted vessel and a more complex "three-motor" system by Vesely et al. (1981).

Keywords	JEL code
Reliability, time to failure, fault tree, stochastic Petri net, exponential distribution	*C10, C18, C60*

INTRODUCTION

The demand for a more precise estimation of the reliability of complex systems has steadily increased because of both legal regulation and the growing complexity of real industrial systems. Such a complex system is for example a power plant, airplane, machine, reactor, large computer or transportation network, a human body. Applications of the methods range from energetics, engineering, transportation, computer science to safety studies, and medicine.

For reliability analysis of complex systems the following standard methodologies are used: i) fault trees (for the state of the art of this approach see Limnios, 2007), ii) Petri Nets (see monograph of Bause and Kritzinger, 2002).

Both methodologies have been further developed mostly assuming exponentially distributed time to failure. This assumption is unrealistic because it implies that parts of the system do not age.

In both approaches a system is represented as a tree in the language of the graph theory. If the fault tree method is applied, it is vital to know minimal cuts and minimal paths sets of a fault tree. However, finding all minimal cuts and minimal path sets of a fault tree is an NP-hard problem (Rosenthal,

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1975). On the other hand, a fault tree representation is suitable to derive the probability distribution of the reliability of a system and its estimators (Limnios, 2007).

Liu and Chiou (1997) established a one-to-one relation between a fault tree and a Petri Net of a system. They also proposed a top-down matrix algorithm to find all minimal cuts and minimal paths of the system. They claim without any proof this algorithm is more computationally efficient than algorithms for fault trees. To our best knowledge no complexity analysis or computational studies have been carried out. From the theoretical point of view Petri Nets is not a framework suitable to derive the probability distribution of reliability of a system and its estimators (Bause and Kitzinger, 2002).

If claim of Liu and Chiou (1997) is true, the following strategy for derivation of distribution of reliability functions and its estimators in general setting (gamma, log-logistic distribution of time to failure) would be efficient. First, transformation a fault tree of a system to a Petri Net is carried out to find minimal cut sets and minimal path sets. Then, all derivations are done in the fault tree setting.

The paper is organized as follows. The first part introduces elements of reliability theory. Secondly, concepts of fault trees and Petri Net are introduced. Thirdly, top-down matrix algorithm of Liu and Chiou (1997) to find all minimal cut sets and minimal paths sets is presented. Then, all the methods are illustrated in the model example of a three-masted vessel (Kubelka, 2016). Finally, both methods are applied to a more complex case of the "three-motor" system (Vesely et al., 1981) to assess Petri Nets based method to find minimal cut/minimal path sets.

1 ELEMENTS OF RELIABILITY THEORY

Systems are classified by their complexity to *Single-Component Systems* and *Multi-Component Systems*. The second classification distinguishes *Repairable* and *Non-Repairable Systems*. We focus on Non-Repairable Systems in this article only.

1.1 Single Component Systems

Let X be a continuous random variable representing time to failure of the system with cumulative distribution function $F(t) = P(X \le t)$ and its density function $f(t)$.

Survival function (reliability) *is the complementary function to cumulative distribution function*:

$$R(t) = 1 - F(t) = P(X > t) = \int_t^{\infty} f(x)\, dx. \tag{1}$$

Note, that we have $R(0) = 1$ and $R(\infty) = 0$.

Hazard rate (instantaneous failure rate) at time t is defined as:

$$\lim_{\Delta t \to 0+} = \frac{P(t < X \le t + \Delta t \,||\, X > t)}{\Delta t} = \frac{f(t)}{1 - F(t)} = \frac{f(t)}{R(t)}. \tag{2}$$

Mean time to failure (MTTF) is defined as the mean of the time to failure, e.g. $E(X)$.

The exponential distribution is the most used for modeling time failure in reliability theory. It gives a system without memory (a system is not aging) or a Markovian system, e.g. for fixed $x > 0$, $t > 0$ we have:

$$P(X > x + t \,||\, X > x) = P(X > t). \tag{3}$$

For fixed time $t > 0$ and parameter $\lambda > 0$ we get:

$$f(t) = \lambda e^{-\lambda t},\ F(t) = 1 - e^{-\lambda t},\ R(t) = e^{-\lambda t},\ h(t) = \lambda,\ MTTF = E(X) = 1/\lambda. \tag{4}$$

The assumption on exponentially distributed time to failure will be used in the section Application.

1.2 Multiple Components Systems

Let us consider a binary system with n components: $C = \{1, 2, \ldots, n\}$. For each component i we define a binary variable x_i (0: the component is in good state, 1: the component is down).

Let $\boldsymbol{x} = (x_1, x_2, \ldots, x_n) \in \{0,1\}^n$ be the vector jointly describing the states of the components. We define a structural function $\boldsymbol{\varphi}(\boldsymbol{x})$ with values {0,1} as:

$\boldsymbol{\varphi}(\boldsymbol{x}) = 1$, if the system is in a good state, $\boldsymbol{\varphi}(\boldsymbol{x}) = 0$, if the system is down.

A system is said to be series if its good functioning depends on the functioning of all its components. If at least one component fails, then the system also fails. In fault tree setting is modeled by gate OR. The structural function of the *series system* is given by:

$$\boldsymbol{\varphi}(\boldsymbol{x}) = \prod_{i=1}^{n} x_i\,. \tag{5}$$

A system is said to be parallel, if its good functioning is assured by functioning of least one of its components. Only if all components fail, then the system also fails. In fault tree setting is modeled by gate AND. The structural function of the *parallel system* is given by:

$$\boldsymbol{\varphi}(\boldsymbol{x}) = 1 - \prod_{i=1}^{n}(1 - x_i). \tag{6}$$

1.3 Coherent Systems

In reliability theory most of the techniques are limited only to *coherent systems*. A coherent system has these properties:

- it consists only of parallel and series systems (e.g. gates AND and gates OR),
- it has no redundant component (i.e. its states do not affect the state of the system),
- it does not contain a component and its negation simultaneously,
- it contains neither loops nor circuits in its graph representation.

For state assessment of a system following concepts are defined (Limnios, 2007):

- *path:* a subset of components whose simultaneous good functioning assures good functioning of the system regardless of the functioning of the other components,
- *minimal path:* a path which does not contain another path,
- *cut set:* a subset of components whose simultaneous failure leads to the system failure regardless of the failure of the other components,
- *minimal cut set:* a cut set that does not contain another cut set.

Set of the minimal paths is denoted as:

$$C = \{C_1, C_2, \ldots, C_c\}.$$

Set of minimal cuts is denoted as:

$$K = \{K_1, K_2, \ldots, K_k\}.$$

By minimal paths set or minimal cuts set its structural function is simplified as:

$$\boldsymbol{\varphi}(\boldsymbol{x}) = 1 - \prod_{j=1}^{c} (1 - \prod_{i \in C_j} x_i), \tag{7}$$

$$\boldsymbol{\varphi}(\boldsymbol{x}) = \prod_{j=1}^{k} (1 - \prod_{i \in K_j} (1 - x_i)). \tag{8}$$

1.4 Probabilistic Study of Coherent Systems

Let's consider a coherent system $S = (C, \boldsymbol{\varphi})$ of order $n \geq 1$ (number of its components). Let X_i be a Bernoulli random variable with parameter p_i, which describes the state of the component i ($i = 1, 2,\dots, n$) with values $x_i \in \{0,1\}$. Then *reliability of the system* $R(\mathbf{p})$, where $\mathbf{p} = (p_1, p_2, \dots, p_n)$ is the probability, that the system is in a good state. We can express reliability also by minimal cut sets or minimal path sets as:

$$R(\mathbf{p}) = P(C_1 \cup C_2 \cup \cdots C_c) = 1 - P(K_1 \cup K_2 \cup \cdots K_k). \tag{9}$$

Different bounds for reliability function was established (see Limnios, 2007) using knowledge of minimal paths set and minimal cuts set.

Minimal sets bounds are established as follows. A lower bound is derived through minimal cut sets and an upper bound is derived through minimal paths sets:

$$\prod_{j=1}^{k} [1 - \prod_{i \in K_j} (1 - p_i)] \leq R(\mathbf{p}) \leq 1 - \prod_{j=1}^{c} (1 - \prod_{i \in C_j} p_i). \tag{10}$$

If the minimal cuts are 2-by-2 disjoint sets ($K_i \cap K_j$ is empty, if $i \neq j$), then upper bound equals to $R(\mathbf{p})$. The same rule applies to minimal paths and the lower bound of $R(\mathbf{p})$.

Trivial bounds are based on the observation that the reliability of a coherent system lies between the reliability of the series system and reliability of the parallel system:

$$\prod_{\mathbf{i}=1}^{\mathbf{n}} \mathbf{p_i} \leq \mathbf{R}(\mathbf{p}) \leq 1 - \prod_{\mathbf{i}=1}^{\mathbf{n}} (1 - \mathbf{p_i}). \tag{11}$$

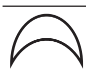
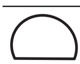
2 FAULT TREES AND PETRI NETS

From now all the methods will be illustrated on the example of a three-masted vessel (Kubelka, 2016). The vessel is maneuverable, if all its three components work (see Figure 1):

- the keel is not broken,
- the helm is maneuverable,
- at least one of the three masts are not broken,
- each mast is not broken, if both sail and spare are not broken.

2.1 Fault Trees

Table 1 Fundamental operators of fault trees

Graphic Symbol	Name	Meaning
	OR	The output is generated if at least one of the inputs exists
	AND	The output is generated if all the inputs exist

Source: Limnios (2007)

The basic notions and symbols of fault trees used in this paper are summarized in Tables 1 and 2.

Note that in this form fault trees representation of a system enables only static analysis of the reliability of the system. It is necessary to extend this setting by adding a time variable. However, this generalization is not the goal of the paper.

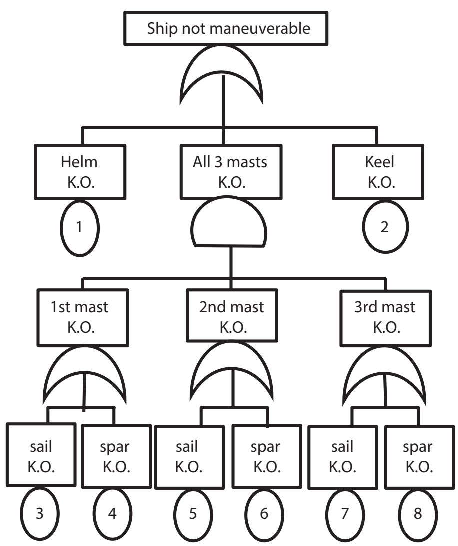
Table 2 Events of fault trees

Graphic Symbol	Meaning
Rectangle	Top or intermediate event (the system is down)
Circle	Basic event
Triangle	Transfer (fault tree is developed further)

Source: Limnios (2007)

Then the fault tree of the three-masted vessel is then constructed as – see Figure 1.

Figure 1 Fault tree of the three-masted vessel



Source: Kubelka (2016)

For such a simple system is easy to find all minimal cuts and paths manually. It has 10 minimal cuts: $K_1 = \{1\}$, $K_2 = \{2\}$, $K_3 =\{3,5,7\}$, $K_4 = \{3,5,8\}$, $K_5 = \{3,6,7\}$, $K_6 = \{3,6,8\}$, $K_7 = \{4,5,7\}$, $K_8 = \{4,5,8\}$, $K_9 = \{4,6,7\}$, $K_{10} = \{4,6,8\}$. For example minimal cut $K1$ means that the vessel is broken because the keel of the vessel is broken.

It has 3 minimal paths: $C_1 = \{1,2,3,4\}$, $C_2 = \{1,2,5,6\}$, $C_3 = \{1,2,7,8\}$. Minimal path C_1 means that the vessel is maneuverable, because of the keel, the helm, and the first mast (i.e. both its spar and sail are not broken) of the vessel work well.

Note that finding all minimal cuts of a coherent system is an NP-hard problem (Rosenthal, 1975). It means that the computational complexity of any algorithm grows exponentially with the number of components of a system. Even for a system with moderate size (a system with more than 20 components) is quite computationally demanding.

2.2 Petri Nets

Petri Nets (Petri, 1962) was designed to study information systems in computer science. They have been further developed and applied in many areas, also in modeling reliability of complex systems (see monograph of Bause and Kritzinger, 2002, among others). The basic notions and graphic symbols of Petri Nets are summarized in Table 3.

Table 3 Symbols and notions of Petri Nets

Graphic Symbol	Notion	Meaning
Circle	Place	Objects, components of a system
Dot	Token	Specific value, state of the object, component
Rectangle	Transition	Activities changing state or value of the object
Arrow	Arc	Connection of places and transitions

Source: Own construction by Bause and Kritzinger (2002)

For reliability modeling, Place-Transition Petri Nets and Stochastic Petri Net are used.

A Place-Transition Petri Net (Bause and Kritzinger, 2002) is defined as 5-tuple $PN = (P, T, I^{+}, I^{-}, M_0)$, where:

- $P = \{p_1, p_2, \dots, p_m\}$ is a finite and non-empty set of places,
- $T = \{t_1, t_2, \dots, t_n\}$ is a finite and non-empty set of transitions,
- $P \cap T = \emptyset$
- $I^{+}, I^{-}: P \times T \rightarrow N_0$ are oriented incidence functions (arcs),
- $M_0: P \longrightarrow N_0$ is a vector of the initial state of the systems.

Note that places represent for example a server or a hardware/software component of a system or a module of a software system. Transitions represent relations of different components of a system (i.e. transactions between servers or software components).

Place-Transition Petri Nets enable an only static analysis of the coherent system. To study the reliability of the system in time domain Place-Transition Petri Nets were generalized to Stochastic Petri Nets.

Stochastic Petri Net (Natkin, 1980; Molloy, 1981) with continuous time $SPN = (PN, \Lambda)$ is defined as Place-Transition Petri Net $PN = (P, T, I^{+}, I^{-}, M_0))$ equipped by a parameter set $\Lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)$.

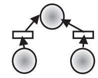
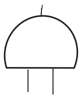
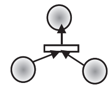
The role of a parameter set Λ is as follows:

- the parameter λ_i serves for modeling activation time of transition t_i,
- the transition t_i can be then activated only if there is a token in the corresponding place p_i,
- transition times T_i of a transition t_i are usually modeled as independent exponential random variables $T_i \sim Exp(\lambda_i)$.

2.3 Equivalence between fault tree and Place-Transition Petri Net

Liu and Chiou (1997) found in their seminal paper one-to-one relationship between Place-Transition Petri Net and corresponding fault tree.

Table 4 Fundamental operators of fault trees and its Petri Nets representation

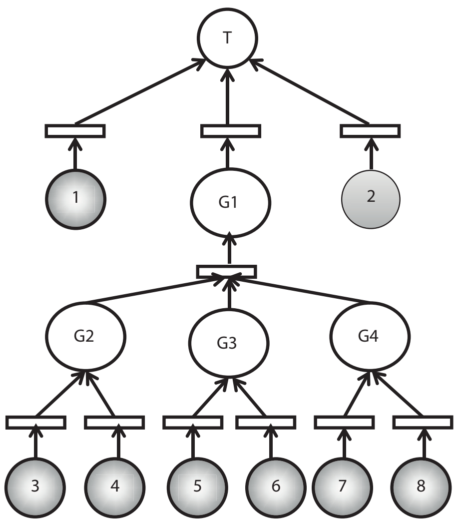
Fundamental Operator		**Structural Function φ**	**Petri Net**
Graphic Symbol	**Name**		
	OR	$\boldsymbol{\varphi}(\boldsymbol{x}) = 1 - (1 - x_1)(1 - x_2)$	
	AND	$\boldsymbol{\varphi}(\boldsymbol{x}) = x_1 x_2$	

Source: Own construction by Liu and Chiou (1997)

Both top and intermediate events and basic events are modeled as places in Petri Nets setting. This is why these events in the fault tree setting resemble a hardware/software component of a system.

Fault tree of the three-masted vessel (see Figure 1) is transformed into Stochastics Petri Net – see Figure 2.

Figure 2 Stochastic Petri Net of the three-masted vessel



Note: T: top event, G: gate (fundamental operators AND, OR), a place with token: .
Source: Own construction

Liu and Chiou (1997) also developed a recursive top-down matrix algorithm to find both minimal cut sets and minimal path sets simultaneously.

This method proceeds as follows:

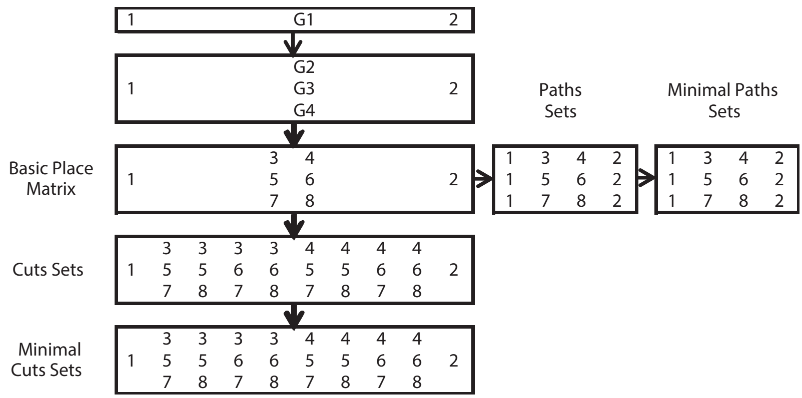
- write down the numbers of places horizontally if the output place is connected by multi-arcs to transitions,
- write down the numbers of places vertically if the output place is connected by an arc to a common transition,

- when all places are replaced by places representing basic events, a matrix is created. If there is a common entry located between rows or columns, it is also the entry present in each row or column. The column vectors of the matrix contain cuts sets, the row vectors then paths sets,
- finally select the minimal cuts sets and minimal path sets.

Liu and Chiou (1997) claim without any proof that this algorithm is much simpler than the corresponding ones for fault trees. By my best knowledge there have not been done any computational studies of algorithms to find minimal cut/path sets. Also, the efficient implementation of the algorithm is still an open problem. If the conjecture of the authors is true, then it would be efficient to transform the fault tree to Petri Net and find the minimal cuts/path sets. Anyway, to find minimal cut sets for a Petri Net is also an NP-hard problem. It follows from a one-to-one relation between a fault tree and a Petri Net and NP-hardness of minimal cut sets problem for a fault tree.

The schematic description of the method for the three-masted vessel is given in Figure 3.

Figure 3 Method for finding minimal cuts/paths sets by Liu and Chiou (1997)



Source: Own construction

The algorithm browses the Petri Net from its top places to down. The places from the current level are written down in one row of a matrix for a series system. On the other hand, the places from a parallel system are written down in one column of a matrix. Basic event (a place in a Petri Net) is represented by a number, intermediate event (logical gate of a Petri Net) is represented by symbols like G1, G2. It symbolizes another level of the Petri Net to be browsed in the next step. The algorithm stops, if we reached the lowest level of a Petri Net, i.e. it browsed all logical gates of a system. The all minimal cut/path sets of the three-masted vessel are found to derive its reliability function.

3 APPLICATIONS

Methods presented above are applied to the example of a three-masted vessel (Kubelka, 2016) and a more complex "three motors system" (Vesely et al., 1981). Time to failure of the components of both systems is assumed to be independent exponentially distributed random variables as well.

3.1 Reliability Function of the Three-Masted Vessel

Let us consider that time to failure all components of the three-masted vessel (Kubelka, 2016) are independent exponentially distributed with parameters given in Table 5.

Table 5 Time to failure distribution of the components

i	Component	MTTF (hours)	Ri(t)
1	Keel	2 000	$R_1(t) = e^{-t/2\,000}$
2	Helm	1 000	$R_2(t) = e^{-t/1\,000}$
3	Sail of the 1st mast	200	$R_3(t) = e^{-t/200}$
4	Spar of the 1st mast	500	$R_4(t) = e^{-t/500}$
5	Sail of the 2nd mast	200	$R_5(t) = e^{-t/200}$
6	Spar of the 2nd mast	500	$R_6(t) = e^{-t/500}$
7	Sail of the 3rd mast	200	$R_7(t) = e^{-t/200}$
8	Spar of the 3rd mast	500	$R_8(t) = e^{-t/500}$

Source: Own construction

The reliability functions of the keel and the helm are known. To derive the reliability function of the vessel the survival function of the three masts must be derived first.

The survival function of one mast is a product of survival functions of its sail and spar. It gives for $t > 0$:

$$R_m(t) = e - \frac{t}{200e} - \frac{t}{500} = e - \frac{7t}{1\,000}. \tag{12}$$

The system of all three masts is a parallel system of the masts, therefore its survival function is for $t > 0$:

$$R_{3m}(t) = 1 - (1 - e - \frac{7t}{1\,000}). \tag{13}$$

The three-masted vessel is a series system consisting of the keel, the helm, and three masts. Therefore, the survival function of the vessel is the product of survival functions of these three components, i.e. for $t > 0$:

$$R_{vessel}(t) = e - \frac{t}{2\,000}\; e - \frac{t}{1\,000} \left(1 - (1 - e - \frac{7t}{1\,000})^3\right) = e - \frac{3t}{2\,000} \left(1 - (1 - e - \frac{7t}{1\,000})^3\right). \tag{14}$$

The reliability of the vessel is strongly affected by the reliability of the system of three masts. Therefore, safety measures should focus mainly on the reliability of the sails and spars (see Table 6).

Table 6 Reliability function of the vessel

Time (hours)	Reliability function of			
	Keel	Helm	3 masts	Total
100	0.9512	0.9048	0.8724	0.7509
200	0.9048	0.8187	0.5724	0.4240

Table 6 (continuation)

Time (hours)	Reliability function of			
	Keel	Helm	3 masts	Total
300	0.8607	0.7408	0.3242	0.2067
400	0.8187	0.6703	0.1716	0.0942
500	0.7788	0.6065	0.0879	0.0415
600	0.7408	0.5488	0.0443	0.0180
700	0.7047	0.4966	0.0222	0.0078
800	0.6703	0.4493	0.0111	0.0033
900	0.6376	0.4066	0.0055	0.0014
1 000	0.6065	0.3679	0.0027	0.0006

Source: Own construction

Trivial bounds of the reliability function provide an extremely poor approximation of the reliability function (see Table 7). Minimal cuts set bound provide quite tight intervals for reliability function. Neither minimal cuts set, nor minimal paths sets are 2-by-2 disjoint, therefore neither lower bound nor upper bound equals the reliability function.

Table 7 Minimal sets bounds and trivial bounds of reliability function of the vessel

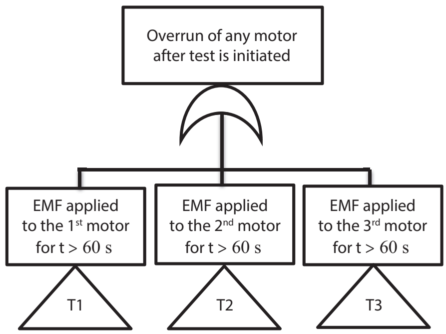
Time (hours)	Lower bound		Reliability function R(t)	Upper bound	
	Trivial	Minimal cuts sets		Minimal path sets	Trivial
100	0.10539922	0.70945428	0.75090010	0.81227589	0.99999832
200	0.01110900	0.28226174	0.42401196	0.45402751	0.99984388
300	0.00117088	0.07071146	0.20673107	0.21643080	0.99844529
400	0.00012341	0.01321632	0.09415493	0.09681565	0.99354886
500	0.00001301	0.00207403	0.04151348	0.04218520	0.98299794
600	0.00000137	0.00029443	0.01801734	0.01817896	0.96576308
700	0.00000014	0.00003957	0.00775945	0.00779717	0.94201188
800	0.00000002	0.00000517	0.00332898	0.00333761	0.91268718
900	0.00000000	0.00000067	0.00142551	0.00142745	0.87906209
1 000	0.00000000	0.00000009	0.00060985	0.00061028	0.84244015

Source: Own construction

3.2 Three-Motor System

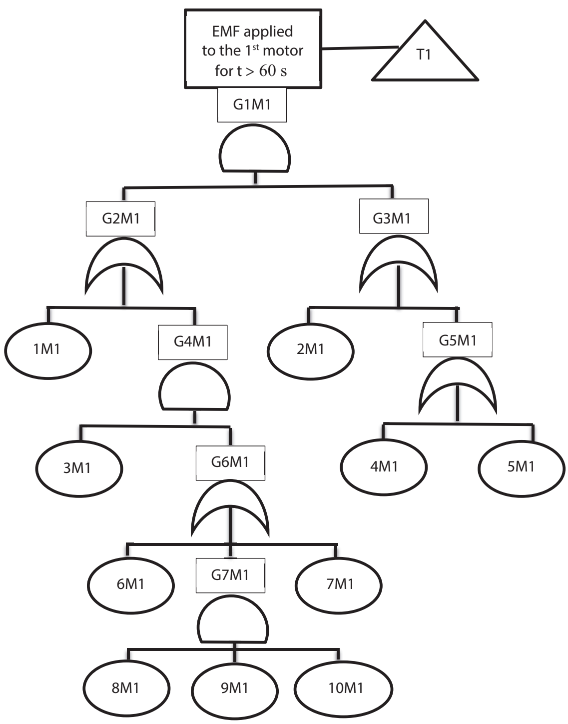
The three-motor system by Vesely et al. (1981) is often used as a benchmark for the assessment of algorithms in the field of reliability. It models a real-life control system of three motors. To shut down the system, it impresses a 60-second signal test. After 60 seconds, it is supposed to shut down all three

Figure 4 Fault tree for "three-motor" system



Source: Own construction, by Vesely et al. (1981)

Figure 5 Fault tree for the 1st motor in "three-motor" system

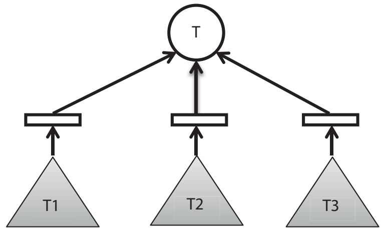


Source: Own construction by Vesely et al. (1981)

motors. The control then reveals failure of the system if the electromagnetic force (EMF) is applied to any of the three motors for more than 60 seconds after the signal test started.

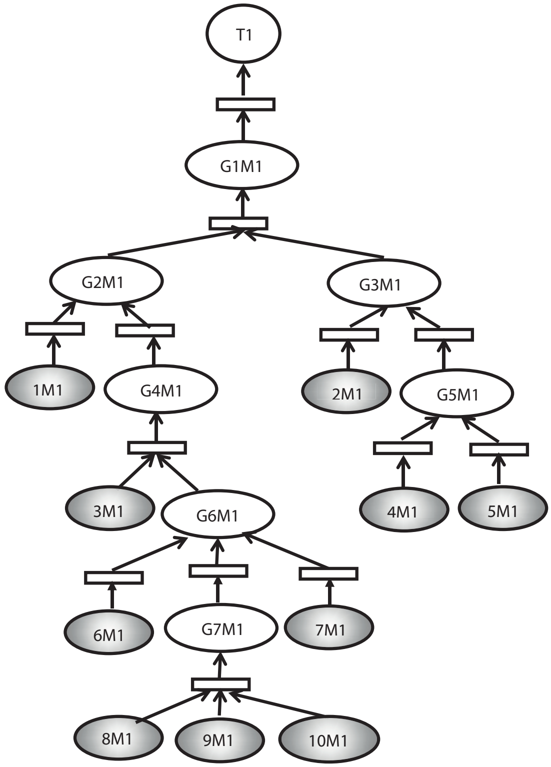
The motors are in series wiring to the control system (see Figure 4 for its fault tree and Figure 6 for its Petri Net). All three motors are the same (see Figure 5 for its fault tree and Figure 7 for its Petri Net). We refer readers to Vesely et al. (1981), p. 116 for the technical details of the motor components.

Figure 6 Place-Transition Petri Net for "three-motor" system



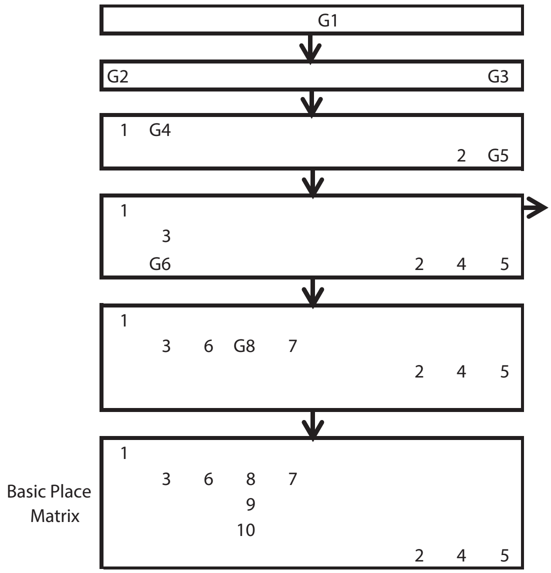
Source: Own construction by Vesely et al. (1981)

Figure 7 Place-transition Petri Net for the 1st motor in "three-motor" system



Source: Own construction by Vesely et al. (1981)

Figure 8 Basic Place Matrix of the 1st motor in "three-motor" system for finding minimal cut/path sets by method Liu and Chiou (1997)

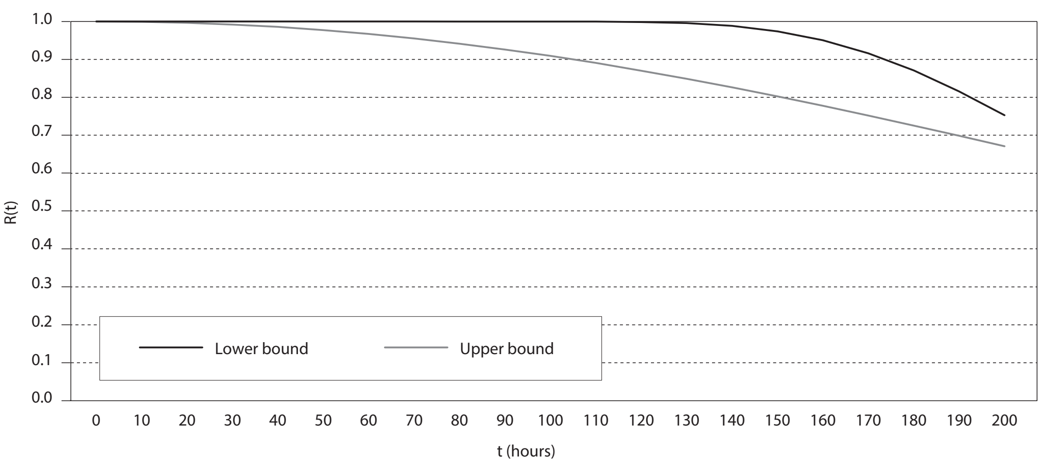


Source: Own construction

For simplicity, only a schematic description of the method of Liu and Chiou (1997) to find the Basic Place Matrix of the first motor is presented below (see Figure 8).

The first motor has 12 minimal cuts: $K_1 = \{1,2\}$, $K_2 = \{1,4\}$, $K_3 = \{1,5\}$, $K_4 = \{2,3,6\}$, $K_5 = \{3,4,6\}$, $K_6 = \{3,5,6\}$, $K_7 = \{2,3,7\}$, $K_8 = \{3,4,7\}$, $K_9 = \{3,5,7\}$, $K_{10} = \{2,3,8,9,10\}$, $K_{11} = \{3,4,8,9,10\}$, $K_{12} = \{3,5,8,9,10\}$. It has 4 minimal paths can: $C_1 = \{1,2,3,4,5\}$, $C_2 = \{1,2,4,5,6,7,8\}$, $C_3 = \{1,2,4,5,6,7,9\}$,$C4 = \{1,2,4,5,6,7,10\}$. We skip subscript of the first motor $1M$ to keep notation simple.

Figure 9 Bounds of the survival function of "three-motor" system



Source: Own construction

The system is a series system of three identical motors with $n = 30$ components. It means that the minimal cut sets of the system are a union of the minimal cut sets of the three identical motors. The minimal path sets of the system are the Cartesian product of the minimal path sets of the three identical motors. Therefore "three-motor" system has $3 \times 12 = 36$ minimal cuts and $4^3 = 64$ minimal paths. In such a complex case reliability function cannot be derived analytically.

Trivial bounds of the survival function (Formula (11)) usually fail for the systems with a high number of components. The bounds by Formula (10) often work quite well. For the system with independent, identically exponentially distributed times to failure (with MTTF = 1 000 hours, i.e. $\lambda = 1/(1\ 000)$ of the component's bounds are very tight (see Figure 9). After time t = 250 hours lower and upper bounds are almost identical.

CONCLUSION

Firstly, we reviewed standard approaches to modeling the reliability of complex systems – *fault trees* and *Petri Nets*. One-to-one relation between a fault tree and Petri Net of Liu and Chiou (1997) and their recursive top-down algorithm to find minimal cut sets and minimal paths sets were presented.

The algorithm was illustrated on a simple model example of the three-masted vessel and a more complex "three-motor" system of Vesely et al. (1981). The bounds of the reliability functions were established. Trivial bounds fail in the real complex systems. The minimal sets bounds seems to be a good approximation.

The two examples showed, that the recursive top-down matrix algorithm by Liu and Chiou (1997) is computationally demanding. It seems that the transformation of the fault tree to Petri Net to find minimal cuts and minimal paths sets more efficiently is not effective.

In future work we focus on comparison, computational complexity assessment, and computational study of minimal cut/minimal path sets algorithms for both settings.

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References

BAUSE, F. AND KRITZINGER, P. S. *Stochastic Petri Nets. An Introduction to Theory.* Braunschweig/Wiesbaden: Friedr. Wieweg & Sohn Verlagsgesellschaft GmBh, 2002. ISBN 3-525-15535-3

KUBELKA, V. *Use of random processes for calculation of risk, reliability and availability of complex industrial systems.* Master thesis, Prague: Faculty of Mathematics and Physics, Charles University, Department of Probability and Mathematical Statistics, 2016.

LIMNIOS, N. *Fault trees.* Newport Beach, CA: ISTE Ltd., 2007. ISBN 978-1-905209-30-9

LIU, T. S. AND CHIOU, S. B. The application of Petri Nets to failure analysis. *Reliability Engineering and System Safety*, 1997, 57, pp. 12–142.

MOLLOY, M. K. *On the Integration of Delay and Throughput Measures in Distributed Processing Models.* PhD thesis, University of California, 1981.

NATKIN. S. *Les Reseaux de Petri Stochastiques et leur Application a l'Evaluation des Systemes Informatiques.* PhD thesis, Paris: CNAM, 1980.

PETRI, C. A. *Kommunikation mit Automaten.* PhD thesis, Bonn: Universitaet Bonn, 1962.

ROSENTHAL, A. A computer scientist looks at reliability computations. In: *Reliability and Fault Tree Analysis*, SIAM, Philadelphia, 1975, pp. 133–152.

VESELY, W. E. et al. *Fault tree handbook.* Washington, D.C.: U.S. Nuclear Regulatory Commission, 1981. NUREG-0492